Reconstruction of Block-Sparse Signals by Using an $\ell_{2/p}$-Regularized Least-Squares Algorithm

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University of Victoria

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Outline

- Compressive Sensing

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- Signal Recovery by Using ℓ_1 or ℓ_p Minimization

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- Performance Evaluation

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- Performance Evaluation
- Conclusions

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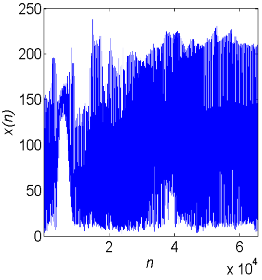
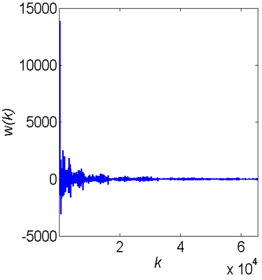
- Example: an image with near sparse wavelet coefficients:

An image of Lena



An equivalent 1-D signal



Wavelet coefficients

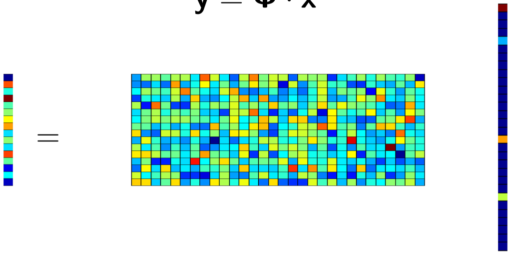
Compressive Sensing, cont'd

- Compressive sensing (CS) is a data acquisition process whereby a sparse signal $\mathbf{x}(n)$ represented by a vector $\mathbf{x}$ of length N is determined using a small number of projections represented by a matrix $\mathbf{\Phi}$ of dimension $M \times N$.

Compressive Sensing, cont'd

- Compressive sensing (CS) is a data acquisition process whereby a sparse signal $\mathbf{x}(n)$ represented by a vector $\mathbf{x}$ of length N is determined using a small number of projections represented by a matrix $\mathbf{\Phi}$ of dimension $M \times N$.
- In such a process, measurement vector $\mathbf{y}$ and signal vector $\mathbf{x}$ are interrelated by the equation

$$\mathbf{y} = \mathbf{\Phi} \cdot \mathbf{x}$$



- The inverse problem of recovering signal $\mathbf{x}$ from measurement $\mathbf{y}$ such that

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is an ill-posed problem.

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- ℓ_2 minimization often fails to yield a sparse $\mathbf{x}$, i.e., a signal obtained as

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} ||\mathbf{x}||_2 \quad \text{subject to} \quad \boldsymbol{\Phi}\mathbf{x} = \mathbf{y}$$

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Signal Recovery by Using ℓ_1 or ℓ_p Minimization

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- A sparse $\mathbf{x}$ can be recovered using ℓ_1 minimization as

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} ||\mathbf{x}||_1 \quad \text{subject to} \quad \boldsymbol{\Phi}\mathbf{x} = \mathbf{y}$$

Signal Recovery by Using ℓ_1 or ℓ_p Minimization, cont'd

- Recently, ℓ_p minimization based algorithms have been shown to recover sparse signals using fewer measurements.
- In these algorithms, the signal is recovered by using the optimization problem

$$\begin{array}{ll} \underset{\mathbf{x}}{\text{minimize}} & ||\mathbf{x}||_p^p = \sum_{i=1}^{N} |x_i|^p \\ \text{subject to} & \mathbf{\Phi x} = \mathbf{y} \end{array}$$

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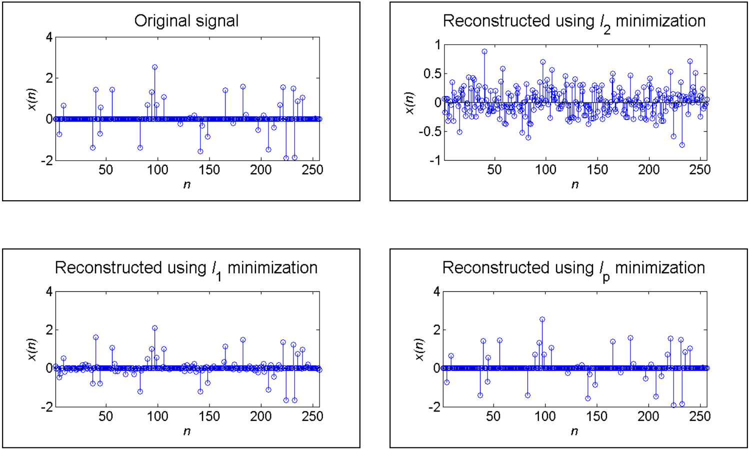
  where $p < 1$.

- Note that the objective function $||\mathbf{x}||_p^p$ in the above problem is nonconvex and nondifferentiable.
- Despite this, it has been shown in the literature that if the above problem is solved with sufficient care, improved reconstruction performance can be achieved.

- Example: $N = 256$, $K = 35$, $M = 100$.



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$$\mathbf{x} = \left[\tilde{\mathbf{x}}_1 \ \tilde{\mathbf{x}}_2 \ \cdots \ \tilde{\mathbf{x}}_{N/d}\right]^T$$

where

$$\tilde{\mathbf{x}}_i = \left[x_{(i-1)d+1} \ x_{(i-1)d+1} \ \cdots \ x_{(i-1)d+d}\right]^T$$

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- Note that the definition of K-sparse in the conventional CS is the special case of K-block sparse with $d = 1$.

Recovery of Block-Sparse Signals, cont'd

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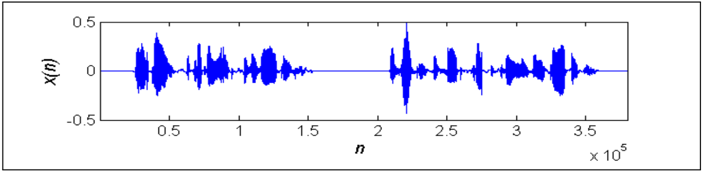

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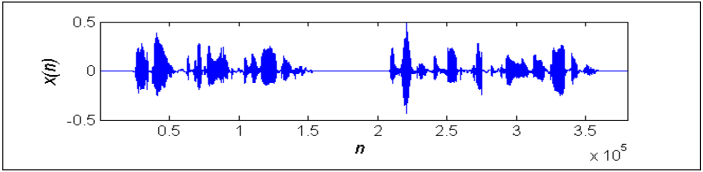
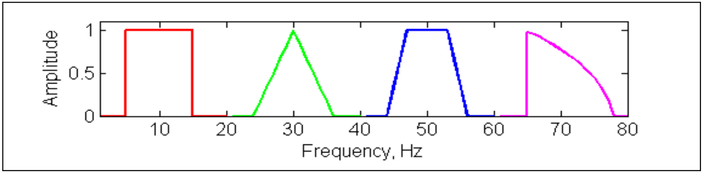
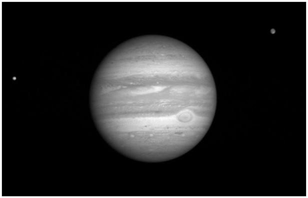


- Multiband spectrum:



- An image of Jupiter:



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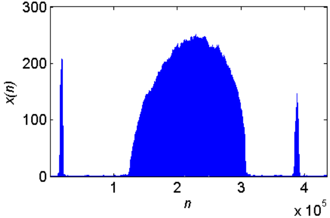

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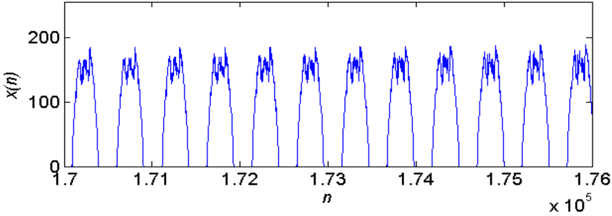

The 1-D signal from $n = 170000$ to 176000

- The block sparsity of a signal can be measured using the $\ell_{2/0}$-pseudonorm which is given by

$$||\mathbf{x}||_{2/0} = \sum_{i=1}^{N/d} \left(||\tilde{\mathbf{x}}_i||_2\right)^0$$

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- A block-sparse signal can therefore be recovered by solving the optimization problem

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- Unfortunately, this problem is nonconvex with combinatorial complexity.

- A practical method for recovering a block sparse signal is to solve the problem

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$$\left[||\mathbf{x}_1||_2 \;\; ||\mathbf{x}_2||_2 \;\; \cdots \;\; ||\mathbf{x}_{N/d}||_2 \right]^T,$$

  which essentially gives a measure of the inter-block sparsity of $\mathbf{x}$.

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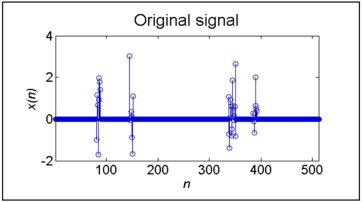
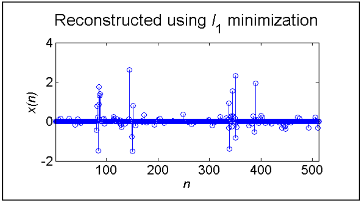
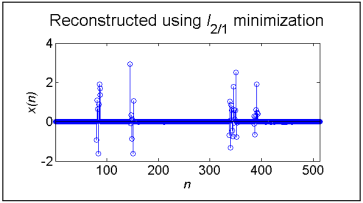
which essentially gives a measure of the inter-block sparsity of $\mathbf{x}$.
- The above problem is a convex programming problem which can be solved using a *semidefinite-programming* or a *second-order cone-programming* (SOCP) solver.

- Example: $N = 512,\ d = 8,\ K = 5,\ M = 100.$







Block-Sparse Signal Recovery by Using $\ell_{2/p}$ Minimization

- We propose reconstructing a block-sparse signal $\mathbf{x}$ from measurement $\mathbf{y}$ by solving the $\ell_{2/p}$-regularized least-squares problem

$$\underset{\mathbf{x}}{\text{minimize}} \quad F_\epsilon(\mathbf{x}) = \frac{1}{2}\,||\mathbf{\Phi x} - \mathbf{y}||_2^2 + \lambda||\mathbf{x}||_{2/p,\epsilon}^p \qquad \textbf{(P)}$$

with $p < 1$ for a small ϵ where

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- Note that

$$\lim_{\epsilon \to 0} ||\mathbf{x}||_{2/p,\epsilon}^p = ||\mathbf{x}||_{2/p}^p$$

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- Good signal reconstruction performance is expected when problem **P** on slide 14 is solved with a sufficiently small ϵ.

Block-Sparse Signal Recovery by Using $\ell_{2/p}$ Minimization, cont'd

- Good signal reconstruction performance is expected when problem **P** on slide 14 is solved with a sufficiently small ϵ.
- However, for small ϵ the objective function $F_\epsilon(\mathbf{x})$ becomes highly nonconvex and nearly nondifferentiable.

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- Good signal reconstruction performance is expected when problem **P** on slide 14 is solved with a sufficiently small ϵ.
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- The larger the ϵ, the easier the optimization of $F_\epsilon(\mathbf{x})$.

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- However, for small ϵ the objective function $F_\epsilon(\mathbf{x})$ becomes highly nonconvex and nearly nondifferentiable.
- The larger the ϵ, the easier the optimization of $F_\epsilon(\mathbf{x})$.
- Therefore, we propose to solve problem **P** on slide 14 by using the following sequential optimization procedure:
  - Choose a sufficiently large value of ϵ and solve problem **P** using Fletcher-Reeves' conjugate-gradient (CG) algorithm. Set the solution to $\mathbf{x}$.
  - Reduce the value of ϵ, use $\mathbf{x}$ as an initializer, and solve problem **P** again.
  - Repeat this procedure until problem **P** is solved for a sufficiently small value of ϵ. Output the final solution and stop.

Block-Sparse Signal Recovery by Using $\ell_{2/p}$ Minimization, cont'd

- In the kth iteration of Fletcher-Reeves' CG algorithm, iterate $\mathbf{x}_k$ is updated as

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k \mathbf{d}_k$$

where

$$\begin{aligned} \mathbf{d}_k &= -\mathbf{g}_k + \beta_{k-1}\mathbf{d}_{k-1} \\ \beta_{k-1} &= \frac{||\mathbf{g}_k||_2^2}{||\mathbf{g}_{k-1}||_2^2} \end{aligned}$$

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- Given $\mathbf{x}_k$ and $\mathbf{d}_k$, the step size α_k is obtained by solving the optimization problem

$$\underset{\alpha}{\text{minimize}} \quad f(\alpha) = F_\epsilon(\mathbf{x}_k + \alpha \mathbf{d}_k)$$

Block-Sparse Signal Recovery by Using $\ell_{2/p}$ Minimization, cont'd

- By setting the first derivative of $f(\alpha)$ to zero, we get

$$\alpha = G(\alpha)$$

where

$$G(\alpha) = -\frac{\mathbf{d}_k^T \mathbf{\Phi}^T (\mathbf{\Phi x}_k - \mathbf{y}) + \lambda \cdot p \cdot \sum_{i=1}^{N/d} \gamma_i \cdot \left(\tilde{\mathbf{x}}_{ki}^T \tilde{\mathbf{d}}_{ki}\right)}{||\mathbf{\Phi d}_k||_2^2 + \lambda \cdot p \cdot \sum_{i=1}^{N/d} \gamma_i \cdot \left(\tilde{\mathbf{d}}_{ki}^T \tilde{\mathbf{d}}_{ki}\right)}$$

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- In the above equations, $\tilde{\mathbf{x}}_{ki}$ and $\tilde{\mathbf{d}}_{ki}$ are the ith blocks of vectors $\mathbf{x}_k$ and $\mathbf{d}_k$, respectively.

Block-Sparse Signal Recovery by Using $\ell_{2/p}$ Minimization, cont'd

- Step size α_k is determined by using the recursive relation

$$\alpha_{l+1} = G(\alpha_l) \quad \text{for} \quad l = 1, 2, \ldots$$

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- According to Banach's fixed-point theorem, if $|dG(\alpha)/d\alpha| < 1$ then function $G(\alpha)$ is a contraction mapping, i.e.,

$$|G(\alpha_1) - G(\alpha_2)| \leq \eta \, |\alpha_1 - \alpha_2|$$

  with $\eta < 1$ and, as a consequence, the above recursion converges to a solution.

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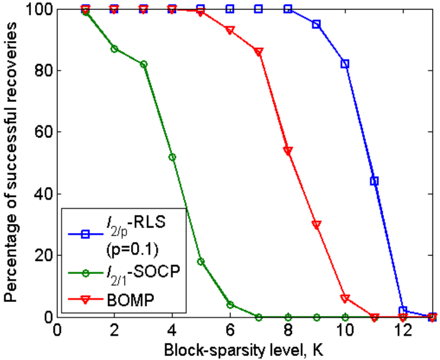
- Extensive experimental results have shown that function $G(\alpha)$ for function $f(\alpha)$ is, in practice, a contraction mapping.

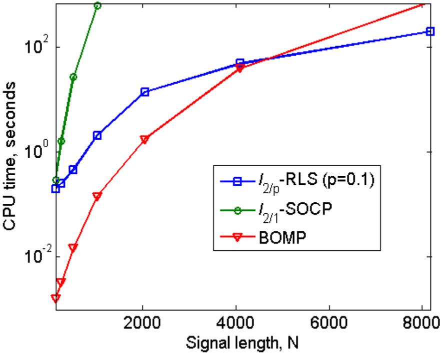
Performance Evaluation

- Number of perfectly recovered instances with $N = 512$, $M = 100$, and $d = 8$ over 100 runs.



$\ell_{2/p}$-RLS: Proposed $\ell_{2/p}$-Regularized Least-Squares
$\ell_{2/1}$-SOCP: $\ell_{2/1}$ Second-Order Cone-Programming (Eldar and Mishali, 2009)
BOMP: Block Orthogonal Matching Pursuit (Eldar et. al., 2010)

- Average CPU time with $M = N/2$, $K = M/2.5d$, and $d = 8$ over 100 runs.



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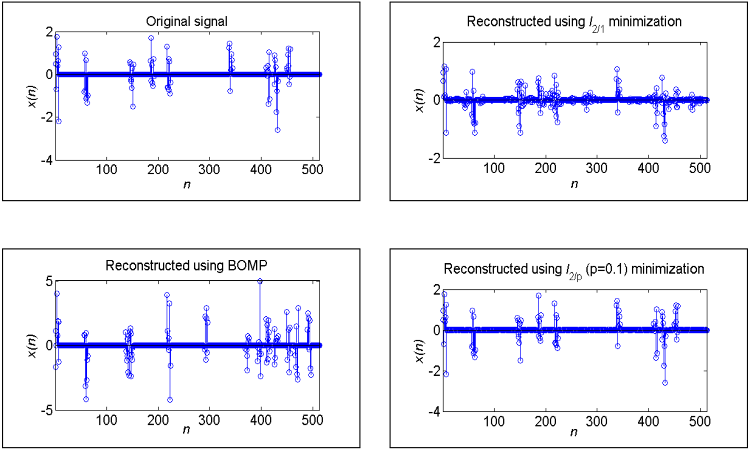
- Example: $N = 512,\ d = 8,\ K = 9,\ M = 100.$



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- ℓ_1-minimization and ℓ_p-minimization with $p < 1$ work well for the reconstruction of sparse signals.
- $\ell_{2/1}$-minimization offers improved reconstruction performance for block-sparse signals.

Conclusions

- Compressive sensing is an effective sampling technique for sparse signals.
- ℓ_1-minimization and ℓ_p-minimization with $p < 1$ work well for the reconstruction of sparse signals.
- $\ell_{2/1}$-minimization offers improved reconstruction performance for block-sparse signals.
- The proposed $\ell_{2/p}$-regularized least-squares algorithm offers improved reconstruction performance for block-sparse signals relative to the $\ell_{2/1}$-SOCP and BOMP algorithms.

Thank you for your attention.