Human-Like Biomechanics

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Human-Like Biomechanics

A Unified Mathematical Approach to Human Biomechanics and Humanoid Robotics

by

VLADIMIR G. IVANCEVIC
Defence Science and Technology Organisation, Adelaide, Australia

and

TIJANA T. IVANCEVIC
The University of Adelaide, SA, Australia

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Dedicated to Nitya, Atma and Kali

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Preface

This monographic textbook is a comprehensive introduction into modern geometric methods to be used as a unified research approach in two apparently separate and rapidly growing fields: *mathematical biomechanics* & *humanoid robotics*. The term *human–like biomechanics* is used to denote this unified modelling and control approach based on: theoretical mechanics, differential geometry and topology, nonlinear dynamics and control as well as modern path–integral methods. This approach has been realized in the form of the world–leading human–motion simulator called *Human Biodynamics Engine* (developed in the Land Operations Division, Defence Science & Technology Organisation, Australia).

This book is designed for a rigorous, one–semester course at the graduate level. The intended audience includes mechanical, control and biomedical engineers (with stronger mathematical background), mathematicians, physicists, computer scientists and mathematical biologists, as well as all researchers and technical professionals interested in modelling and control of humanoid robots and biomechanical systems.

From this *geometry–mechanics–control* modelling perspective, 'human' and 'humanoid' means the *same*. This unified approach, locally tensorial and globally functorial, enables both design of humanoid systems of immense complexity and prediction/prevention of subtle neuro–musculo–skeletal injuries. Consider, for example, human spine. Even if we ignore the highly irregular multi–vertebral geometry, a popular 'spinal column' is not columnar at all; it is a chain of 26 flexibly–coupled rigid bodies, including 25 movable spinal joints, each with three rotations (restricted to about 6–7 angular degrees in average) as well as three translations (each restricted to about 5 millimeters), which gives a total number of 150 constraint degrees of freedom (DOF). To perform a predictive analysis of the mechanism of a very common 'back pain' syndrome, in which there is no evidence at all for any tissue damage (measured by X–ray, CT, PET or functional MRI scans), and yet the patient suffers from spastic pain and drastically reduced motion capability — we need to take into account *all* 150 DOF. Dynamically speaking, we have a chain of 25 constrained

Euclidean $SE(3)$ groups acting in all movable spinal joints, and we need to develop a rigorous kinematics and dynamics, as well as a hierarchical control model for this chain — to be able to accurately predict/control all possible motions (between any initial and final body configurations) and prevent spinal injuries, and thus efficiently cope with the back pain syndrome.

Spine is just one of many examples in which current biomechanics (or biomedical engineering) tries to *predict* and *control* the behavior of highly complex physiological systems using trivial models, like a very popular 1 DOF *inverted pendulum model* for the whole human body that has more than 300 DOF driven by more than 600 muscles, or similarly popular 'Hybrid III" crash–test dummy, that has one cylinder for the spine and one for the neck.[1]

In all monograph we try to follow the path–showing words of Paul Dirac [Dir30]: "...The main object of physical science is not the provision of pictures, but is the formulation of *laws* governing phenomena and the application of these *laws* to the discovery of new phenomena..." Modern scientific way of implementing this idea is to follow the slogan of Ralph Abraham [AS92]: "...*dynamics* is *geometry* of *behavior*..." Therefore, the whole text is dominated by tensorial geometry and topology, as a set of variations to the central theme of the book – our *covariant force law*, that states:

Force 1–form–field = Mass distribution × Acceleration vector–field

This *law* is the *core* of human–like biomechanics. It is essentially mechanical, but at the same time it makes necessary three other related mathematical fields: differential geometry and topology, as well as nonlinear control theory.

[1] *Human Biodynamics Engine* (HBE) is a generalized Hamiltonian system with 264 DOF, including 132 rotational DOF (considered active) and 132 translational DOF (considered passive). Passive joint dynamics models visco–elastic properties of intervertebral discs, joint tendons and muscle ligaments as a nonlinear spring–damper system. Active joint dynamics is driven by 264 *nonlinear muscular actuators*, each with its own excitation–contraction dynamics (following traditional biomechanical models) and two–level neural–like control. The lower control level resembles spinal–reflex positive and negative force feedbacks (stretch and Golgi reflexes). The higher control level resembles cerebellum's postural stabilization and velocity control (modelled as a high–dimensional Lie–derivative controller). The HBE includes over 2000 body parameters, all derived from individual user data, using standard biomechanical tables. It models stabilizing and reaching body movements at a spot, walking and running with any speed and a generic crash simulator. The HBE incorporates a new theory of soft neuro–musculo–skeletal injuries, much more sensitive than the traditional Principal Loading Hypothesis (of tension, compression, bending and shear) for spinal and other neuro–musculo–skeletal injuries. It is based on the concept of the local Jolts and Torque–Jolts, which are the time derivatives of the total forces and torques localized in each joint at a particular time instant.

The book contains six Chapters and Appendix. The first Chapter is Introduction, giving the brief review of mathematical techniques to be used in the text. The second Chapter develops geometric basis of human–like biomechanics, while the third Chapter develops its mechanical basis, mainly from generalized Lagrangian and Hamiltonian perspective. The fourth Chapter develops topology of human–like biomechanics, while the fifth Chapter reviews related nonlinear control techniques. The sixth Chapter develops covariant biophysics of electro–muscular stimulation. The Appendix includes three parts: (i) basic formulas from tensor calculus, including the derivation of our *covariant force law*, (ii) classical muscular mechanics, and (iii) modern path integral methods, which are all used frequently in the main text. The whole book is based on authors' own research papers in human–like biomechanics.

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V. Ivancevic, Defence Science & Technology Organisation, Australia, e-mail: Vladimir.Ivancevic@dsto.defence.gov.au

T. Ivancevic, School of Mathematics, The University of Adelaide, e-mail: Tijana.Ivancevic@adelaide.edu.au

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0.1 Glossary of Frequently Used Symbols

General

– ‘iff’ means ‘if and only if’;
– ‘r.h.s’ means ‘right hand side’; ‘l.h.s’ means ‘left hand side’;
– *Einstein’s summation convention over repeated indices* (not necessarily one up and one down) *is assumed in the whole text,* unless explicitly stated otherwise.

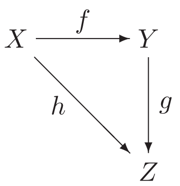
Sets

$\mathbb{N}$ – natural numbers;
$\mathbb{Z}$ – integers;
$\mathbb{R}$ – real numbers;
$\mathbb{C}$ – complex numbers;
$\mathbb{H}$ – quaternions;
$\mathbb{K}$ – number field of real numbers, complex numbers, or quaternions.

Maps

$f : A \to B$ – a function, (or map) between sets $A \equiv \mathrm{Dom}\, f$ and $B \equiv \mathrm{Cod}\, f$;

$$\begin{aligned} \mathrm{Ker}\, f &= f^{-1}(e_B) - \text{a kernel of } f; \\ \mathrm{Im}\, f &= f(A) - \text{an image of } f; \\ \mathrm{Coker}\, f &= \mathrm{Cod}\, f / \mathrm{Im}\, f - \text{a cokernel of } f; \\ \mathrm{Coim}\, f &= \mathrm{Dom}\, f / \mathrm{Ker}\, f - \text{a coimage of } f; \end{aligned}$$



– a commutative diagram, requiring $h = g \circ f$.

Derivatives

$C^k(A, B)$ – set of $k-$times differentiable functions between sets A to B;
$C^\infty(A, B)$ – set of *smooth* functions between sets A to B;
$C^0(A, B)$ – set of *continuous* functions between sets A to B;
$f'(x) = \frac{df(x)}{dx}$ – derivative of f with respect to x;
$\dot{x}$ – total time derivative of x;
$\partial_t \equiv \frac{\partial}{\partial t}$ – partial time derivative;
$\partial_{x^i} \equiv \partial_i \equiv \frac{\partial}{\partial x^i}$ – partial coordinate derivative;

$\dot{f} = \partial_t f + \partial_{x^i} f\, \dot{x}^i$ – total time derivative of the scalar field $f = f(t, x^i)$;
$u_t \equiv \partial_t u,\ u_x \equiv \partial_x u,\ u_{xx} \equiv \partial_{x^2} u$ – only in partial differential equations;
$L_{x^i} \equiv \partial_{x^i} L,\ L_{\dot{x}^i} \equiv \partial_{\dot{x}^i} L$ – coordinate and velocity partial derivatives of the Lagrangian function;
d – exterior derivative;
d^n – coboundary operator;
∂_n – boundary operator;
$\nabla = \nabla(g)$ – Levi–Civita affine connection on a smooth manifold M with Riemannian metric tensor $g = g_{ij}$;
Γ^i_{jk} – Christoffel symbols of the connection ∇;
$\nabla_X T$ – covariant derivative of the tensor–field T with respect to the vector–field X, defined by means of Γ^i_{jk};
$T_{;x^i}$ – covariant derivative of the tensor–field T with respect to the coordinate basis $\{x^i\}$;
$\dot{T} \equiv \frac{DT}{dt}$ – absolute (intrinsic, or Bianchi) derivative of the tensor–field T upon the parameter t; e.g., acceleration vector is the *absolute time derivative* of the velocity vector, $a^i = \dot{\bar{v}}^i \equiv \frac{Dv^i}{dt}$; note that in general, $a^i \neq \dot{v}^i$ – this is crucial for *proper definition of Newtonian force* (see Appendix);
$\mathcal{L}_X T$ – Lie derivative of the tensor–field T in direction of the vector–field X;
$[X, Y]$ – Lie bracket (commutator) of two vector–fields X and Y;
$[F, G]$ – Poisson bracket of two functions F and G;
$\{F, G\}$ – Lie–Poisson bracket of two functions F and G.

Manifolds and Fibre Bundles

M – manifold, usually the biomechanical configuration manifold;
TM – tangent bundle of the manifold M;
$\pi_M : TM \to M$ – natural projection;
T^*M – cotangent bundle of the manifold M;
(E, π, M) – a vector bundle with total space E, base M and projection π;
(E, p, M, F) – a fibre bundle with total space E, base M, projection p and standard fibre F;
$J^k(M, N)$ – bundle of k–jets of smooth functions between manifolds M, N.

Groups

G – usually a general Lie group;
$GL(n)$ – general linear group with real coefficients in dimension n;
$SO(n)$ – group of rotations in dimension n;
T^n – toral (Abelian) group in dimension n;
$Sp(n)$ – symplectic group in dimension n;
$T(n)$ – group of translations in dimension n;
$SE(n)$ – Euclidean group in dimension n;
$H_n(M) = \mathrm{Ker}\,\partial_n / \mathrm{Im}\,\partial_{n-1}$ – nth homology group of the manifold M;
$H^n(M) = \mathrm{Ker}\, d^n / \mathrm{Im}\, d^{n+1}$ – nth cohomology group of the manifold M.

Other Spaces and Operators

$C^k(M)$ – space of $k-$differentiable functions on the manifold M;
$\Omega^k(M)$ – space of $k-$forms on the manifold M;
$\mathfrak{g}$ – Lie algebra of a Lie group G, i.e., the tangent space of G at its identity element;
$Ad(g)$ – adjoint endomorphism; recall that *adjoint representation* of a Lie group G is the linearized version of the action of G on itself by conjugation, i.e., for each $g \in G$, the inner automorphism $x \mapsto gxg^{-1}$ gives a linear transformation $Ad(g) : \mathfrak{g} \to \mathfrak{g}$, from the Lie algebra $\mathfrak{g}$ of G to itself;
nD space (group, system) means $n-$dimensional space (group, system), for $n \in \mathbb{N}$;
$\lrcorner$ – interior product, or contraction, of a vector–field and a 1−form;
$\triangleright$ – semidirect (noncommutative) product; e.g., $SE(3) = SO(3) \triangleright \mathbb{R}^3$;
$\displaystyle\sum\!\!\!\!\!\!\int$ – Feynman path integral symbol, denoting integration over continuous spectrum of smooth paths and summation over discrete spectrum of Markov chains; e.g., $\sum\!\!\!\!\!\!\int \mathcal{D}[x]\, e^{iS[x]}$ denotes the path integral (i.e., sum–over–histories) over all possible paths $x^i = x^i(t)$ defined by the Hamilton action, $S[x] = \frac{1}{2}\int_{t_0}^{t_1} g_{ij}\, \dot{x}^i \dot{x}^j\, dt$, while $\sum\!\!\!\!\!\!\int \mathcal{D}[\Phi]\, e^{iS[\Phi]}$ denotes the path integral over all possible fields $\Phi^i = \Phi^i(x)$ defined by some field action $S[\Phi]$.

Categories

$\mathcal{S}$ – all sets as objects and all functions between them as morphisms;
$\mathcal{PS}$ – all pointed sets as objects and all functions between them preserving base point as morphisms;
$\mathcal{V}$ – all vector spaces as objects and all linear maps between them as morphisms;
$\mathcal{B}$ – Banach spaces over $\mathbb{R}$ as objects and bounded linear maps between them as morphisms;
$\mathcal{G}$ – all groups as objects, all homomorphisms between them as morphisms;
$\mathcal{A}$ – Abelian groups as objects, homomorphisms between them as morphisms;
$\mathcal{AL}$ – all algebras (over a given number field $\mathbb{K}$) as objects, all their homomorphisms between them as morphisms;
$\mathcal{T}$ – all topological spaces as objects, all continuous functions between them as morphisms;
$\mathcal{PT}$ – pointed topological spaces as objects, continuous functions between them preserving base point as morphisms;
$\mathcal{TG}$ – all topological groups as objects, their continuous homomorphisms as morphisms;
$\mathcal{M}$ – all smooth manifolds as objects, all smooth maps between them as morphisms;
$\mathcal{M}_n$ – nD manifolds as objects, their local diffeomorphisms as morphisms;

$\mathcal{LG}$ – all Lie groups as objects, all smooth homomorphisms between them as morphisms;
$\mathcal{LAL}$ – all Lie algebras (over a given field $\mathbb{K}$) as objects, all smooth homomorphisms between them as morphisms;
$\mathcal{TB}$ – all tangent bundles as objects, all smooth tangent maps between them as morphisms;
$\mathcal{T}^*\mathcal{B}$ – all cotangent bundles as objects, all smooth cotangent maps between them as morphisms;
$\mathcal{VB}$ – all smooth vector bundles as objects, all smooth homomorphisms between them as morphisms;
$\mathcal{FB}$ – all smooth fibre bundles as objects, all smooth homomorphisms between them as morphisms;
Symplec – all symplectic manifolds (i.e., physical phase–spaces), all symplectic maps (i.e., canonical transformations) between them as morphisms;
Hilbert – all Hilbert spaces and all unitary operators as morphisms.