

Symbol Table

Numbers in parentheses refer to chapters where the symbol is used as indicated.

b, \mathbf{b}	regression coefficient function(s) estimates
c, \mathbf{c}	basis expansion coefficient(s)
d	discriminant of a second-order system; eigenvalue for a first-order system
g	forcing function
h, \mathbf{h}	warping function(s)
i, j, k, ℓ	indices
I, J, K, m, n, N	dimensions of vectors or matrices
s, \mathbf{s}	value(s) on the domain of a function
t, \mathbf{t}	value(s) on the domain of a function
w, W	log derivative of monotone or warping function
x, \mathbf{x}	functional data observation(s)
y, \mathbf{y}	functional data observation
z, \mathbf{z}	covariate scalar or functional data observation(s)
α	rate constant in an exponent (3); an intercept (9); forcing function (11)
$\beta, \boldsymbol{\beta}$	regression coefficient function (scalar or vector)
γ	rate constant in an exponent
δ	time shift (8, 10); statistical technique (10)
ε	error or residual
θ	latent ability value (1); parameter (11)
λ	smoothing parameter value
μ	mean function (9,10,1); eigenvalue (7)
ν	eigenvalue (7)
ξ	weight function (6); exponential basis function (11)
η	weight function (7)
π	trigonometric constant
ρ	correlation (4, 6); probe functional (6, 7)
σ, Σ	standard deviation, variance, covariance

ϕ, ϕ	basis function
ψ, ζ	basis function
Θ	matrix of basis function values
Φ	matrix of basis function values
Ψ	matrix of basis function values

References

- Adler, D. and D. Murdoch (2009). *rgl: 3D visualization device system (OpenGL)*. R package version 0.82. <http://rgl.neoscientists.org>.
- Bellman, R. and R. S. Roth (1971). The use of splines with unknown end points in the identification of systems. *Journal of Mathematical Analysis and Applications* 34, 26–33.
- Bookstein, F. L. (1991). *Morphometric Tools for Landmark Data: Geometry and Biology*. Cambridge: Cambridge University Press.
- Borrelli, R. L. and C. S. Coleman (2004). *Differential Equations: A Modelling Perspective*. New York: Wiley.
- Brumback, B. A. and J. A. Rice (1998). Smoothing spline models for the analysis of nested and crossed samples of curves. *Journal of the American Statistical Association* 93, 961–994.
- Brunel, N. (2008). Parameter estimation of ODEs via nonparametric estimators. *Electronic Journal of Statistics* 2, 1242–1267.
- Cardot, H., F. Ferraty, A. Mas, and P. Sarda (2003b). Testing hypotheses in the functional linear model. *Scandinavian Journal of Statistics* 30, 241–255.
- Cardot, H., F. Ferraty, and P. Sarda (1999). Functional linear model. *Statistics and Probability Letters* 45, 11–22.
- Cardot, H., F. Ferraty, and P. Sarda (2003a). Spline estimators for the functional linear model. *Statistica Sinica* 13, 571–591.
- Cardot, H., A. Goia, and P. Sarda (2004). Testing for no effect in functional linear models, some computational approaches. *Communications in Statistics—Simulation and Computation* 33, 179–199.
- Chambers, J. M. (2008). *Software for Data Analysis*. New York: Springer.
- Chambers, J. M. and T. J. Hastie (1991). *Statistical Models in S*. New York: Chapman and Hall.
- Chaudhuri, P. and J. S. Marron (1999). SiZer for exploration of structures in curves. *Journal of the American Statistical Association* 94, 807–823.
- Chen, J. and H. Wu (2008). Estimation of time-varying parameters in deterministic dynamic models. *Statistica Sinica* 18, 987–1006.

- Chiou, J. M. and H. G. Müller (2009). Modeling hazard rates as functional data for the analysis of cohort lifetables and mortality forecasting. *Journal of the American Statistical Association*, in press.
- Craven, P. and G. Wahba (1979). Smoothing noisy data with spline functions: Estimating the correct degree of smoothing by the method of generalized cross-validation. *Numerische Mathematik* 31, 377–403.
- Cuevas, A., M. Febrero, and R. Fraiman (2002). Linear functional regression: The case of fixed design and functional response. *Canadian Journal of Statistics* 30, 285–300.
- de Boor, C. (2001). *A Practical Guide to Splines, Revised Edition*. New York: Springer.
- Delsol, L., F. Ferraty, and P. Vieu (2008). Structural test in regression on functional variables. to appear.
- Escabias, M., A. Aguilera, and M. J. Valderrama (2004). Principal component estimation of functional logistic regression: Discussion of two different approaches. *Nonparametric Statistics* 16, 365–384.
- Eubank, R. L. (1999). *Spline Smoothing and Nonparametric Regression, Second Edition*. New York: Marcel Dekker.
- Fan, J. and I. Gijbels (1996). *Local Polynomial Modelling and Its Applications*. London: Chapman and Hall.
- Ferraty, F. and P. Vieu (2001). The functional nonparametric model and its applications to spectrometric data. *Computational Statistics* 17, 545–564.
- Fisher, N. I., T. L. Lewis, and B. J. J. Embleton (1987). *Statistical Analysis of Spherical Data*. Cambridge: Cambridge University Press.
- Gasser, T. and A. Kneip (1995). Searching for structure in curve samples. *Journal of the American Statistical Association* 90, 1179–1188.
- Gervini, D. and T. Gasser (2004). Self-modeling warping functions. *Journal of the Royal Statistical Society, Series B* 66, 959–971.
- Hastie, T. and R. Tibshirani (1993). Varying-coefficient models. *Journal of the Royal Statistical Society, Series B* 55, 757–796.
- Hiebeler (2009). Matlab / R reference. <http://www.math.umaine.edu/faculty/hiebeler/comp/matlabR.pdf>, accessed 2009.02.06.
- James, G., J. Wang, and J. Zhu (2009). Functional linear regression that's interpretable. *Annals of Statistics*, in press.
- James, G. M. (2002). Generalized linear models with functional predictors. *Journal of the Royal Statistical Society, Series B* 64, 411–432.
- James, G. M. and T. Hastie (2001). Functional linear discriminant analysis for irregularly sampled curves. *Journal of the Royal Statistical Society, Series B* 63, 533–550.
- James, G. M., T. J. Hastie, and C. A. Sugar (2000). Principal component models for sparse functional data. *Biometrika* 87, 587–602.
- James, G. M. and C. A. Sugar (2003). Clustering sparsely sampled functional data. *Journal of the American Statistical Association* 98, 397–408.
- Jolliffe, I. T. (2002). *Principal Components Analysis, Second Edition*. New York: Springer.

- Kneip, A. and T. Gasser (1992). Statistical tools to analyze data representing a sample of curves. *Annals of Statistics* 20, 1266–1305.
- Kneip, A. and J. O. Ramsay (2008). Combining registration and fitting for functional models. *Journal of the American Statistical Association* 20, 1266–1305.
- Kuznetsov, Y. A. (2004). *Elements of Applied Bifurcation Theory*. New York: Springer.
- Liu, X. and H. G. Müller (2004). Functional convex averaging and synchronization for time-warped random curves. *Journal of the American Statistical Association* 99, 687–699.
- Malfait, N. and J. O. Ramsay (2003). The historical functional linear model. *Canadian Journal of Statistics* 31, 115–128.
- Müller, H.-G. and U. Stadtmüller (2005). Generalized functional linear models. *Annals of Statistics* 33, 774–805.
- Olshen, R. A., E. N. Biden, M. P. Wyatt, and D. H. Sutherland (1989). Gait analysis and the bootstrap. *Annals of Statistics* 17, 1419–1440.
- Pascual, M. and S. P. Ellner (2000). Linking ecological patterns to environmental forcing via nonlinear time series models. *Ecology* 81(10), 2767–2780.
- Ramsay, J. O., R. D. Bock, and T. Gasser (1995a). Comparison of height acceleration curves in the Fels, Zurich, and Berkeley growth data. *Annals of Human Biology* 22, 413–426.
- Ramsay, J. O., G. Hooker, D. Campbell, and J. Cao (2007). Parameter estimation in differential equations: A generalized smoothing approach. *Journal of the Royal Statistical Society, Series B* 16, 741–796.
- Ramsay, J. O. and B. W. Silverman (2005). *Functional Data Analysis, Second Edition*. New York: Springer.
- Ramsay, J. O., X. Wang, and R. Flanagan (1995b). A functional data analysis of the pinch force of human fingers. *Applied Statistics* 44, 17–30.
- Rossi, N., X. Wang, and J. O. Ramsay (2002). Nonparametric item response function estimates with the em algorithm. *Journal of the Behavioral and Educational Sciences* 27, 291–317.
- Rupert, D., M. P. Wand, and R. J. Carroll (2003). *Semiparametric Regression*. Cambridge: Cambridge University Press.
- Sakoe, H. and S. Chiba (1978). Dynamic programming algorithm optimization for spoken word recognition. *IEEE Transactions, ASSP-26* 1, 43–49.
- Sarkar, D. (2008). *lattice: Lattice Graphics*. R package version 0.17-13.
- Schumaker, L. (1981). *Spline Functions: Basic Theory*. New York: Wiley.
- Silverman, B. W. (1986). *Density Estimation for Statistics and Data Analysis*. London: Chapman and Hall.
- Simonoff, J. S. (1996). *Smoothing Methods in Statistics*. New York: Springer.
- Tuddenham, R. D. and M. M. Snyder (1954). Physical growth of California boys and girls from birth to eighteen years. *University of California Publications in Child Development* 1, 183–364.
- Varah, J. M. (1982). A spline least squares method for numerical parameter estimation in differential equations. *SIAM Journal on Scientific Computing* 3, 28–46.

- Yao, F., H.-G. Müller, and J.-L. Wang (2005). Functional data analysis for longitudinal data. *Annals of Statistics* 33, 2873–2903.
- Zwiefelhofer, D., J. H. Reynolds, and M. Keim (2008). Population trends and annual density estimates for select wintering seabird species on Kodiak Island, Alaska. Technical report, U.S. Fish and Wildlife Service, Kodiak National Wildlife Refuge. Technical Report, no. 08-00x.

Index

- ., 22
- <-, 22
- =, 22
- [] , 22, 23
- \$, 22
- \$fd suffix, 23
- alignment, *see* registration, *see* registration
- amplitude variation, 16, 118, 119, 125
- ANOVA
 - amplitude and phase, 125
 - functional regression, 147
- Applied Psychology Unit, 14
- argument names, 23
- argument passing, 23
- arithmetic, 48
- assessing fit, 77
- assignment operator, 22
- B-spline, 35, 37, 44, 50, 52, 57, 152
- baby's tibia data, 72
- bases, 45
- basis
 - B-spline, *see* B-spline
 - constant, *see* constant basis
 - exponential, *see* exponential basis
 - Fourier, *see* Fourier basis
 - monomial, *see* monomial basis
 - polygonal, *see* polygonal basis
 - power, *see* power basis
 - smooth.basis, 80
- basis, 42
- basis function coefficients, 30
- basis function expansion, 30
- basis function systems, 29
- basisfd, 42
- basisfd object, 29
- Berkeley Growth Study, 1, 67, 73, 91, 119, 166, 179, *see* growth data
- bifd, 57
- biomechanics, 5, 13
- bivariate functional data object, 56
- boundary instability with splines, 38
- break points, 33
- bucket, 180
- c (), 22
- c2rMap, 93
- Cambridge, 14
- Canadian Weather data, 103
- Canadian weather data, 10, 13, 17, 39, 46, 47, 52, 59, 67, 77, 83, 85, 94, 95, 99, 110, 132, 134, 135, 139, 145, 147, 168
- canonical correlation analysis, 17, 88, 99, 110
 - cca . fd, 111, 114
- CCA, *see* canonical correlation analysis
- cca . fd, 111, 114
- Chinese script, *see* handwriting
- class, 24
- climate region, 147
- coefficients, 45
- compact support, 35
- concurrent linear model, *see* functional regression
- concurvity, 155
- conditional covariance matrix, 87
- confidence intervals, 92, 95
 - derivatives, 92
 - functional regression, 140
- confidence intervals for concurrent model, *see* functional regression 157
- confidence limits for probe values, 95
- confidence regions, 83
- constant basis, 30, 39, 134, 137

constant basis function, 30
 constrained smooth, 70
 constructor functions, 42
 container components, 41
 continuous registration, 122
 contour plot, 85
 correlated residuals, 78
 covariance function, 84
 create, 30
`create.fourier.basis`, 32
 cross-covariance function, 85

 data display, 14
 data interpolation, 12
 data registration, 117, *see* registration
 data representation, 12
 degree of a spline, 33
 degrees of freedom, 65
 density estimation, 9, 74
`density.fd`, 74, 75
 Depression, 3
`deriv.fd`, 56
 derivative, 13
 derivatives, *see* principal differential analysis,
 see also smoothing
 use in FDA, 18
 descriptive statistics, 16, 83
 diet effect, 150
 differential equation, 11, 64, 136, 179
 differential operator, *see* linear differential
 operator
 discriminant, 181
 division, 49
 dynamics, 179

 Edmonton, 10
`eigen.pda`, 192
 eigenfunction, 99
 empirical orthogonal functions, 40, 101
 energy, 16, 88
`eval.bifd`, 85
 exponential basis, 40
 exponentiation, 49

`fd`, 57
`fd` object, *see* functional data object
 “fda” script, *see* handwriting
`fdevaluation`, 49
`fdPar` class, 78
 feature alignment, *see* registration: landmark
 fit, 77
 fixed point, 183
 forcing function, 11
 Fourier basis, 32

 Fourier basis functions, 30
 Fourier series, 13
`Fperm.fd`, 145
`fRegress`, *see* functional regression, 149, 169
`fRegress.CV`, 173
`fRegress.stderr`, 173
 functional basis object, 31
 methods, 40
 functional contrast, 88
 functional data, 1, 39
 class, 45
 `fd`, 45
 functional data object, 45
 bivariate, 56
 labels, 46
 methods, 48
 functional *F*-test, 168
 functional linear model, 17, *see* functional
 regression
 functional parameter, 9
 functional parameter object, 39, 66, 133, 134, 137, 148, 185, 188
`fdPar` class, 78
 functional principal components, *see* principal
 components analysis
 functional probe, 83, 87, 100
 functional regression, 17, *see* principal
 differential analysis, *see* regression
 analysis
`ANOVA`, 147
 bivariate regression coefficient function, 162, 165
 concurrent linear model, 154
 confidence intervals, 140
 confidence intervals for concurrent model, 157
`fRegress`, 169
`fRegress.CV`, 173
`fRegress.stderr`, 173
 functional response, 147
 integral, 162, 163
`linmod`, 174
`plotbeta`, 174
 principal components, 141
 roughness penalty, 135, 138, 153
 scalar response, 131
 set up, 132
 statistical tests, 143
`y2cMap`, 141
 functional *t*-Test, 166
 functions, 23, 45

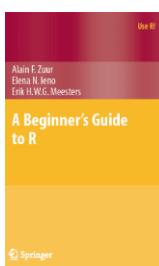
 gait data, 5, 12, 14, 39, 47, 99, 158

- gait data: model for knee angle, 158
generalized cross-validation, 66
generic functions, 25
goodness of fit, 77
goods index, 1, 3, 88
growth data, 1, 13, 15, 38, 47, 59–62, 66, 67, 87, 88, 91, 104, 117, 119, 122, 166, 179, 190, *see* Berkeley Growth Study
- handwriting, 39
 Chinese, 7, 126, 162
 “fda” script, 7, 108, 187
- harmonic acceleration, 12, 55, 136
harmonic acceleration operator, 55, *see* linear harmonic acceleration
- harmonic process, 88
harmonics, 103
“Hat” matrix, 65
hazard rate, 163
hip angle, 5, 158
historical linear model, 163
<http://www.functionaldata.org>,
 see www.functionaldata.org
- hydrolics, 180
- I-splines, 35
index, 1, 3, 88
inner product, 93
inner product function, 93
inprod, 93
int2Lfd, 55
interchild variability, 16
interpolation, 12
- kinetic energy, 89
knee angle, 5, 158
knot spacing, 37
knots, 34
Kodiak Island, 149
Kronecker product, 158
- labels for functional data objects, 46
landmark, 118, 121, 123, 190
landmarkreg, 127
lattice package, 85
leak, 180
Lfd, 57, *see* linear differential operator
line continuation, 22
line termination, 22
linear differential equation, 194
linear differential operator, 11, 18, 55, 65, 68, 94, 140, 185
Lfd, 55
linear differential operators, 55
- linear harmonic acceleration, 55, 56, 63, 64, 139, *see* harmonic acceleration operator
- linear mapping, 93
linear model, *see* functional linear model
- linmod, 174
lip data, 185, 187
list, 24
list object, 23, 47
log hazard rate, 163
logical variables, 22
- M-spline, 35
manufacturing index, 1, 3, 88
Matlab and R syntax, 21
Matlab syntax, 21
mean, 49
mean.fd, 84
mental test, 9
methods, 25, 40, 48
midpubertal age, 15
midspurt, 2
mollusk, 150
monomial basis, 30, 39, 40
monomial basis functions, 30
monotone smooth, 71
Montreal, 10, 52
mortality, 163
Motion Analysis Laboratory, 5
multicollinearity, 155
multivariate function, 45
multivariate functional data, 5, 185, 187
- nbasis, 31
neurophysiology, 13
Newton, 179
nondurable goods index, 1, 3–5, 88–90
nonfunctional data, 9
nonurable goods cycle, 90
normal equations, 156
normalizing constant, 74
number of spline basis function rule, 35
numerical precision, 38
- object, 24
object-oriented programming, 24
oil refinery data, 4, 6, 34
order of a spline, 33
order of spline rule, 36
orthonormal, 101, 102
- PCA, *see* principal components analysis
pca.fd, 103, 108
pca.fd function, 113
PDA, *see* principal differential analysis

- pda.fd, 191
- pda.overlay, 193
- penalized negative log likelihood, 74
- penalized sum of squares, *see* roughness penalty
- period, 21
- permutation tests, 165
- perspective plot, 85
- PGS, *see* pubertal growth spurt
- phase variation, 16, 118, 119, 125
- phase-plane plot, 15, 83, 88, 91, 159, 160
- pinch force data, 13
- pinchforce, 13
- plotbeta, 174
- pointwise confidence intervals, 92
- polygonal basis, 40
- polynomial functions, 30
- polynomial regression, 39
- positive smooth, 70
- potential energy, 89
- power basis, 40
- precipitation data, 12, 67, 71, 85, 94, 95, 99, 103, 110, 120, 132
- predict, 50
- pressure, 180
- Prince Rupert, 10, 95
- principal component scores, 102
- principal components
 - functional regression, 141
- principal components analysis, 3, 16, 17, 40, 88, 99, 133, 141
- pca.fd, 102, 113
- principal differential analysis, 179
 - eigen.pda, 192
 - lip data, 185
 - pda.fd, 185, 187, 191
 - pda.overlay, 193
 - registration, 190
- probe score, 111
- probe weight, 110
- psychometrics, 9
- pubertal growth spurt, 2, 92, 117, 118, 120–122, 190
- R syntax, 21
- rangeval, 31
- refinery, 4, 34
- region effect, 17, 147, 168
- register.fd, 128
- register.newfd, 193
- registration, 3, 13, 14, 117
 - continuous, 122
 - landmark, 118, 121
 - landmarkreg, 121, 127
- principal differential analysis, 190
- register.fd, 122, 128
- register.newfd, 193
- regression analysis, 51, *see* functional regression, *see* smoothing
- regression splines, 60
- regularization, 17
- residual covariance matrix, 87
- Resolute, 10
- rgl package, 85
- rotation matrix, 102
- roughness penalties, 62
- roughness penalty, 12
 - functional regression, 135, 138
- roughness penalty matrix, 64
- rounding errors, 38
- scree plot, 102
- seabird data, 149
- seasonal variation, 88
- semicolon, 22
- Shelikof Strait, 149
- shellfish, 150
- singleton index, 24
- smooth
 - constrained, 70
 - monotone, 71
 - positive, 70
- smoothing, 12, 55, 59
 - density, 74
 - functional parameter object, 66
 - matrix, 65
 - parameter, 66
 - regression splines, 59
 - roughness penalty, *see* roughness penalty
 - smooth.basis, 80
- spline basis, 33
- spline basis functions, 30
- spline function, 36
- splines
 - regression, 59
- stability, 183
- statistical tests, *see* functional regression: statistical tests
- “statistics” script, *see* handwriting: Chinese
- std.fd, 84
- struct, 24
- struct array, 22
- subsec:datadisplay, 14
- subsec:rangeoft, 38
- sum, 49
- sum of B-splines, 37
- support of a spline, 35, 37
- surface plot, 85

- Swedish mortality data, 163
syntax, 21
- temperature, 10, 32, 45, 63, 87, 94, 99, 110,
 120, 132, 147, 168
- test data, 9
- tests
- F*-test, 168
 - Fperm.fd*, 175
 - permutation, 165
 - t*-test, 166
 - tperm.fd*, 176
- tibia, 72
- time warping, *see* registration
- total curvature, 63
- transect, 149
- tray 47, 4
- truncated power basis, 35
- Uganik, 149
- underscore, 21
- US nondurable goods manufacturing index, 1,
 3, 88
- Uyak, 149
- var.fd*, 84
- variance-covariance surface, 84
- VARIMAX, 102
- varmx.pca.fd*, 104, 108
- vec2Lfd*, 56
- Vietnam War, 3
- walking, *see* gait data
- web site, 19
- weight function, 87
- World War II, 3
- www.functionaldata.org, 19
- y2cMap*, 93, 141, *see* functional regression
- y2rMap*, 93

A Beginner's Guide to R



Alain F. Zuur, Elena N. Ieno, Erik H.W.G. Meesters, and Den Burg

The text covers how to download and install R, import and manage data, elementary plotting, an introduction to functions, advanced plotting, and common beginner mistakes.

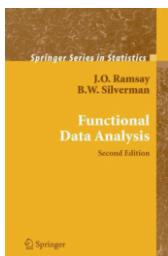
Content: Introduction.- Getting data into R.- Accessing variables and managing subsets of data.- Simple commands.- An introduction to basic plotting tools.- Loops and functions.- Graphing tools.- An introduction to lattice package.- Common R mistakes.

2009. Approx. 215 p. Softcover (Use R)

ISBN: 978-0-387-93836-3

Functional Data Analysis

J. Ramsay
B. W. Silverman



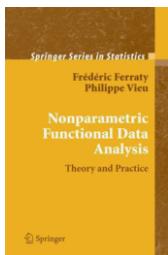
Content: Introduction .- Tools for Exploring Functional Data .- From Functional Data to Smooth Functions .- Smoothing Functional Data by Least Squares .- Smoothing Functional Data with a Roughness Penalty .- Constrained Functions .- The Registration and Display of Functional Data .- Principal Components Analysis for Functional Data .- Regularized Principal Components Analysis .- Principal Components Analysis of Mixed Data .- Canonical Correlation and Discriminant Analysis .- Functional Linear Models .- Modelling Functional Responses with Multivariate Covariats .- Functional Responses, Functional Covariates and the Concurrent Model .- Functional Linear Models for Scalar Responses .- Functional Linear Models for Functional Responses .- Derivatives and Functional Linear Models .- Differential Equations and Operators .- Principal Differential Analysis .- Green's Functions and Reproducing Kernels .- More General Roughness Penalties .- Some Perspectives on FDA.

2005. 2nd ed. XX, 430 p. 151 illus. Hardcover (Springer Series in Statistics)

ISBN: 978-0-387-40080-8

Nonparametric Functional Data Analysis Theory and Practice

Frédéric Ferraty
Philippe Vieu



Content: Introduction to functional nonparametric statistics.- Some functional datasets and associated statistical problematics.- What is a well adapted space for functional data?- Local weighting of functional variables.- Functional nonparametric prediction methodologies.- Some selected asymptotics.- Computational issues.- Nonparametric supervised classification for functional data.- Nonparametric unsupervised classification for functional data.- Mixing, nonparametric and functional statistics.- Some selected asymptotics.- Application to continuous time processes prediction.- Small ball probabilities, semi-metric spaces and nonparametric statistics.- Conclusion and perspectives.

2006. XX, 268 p. 29 illus. Hardcover (Springer Series in Statistics)

ISBN: 978-0-387-30369-7

Easy Ways to Order ►

Call: Toll-Free 1-800-SPRINGER • E-mail: orders-ny@springer.com • Write:
Springer, Dept. S8113, PO Box 2485, Secaucus, NJ 07096-2485 • Visit: Your
local scientific bookstore or urge your librarian to order.