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Comments on:
Natural Induction: An Objective Bayesian Approach

Christian P. Robert

This is a quite welcomed addition to the multifaceted literature on this topic of natural induction that keeps attracting philosophers and epistemologists as much as statisticians. The authors are to be congratulated on their ability to reformulate the problem in a new light that makes the law of natural induction more compatible with the law of succession. Their approach furthermore emphasize the model choice nature of the problem.

First, I have always been intrigued by the amount of attention paid to a problem which, while being formally close to Bayes' own original problem of the binomial posterior, did seem quite restricted in scope. Indeed, the fact that the population size N is supposed to be known is a strong deterrent to see the problem as realistic, as shown by the (neat!) Galapagos example. My first question is then to wonder how the derivation of the reference prior by Berger, Bernardo, and Sun extends to the case when N is random, in a rudimentary capture-recapture setting. An intuitive choice for $\pi_r(N)$ is $1/N$ (since N appears as a scale parameter), but is

$$\frac{\binom{R}{r}\binom{N-R}{n-r}}{\binom{N}{n}} \frac{\Gamma(R+1/2)\Gamma(N-R+1/2)}{R!(N-R)!} \frac{1}{N}$$

summable in both R and N? (Obviously, improperness of the posterior does not occur for a fixed N.)

As exposed in the paper, one reason for this special focus on natural induction may be that it leads to such a different outcome when compared with the binomial situation. Another reason is certainly that Laplace succession's rule seems to summarise in the simplest possible problem the most intriguing nature of inference. And to attract its detractors, from the classical Hume's (1748) ([1]) to the trendy Taleb's (2007) ([2]) "black swan" argument (which is not the issue here, since the "black swan" criticism deals with the possibility of model changes).

Second, the solution adopted in the paper follows Jeffreys' approach and I find this perspective quite meaningful for the problem at hand. Indeed, while N can be seen as $(N-1)+1$, i.e. as one of the $N+1$ possible values for R, the consequence of having R equal to either N or 0 lead to atomic distributions for the number of successes. Thus, to distinguish those two values from the other makes sense even outside a testing perspective. In Jeffreys' (1939) original formulation, both extreme values, 0 and N, are kept separate, with a prior probability k between $1/3$ and $1/2$. I thus wonder why the authors moved away from this original perspective. The computation for this scenario does not seem much harder since $\pi_r(0|N) = f(N)$ as well and the equivalent of (22) would then be

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$$\pi_\phi(\text{All} + | n, N) = \left(1 + \frac{k}{1-2k}\,\frac{f(n) - 2f(N)}{1 - 2f(N)}\right)^{-1},$$

which is then $(1 + 0.5f(n))^{-1}$ for N large. In this case, (24) is replaced with $\sqrt{n}/(\sqrt{n} + 2/\sqrt{\pi})$, not a considerable difference.

In conclusion, I enjoyed very much reading this convincing analysis of a hard "simple problem"! It is unlikely to close the lid on the debate surrounding the problem, especially by those more interested in the philosophic side of it, but rephrasing natural induction as a model choice issue and advertising the relevance of Jeffreys' approach to this very problem have bearings beyond the "simple" hypergeometric model.

References

[1] HUME, D., (1748). *A Treatise of Human Nature: Being an Attempt to Introduce the Experimental Method of Reasoning into Moral Subjects*, (2000 version: Oxford University Press).

[2] TALEB, N. N., (2007). *The Black Swan: The Impact of the Highly Improbable*. Penguin Books, London.

Christian P. Robert
CEREMADE
Université Paris Dauphine,
France
xian@ceremade.dauphine.fr