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AN ALGORITHM FOR THE GENERATION OF CURVILINEAR WRIST MOTION IN AN ARBITRARY PLANE IN THREE-DIMENSIONAL SPACE

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Abstract—The elements of an algorithm are presented which predicts for some simple forms (circles and ellipses) the kinematic and figural aspects of the trajectories of the human wrist when these are drawn in any arbitrary plane of free, three-dimensional space. The algorithm is based on theoretical considerations and experimental data and specifies in a unique way the angular motion at the shoulder and elbow joints by utilizing a coordinate transformation, which is only approximate, between the chosen extrinsic (trajectory) and intrinsic (joint angles) parameters. A way to extend the use of this algorithm to generate any arbitrary complex movement in all possible planes of space is also suggested.

It is a matter of everyday experience that once a particular skilled movement has been learned, movements which are equivalent to it in terms of its finality but which differ in terms of the kinematic and dynamic details of joint motion can be produced with apparent ease. For example, once handwriting has been learned, script of different sizes and in different planes (e.g. horizontal or vertical, as on a blackboard) can be generated with little degradation in performance. Yet in any given instance the relative amount of angular motion at a particular joint and the amount and timing of the muscular contractions, which give rise to it, can be expected to be unique to that particular instantiation. This aspect of motor behavior has been recognized for a long time and has been termed "motor equivalence".[6,10]

However, little is known about the algorithm, that is the set of rules, according to which a specified movement can be generalized to an arbitrary size, speed, location and orientation in space. In this paper, we shall present some of the elements of a possible algorithm according to which a specified curve can be traced by the hand in an arbitrary plane in space. To simplify the problem we shall consider only motion at the shoulder and elbow joints, neglecting any contribution of wrist and hand motion to the desired trajectory. Furthermore, we shall consider only simple, closed trajectories such as circles and ellipses, but in the Discussion we shall take up the question of how the proposed algorithm could be generalized to generate arbitrary curved motion of the hand in space.

To approach the subject, it is useful to break down the problem into a set of sequential operations. Suppose that the task is to draw an ellipse of a given size in a particular plane, i.e. that the path of the hand in space is specified. In addition to the path, the speed of the hand along the path would also have to be specified. Secondly, the trajectory of the hand in space must be converted into a trajectory of angular motions at the shoulder and elbow joints. Thirdly, given a specified trajectory of joint angular motions, torques acting at each of the joints adequate to produce the desired motion must be determined.

In this paper, we shall focus on the kinematic aspects of the problem, namely the first two operations described above, and each of these steps will be considered in detail below.

The speed of the hand along the path

The simplest way to specify the speed of the movement would be merely to require that a given distance be traversed in a certain time. For a closed trajectory, such as an ellipse, this could be equivalent to requiring that one cycle of the movement be completed in a particular time. Note that such a constraint would specify the average speed but not the instantaneous speed of the movement. Experimentally it has been found, however, that the instantaneous speed of the hand does depend on the local spatial characteristics of the movement. In particular, Viviani and Terzuolo[22] found that the tangential velocity at the hand was inversely related to the curvature of the path traced by it. This relationship held true for handwriting and drawing movements (circles and ellipses) and even for scribbles drawn in the horizontal plane. Qualitatively, their observations have been confirmed by Abend *et al.*[1] and by Flash and Hogan[5] for point-to-point arm movements in the horizontal plane.

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Empirically, Lacquanti *et al.*[9] described this relation by a power law relating the instantaneous angular velocity ω to the curvature κ

$$\omega = K\kappa^{2/3}, \tag{1}$$

where K is a constant. The implications of this relationship for arm motion in three-dimensional space, which will be derived in the following, are: (1) motion at the wrist is confined to a plane, and (2) sinusoidal wrist motion satisfies this relation identically (see also Ref. 21).

From analytical geometry the tangential velocity $\mathbf{V}$ is given by

$$\mathbf{V} = v\,\mathbf{t} = \omega/\kappa\,\mathbf{t} \tag{2}$$

where v is the instantaneous speed and $\mathbf{t}$ is the tangent vector of unit length. Substituting for ω in (1) and rearranging terms, one obtains

$$v^3\kappa = K^3. \tag{3}$$

Since the acceleration $\mathbf{A}$ is given by

$$\mathbf{A} = \mathrm{d}v/\mathrm{d}t\,\mathbf{t} + v^2\kappa\mathbf{n}, \tag{4}$$

where $\mathbf{n}$ is the unit normal to the trajectory and the two terms represent the tangential and normal components of the acceleration, the vector cross-product of velocity and acceleration is

$$\mathbf{V} \times \mathbf{A} = v^3\kappa\mathbf{b}, \tag{5}$$

where $\mathbf{b}$ is the instantaneous normal to the plane of motion (binormal). Thus, equation (1) is equivalent to

$$\mathbf{V} \times \mathbf{A} = K\mathbf{b} \tag{6}$$

where K and $\mathbf{b}$ are constants. (For planar motion, $\mathbf{b}$ is constant by definition.)

Differentiating (6) one obtains

$$\mathbf{V} \times \mathrm{d}\mathbf{A}/\mathrm{d}t = 0, \tag{7}$$

which implies that the velocity $\mathbf{V}$ and its second derivative $\ddot{\mathbf{V}}$ are parallel:

$$\ddot{\mathbf{V}} = c(t)\mathbf{V}. \tag{8}$$

In a Cartesian coordinate system $\{x, y, z\}$, the components of (8) are

$$\ddot{v}_x = cv_x$$
$$\ddot{v}_y = cv_y$$
$$\ddot{v}_z = cv_z. \tag{9}$$

For periodic motion, under the assumption that c is constant, equation (9) has the solution

$$v_x = x_0 \sin pt$$
$$v_y = y_0 \sin(pt - \delta_1)$$
$$v_z = z_0 \sin(pt - \delta_2) \tag{10}$$

and by equation (6)

$$\mathbf{V} \times \mathbf{A} = p^3 \begin{Bmatrix} y_0 z_0 \sin(\delta_2 - \delta_1) \\ -x_0 z_0 \sin \delta_2 \\ y_0 x_0 \sin \delta_1 \end{Bmatrix}. \tag{11}$$

Parametrization of the plane of motion

The conversion from wrist trajectory in space into a trajectory of angular motions at the shoulder and elbow joints implies a coordinate transformation from the reference frame in which the trajectory of the hand is described into the reference frame describing joint angular motions. Both reference frames have to be specified. While a number of different coordinate systems could be chosen, some judicious choice can perhaps be made on the basis of available data[18,19] and results obtained.

To describe the motion of the hand we use the Cartesian coordinate system shown in Fig. 1. In this frame of reference, X represents the anterior direction, Y the lateral direction and Z the vertical, defined relative to the subject. In this coordinate system, the plane of motion of the hand is defined by the unit vector $\mathbf{b}$ perpendicular to the plane of motion. Two scalar parameters are adequate to define $\mathbf{b}$. We have chosen the planar elevation Ψ and its azimuth χ, defined as

$$\tan\chi = b_y/b_x$$
$$\tan\Psi = -b_z/\sqrt{b_x^2 + b_y^2}. \tag{12}$$

χ and Ψ are both zero when the motion of the hand lies in the frontal plane, as in drawing on a blackboard. Note that these two parameters could have been defined differently; the choice was motivated by experimental data to be presented in this paper.

For the trajectories we shall consider, namely circles and ellipses, one additional parameter serves to define the figural characteristics of the motion, namely the slant (σ). Consider for simplicity motion restricted to one of the principal planes such as the sagittal plane. In this instance, $y_0 = 0$ (10), and slant is defined as $\sigma = \delta_2$, that is the phase difference between the vertical and horizontal components of the motion. When this phase difference is 90°, an ellipse

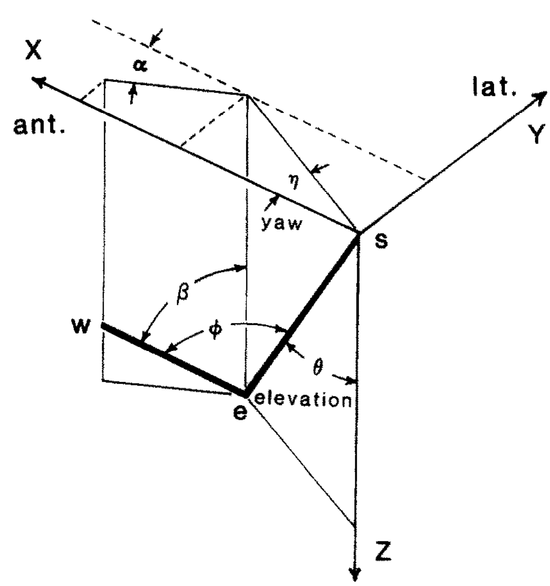

Fig. 1. Parameters used to define the angular orientation of the arm. The angles θ and β represent the angular elevation of the arm and forearm and are measured in a vertical plane relative to the vertical (Z) axis. The yaw angles η and α are measured in the horizontal plane from the anterior (X) direction.

with its major and minor axes oriented horizontally or vertically will result. Slant angles of 0° or 180° produce rectilinear motion and intermediate values yield ellipses which are slanted anteriorly or posteriorly.

When the motion of the hand is in an oblique plane, the slant depends on both δ_1 and δ_2 (10). In such a case, we define slant by the following procedure. We consider the frontal plane (χ and Ψ equal to zero) to be the cardinal plane of motion. A figure in this cardinal plane can be rotated into any oblique plane by two successive rotations: first, by an angle χ about the vertical Z-axis, then by an angle Ψ about the Y-axis defined after the first rotation. If we define a coordinate system $\{x', y', z'\}$ which is rotated by χ and Ψ, the relationship between this primed and the fixed, unprimed coordinate system (Fig. 1) is given by

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \begin{bmatrix} \cos\Psi\cos\chi & \cos\Psi\sin\chi & -\sin\Psi \\ -\sin\chi & \cos\chi & 0 \\ \sin\Psi\cos\chi & \sin\Psi\sin\chi & \cos\Psi \end{bmatrix} \times \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \quad (13a)$$

and

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{bmatrix} \cos\Psi\cos\chi & -\sin\chi & \sin\Psi\cos\chi \\ \cos\Psi\sin\chi & \cos\chi & \sin\Psi\sin\chi \\ -\sin\Psi & 0 & \cos\Psi \end{bmatrix} \times \begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix}. \quad (13b)$$

In the primed coordinate system, the normal **b** is by definition $\{1, 0, 0\}$, and by using (13b) $\mathbf{b} = \{\cos\Psi\cos\chi, \cos\Psi\sin\chi, -\sin\Psi\}$ in the unprimed coordinates. The definition of elevation Ψ and azimuth χ given in (12) follows automatically.

In this coordinate system, slant σ is defined as the phase difference between the vertical ($v_{z'}$) and lateral ($v_{y'}$) components of the velocity and $v_{x'} = 0$. In the unprimed coordinate system, the velocity components $\{v_x, v_y, v_z\}$ are again obtained by using (13b).

Specification of torque

Once the motion of the arm has been specified joint torques adequate to produce this motion must be determined. Mathematically, joint torque depends in a nonlinear manner on the angular accelerations, velocities and displacements and for arm motion in three-dimensional space these equations have a large number of terms. In robotics, efficient schemes to solve these equations have been developed.[7] Tabular solutions have also been proposed.[15] Alternatively, it has also been suggested that the system takes advantage of the visco-elastic properties of muscles by specifying a trajectory of equilibrium points, thus obviating the need to solve the equations analytically.[3]

Note that the required joint torques must be partitioned among the muscles acting at each joint according to some set of rules. Pellionisz and Llinás[13,14] have suggested a way in which this might be accomplished and this suggestion is being investigated experimentally.[2] These problems will not be taken up further in this paper.

The theoretical question and an experimental approach

The problem we wish to consider now is the following: given a desired plane of motion of the hand (i.e. χ and Ψ) and a figure having a given slant (σ), can one derive a simple set of rules to specify the appropriate angular motion at the shoulder and elbow joints?* A solution of this problem can be obtained analytically by solving equations (10), (11) and (12) for the joint angles (e.g. as given by equation 14). Such a solution will not be unique since the arm has four degrees of freedom (three at the shoulder and one at the elbow) while only three parameters are necessary to describe the motion of the hand in space. However, uniqueness could be achieved by imposing some constraint.

If such an exact procedure were used, the execution of a given movement in different planes in space would require a separate solution each time and it would thus appear as if each movement were novel, contrary to everyday experience. Therefore it is appealing to hypothesize that the problem is solved in an approximate manner[11] and that such approximations involve simple relations between extrinsic parameters (such as χ, Ψ, σ) and intrinsic parameters related to the joint angles. We have attempted to uncover such simple relations experimentally by asking subjects to draw circles and ellipses with their arm in different planes in space. In the following sections we will briefly outline the experimental procedures, present the experimental results and the results of some simulations.

EXPERIMENTAL PROCEDURES

The experimental arrangement and details of the analytical procedures have been described fully in a previous publication in which we consider drawing movements restricted to the vertical plane.[18] The location of the elbow and of the wrist in three-dimensional space was determined by means of a pair of ultrasound emitters and a set of three microphones.[16] The elbow angle of flexion extension was measured goniometrically.[4]

From these measures the angular orientation of the arm and of the forearm was calculated. The parameters we have chosen are defined in Fig. 1. They are: θ and β (the angular elevation) and η and α (yaw) of the arm and forearm. Angular elevation is measured in a vertical plane relative to the vertical (Z) and yaw in the horizontal plane relative to the anterior direction (X). These angles were identified previously psychophysically as the preferred coordinate system for the recognition of the orientation of the arm in space.[19]

*Note that the problem as we have formulated it is incomplete. To fully define the motion, one would also need to specify its location in space and its amplitude. This latter aspect of the problem is beyond the scope of this paper.

In these calculations we assumed that the center of rotation at the shoulder remains fixed. This assumption is not strictly valid and introduces uncertainty in the calculation of upper arm orientation. One estimate of this error is given by the difference between the measured and calculated elbow angles;[18] the root-mean-square difference between these values typically ranged from 2° to 4°. Forearm pronation–supination and wrist rotations were not measured. These degrees of freedom affect the motion at the fingers. Therefore, the subjects were asked to perform the task while keeping the wrist rigid. However, given the size of the movements we studied, the potential contribution of motion at the distal joint to the overall motion is negligible.

In terms of these orientation angles, the position of the wrist is given by

$$\begin{aligned} x_w &= l_1 \sin\theta \cos\eta + l_2 \sin\beta \cos\alpha \\ y_w &= l_1 \sin\theta \sin\eta + l_2 \sin\beta \sin\alpha \\ z_w &= l_1 \cos\theta - l_2 \cos\beta. \end{aligned} \tag{14}$$

As we have shown previously[18] and as predicted by the derivation presented above (10), for freely and repetitively drawn circles and ellipses, the horizontal and vertical components of the motion at the wrist are well approximated by sinusoids. The variation in the orientation angles is also close to sinusoidal.[18] Therefore, the mean, amplitude and phase of the fundamental for each of the parameters (wrist motion and orientation angles) was calculated by Fourier analysis. The distortion from true sinusoidal motion was defined conventionally to be the root-mean-square difference between the fundamental component and the experimental value, normalized by the latter's variance. Distortion can thus range from 0 to 1.

The normal to the plane of motion at the wrist (**b**) was calculated according to (6) by averaging several cycles of the motion. The elevation Ψ and azimuth χ of the plane were calculated according to (12). Since the direction of the normal is reversed for clockwise and counterclockwise motion, we adopted the convention that the X-component of the normal (b_x) point in the anterior ($+X$) direction. This restricts the absolute value of χ to be less than 90°. For $b_x < 0$, the sign of each of the components of **b** was inverted. (For example, a circle drawn in the clockwise direction in the frontal plane gives $b_x = 1$ while a counterclockwise rotation gives $b_x = -1$. There is an ambiguity, however, since the latter could be represented by $\chi = 180°$ and a slant $\sigma = -90°$ or by $\chi = 0°$ and $\sigma = 90°$.)

The motion of the wrist was then rotated into the cardinal (frontal) plane using (13a). In this coordinate system, the motion would lie in the plane $x' =$ constant. Slant σ was calculated as the phase difference between the z' and y' fundamental components of the motion at the wrist. The extent (ε) to which wrist motion deviated from the plane was calculated by

$$\varepsilon = \{\Sigma v_x^2 / \Sigma (v_y^2 + v_z^2)\}^{1/2}, \tag{15}$$

where v_x is the out of plane component of the velocity (in the primed coordinate system) and v_y and v_z are the in-plane components.

The data to be reported summarize the results of 8 experiments involving 6 subjects.

EXPERIMENTAL RESULTS

The results of a typical trial are shown in Fig. 2. In this instance the subject was asked to draw a

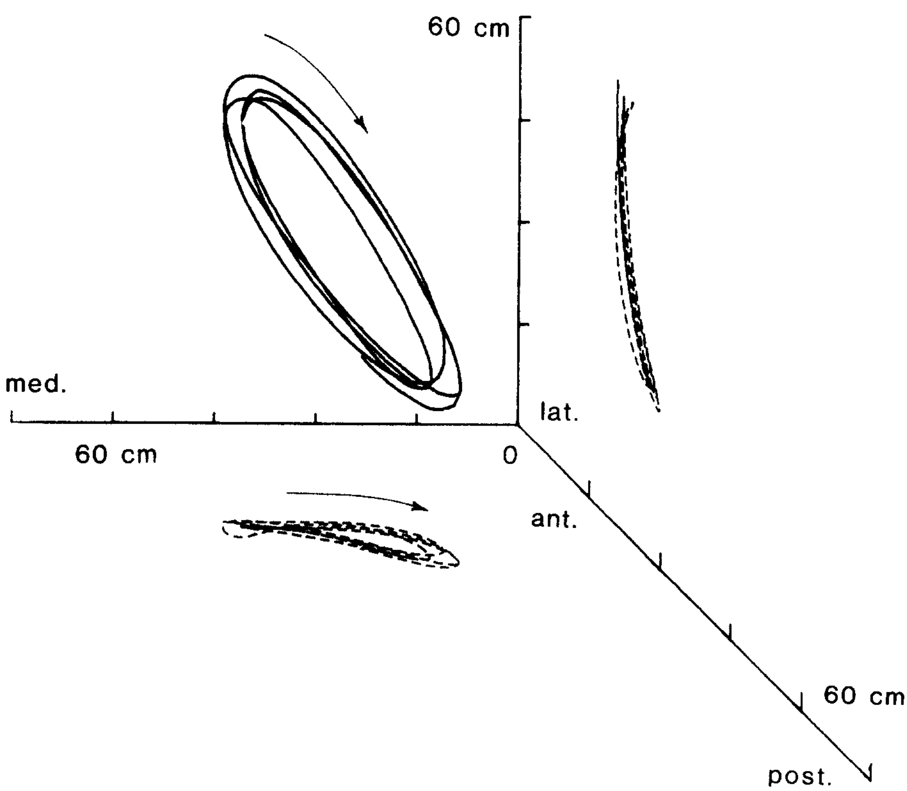

Fig. 2. Perspective view of wrist motion in three-dimensional space. Shown is a typical example in which the subject was asked to draw a slanted ellipse in the frontal plane. The direction of motion is indicated by the arrows and the dashed lines depict the projection of wrist motion onto the horizontal and sagittal planes.

slanted ellipse in the frontal plane. In this trial the deviation from planar motion ε, calculated according to (15), was 0.15. For the 31 trials obtained for this subject the average deviation from planar motion ε was 0.089 ± 0.026. The elevation Ψ of the plane of motion was $-7.4°$. (Negative values of Ψ indicate that the upper portion of the curve is anterior to the lower portion, as may be appreciated from the projection of the ellipse onto the sagittal plane.) The azimuth χ was 2.6°. (Positive values of χ indicate the medial portion of the curve is anterior to its lateral portion, as can be seen from the projection of the ellipse onto the horizontal plane.) Slant σ was $-34°$. The same ellipse drawn in the counterclockwise direction would have given $\sigma = 34°$, while an ellipse slanted in the opposite direction (upper portion lateral) would have resulted in $\sigma > 90°$. The period of the motion was 1.22 s, a value typical for our subjects drawing figures of this size.

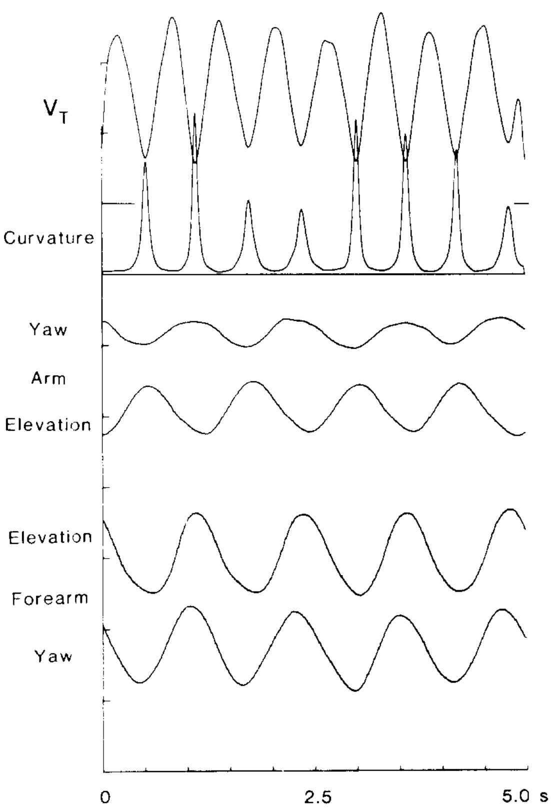

Fig. 3. Kinematics of wrist motion and orientation angles during a drawing movement. The data are for the slanted ellipse shown in Fig. 2. From top to bottom, the traces show the tangential velocity (V_T) and the curvature κ of the wrist, and the yaw (η) and angular elevation (θ) of the arm and of the forearm (β and α). Note that the curvature is largest when the tangential velocity is smallest, the modulation in the orientation angles is close to sinusoidal and the two angular elevations are close to 180° out of phase. Scale (per division): V_T – 50 cm/s, κ – 0.33 cm^{-1} and 45° for the orientation angles. Tangential velocity ranged from about 40–120 cm/s in this trial.

In each trial, we specified the approximate plane of motion, its direction (clockwise or counterclockwise) and the slant. The location in space and the size of the figure to be drawn were not specified. Furthermore, we were not able to determine how accurately the requested plane of motion was reproduced except in the trivial cases when the figure was to be drawn in one of the principal planes. In those instances Ψ and χ were generally within 10° of the specified values.

As we have reported previously, and in agreement with the theoretical predictions (10), the horizontal and vertical components of the motion at the wrist could be well approximated by sinusoids.[18] For example, for the trial illustrated in Fig. 2 distortion in the horizontal component of wrist velocity was 14% and 11% for the vertical component. The modulation of the orientation angles of the arm and forearm was also close to sinusoidal. The variation in the yaw (η, α) and elevation (θ, β) angles of the arm and forearm are shown in Fig. 3. In this instance, arm yaw (η) had the largest distortion (30% for velocity). The tangential velocity of the wrist (V_T) and the curvature of the wrist motion (κ) are also illustrated for this trial. In agreement with previous results, the two variables are inversely related. Tangential velocity is at a minimum when the curvature is largest, as predicted by (1) and (3).

Since the modulation in the orientation angles was generally close to sinusoidal, it was possible to compute the amplitude and phase of the fundamental component for each of the angles. We have previously shown that when the plane of wrist motion is close to vertical ($\Psi \simeq 0$), the modulations in the angular elevation of the forearm (β) and of the arm (θ) are approximately 180° out of phase (Fig. 3), independently of the azimuth (Ψ) and slant (σ) of the motion.[18] Results summarized in Fig. 4 extend these observations to instances in which the plane of wrist motion deviated considerably from the vertical. The figure shows the distribution of the phase differences between the two angles of elevation. Trials have been grouped according to the amount by which the plane of motion at the wrist deviated from the vertical, i.e. according to the absolute value of Ψ. Only those trials in which the distortion in the modulation of both angles of elevation was less than 30% are included. As can be seen from Fig. 4 and Table 1, the values for the phase difference between β and θ clustered around 180° when the plane of motion is

Table 1. Variation of phase difference between β and θ as a function of planar elevation Ψ

Number of trials (distortion < 30%)	Total number of trials	Phase	$\lvert\Psi\rvert$
94	101	189 ± 24	< 15
21	21	194 ± 28	< 30
13	16	176 ± 21	< 45
13	29	200 ± 83	> 45

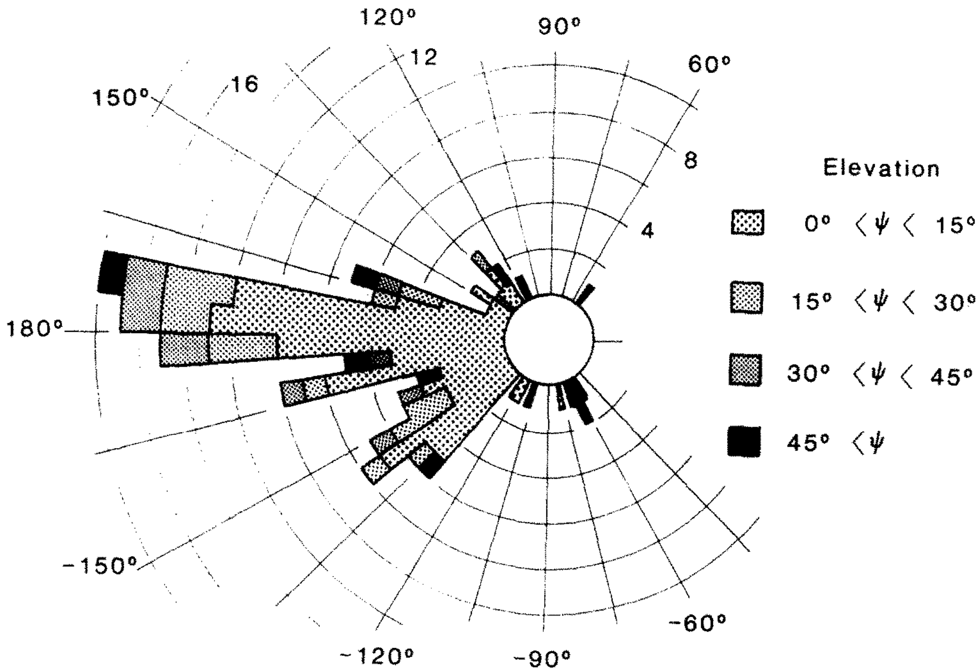

Fig. 4. Distribution of the phase difference between the angular elevations of arm and forearm. The polar histogram shows the distribution of the phase lead (or lag) of the angular elevation of the forearm (β) relative to that of the arm (θ). Trials are grouped according to the planar elevation Ψ of wrist motion (vertical: $\Psi = 0°$; horizontal: $\Psi = 90°$).

within 45° of the vertical. No trend is apparent in the data. The average values reported in Table 1 do not differ significantly from one another ($P > 0.05$). The phase relation between the two angular elevations is not correlated with either the planar elevation Ψ ($r = 0.001$), the azimuth χ ($r = 0.112$) or the slant σ ($r = 0.125$).

For trials in which wrist motion was close to horizontal ($|\Psi| > 45°$) there is a much greater variability in the phase difference between β and θ. Furthermore, in the majority of such trials (16 of 29), the distortion in β and/or θ was greater than 30% and the phase relationship between the two angles could not be estimated. Generally, the amplitude of the modulation in β and/or θ was small. One example is shown in Fig. 5. In this instance, an ellipse was drawn in a plane close to horizontal ($\Psi = -70°$). The distortions in the angular motion of the arm (η, θ) and in forearm yaw (α) were all small, ranging from 9% for θ to 19% for η. The distortion in forearm elevation is much larger (61%), the amplitude of its fundamental component being only 3°.

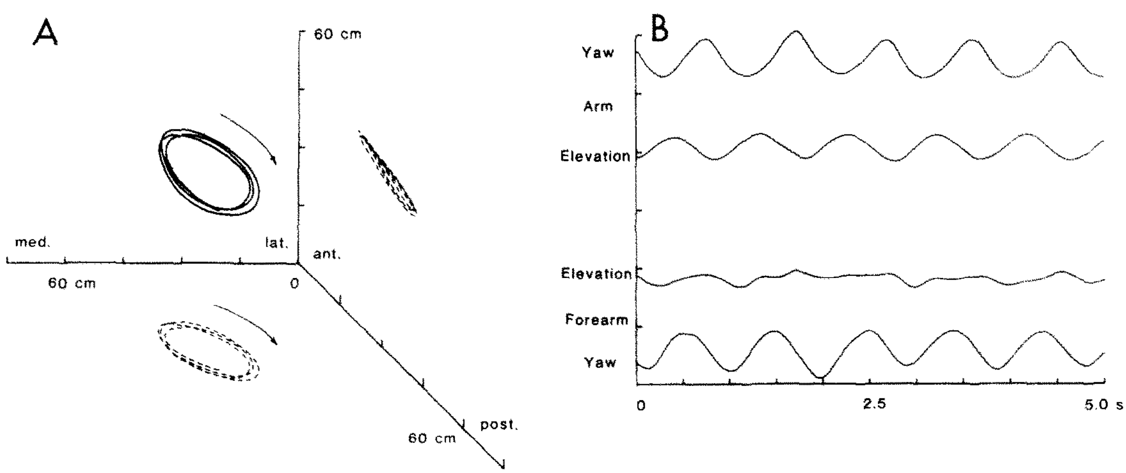

Fig. 5. Ellipse drawn in a plane close to the horizontal. (A) shows in perspective the motion at the wrist, (B) the modulation in the orientation angles. Note that the angular elevation of the forearm varies little during this trial and is highly distorted while the modulation in the other orientation angles is close to sinusoidal.

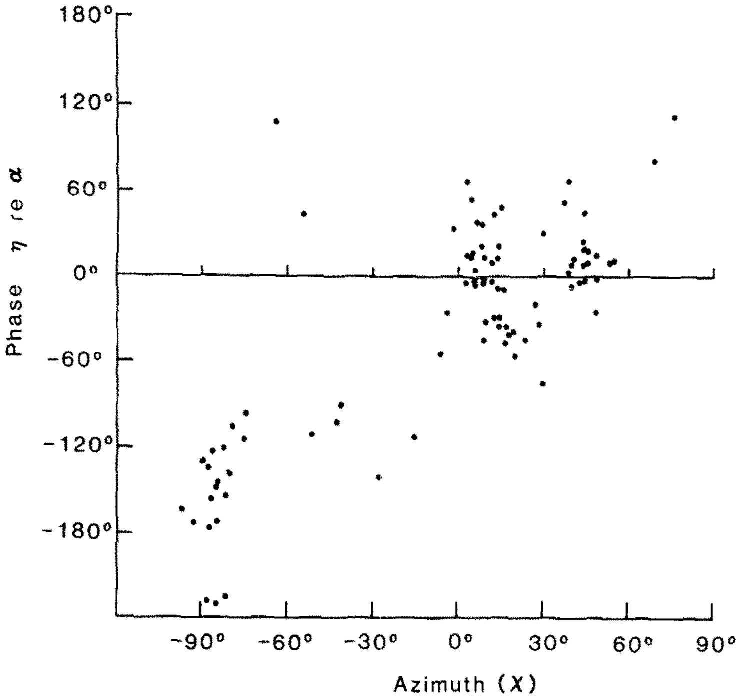

Fig. 6. Correlation of the phase difference between arm (η) and forearm (α) yaw with the azimuth (χ) of wrist motion. When the modulation in the two yaw angles is in phase, wrist motion is close to the frontal plane ($\chi = 0°$) and in the sagittal plane ($\chi = \pm 90°$) when the phase difference between η and α is 180°. The planar elevation of wrist motion ranged from vertical to horizontal in these trials.

We previously showed that for circles and ellipses drawn in the frontal plane ($\chi = 0$), the modulation in forearm yaw (α) and in upper arm yaw (η) was approximately in phase,[18] whereas the modulation in the two yaw angles was about 180° out of phase when the figure was drawn in the sagittal plane ($\chi = \pm 90°$). This can be appreciated in Fig. 6, in which the phase difference between the two yaw angles is plotted as a

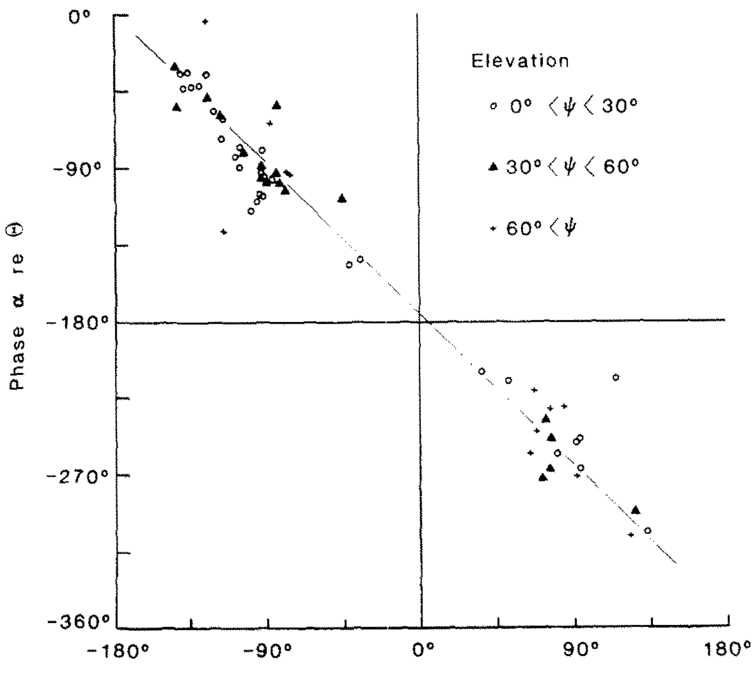

Fig. 7. Correlation of the phase difference between forearm yaw (α) and arm angular elevation (θ) with the slant (σ) of the ellipse. The plane of wrist motion ranged from vertical ($\Psi = 0°$) to horizontal ($\Psi = 90°$), as denoted by the different symbols.

function of the azimuth (χ) of the plane of wrist motion. Data for trials in 5 experiments in which wrist movements in different planes were explored are presented. The data include trials in which the plane of motion was close to the vertical ($\Psi = 0$) as well as those in which the figure was drawn in an oblique plane. Although there is considerable scatter in the data, the phase difference between the two yaw angles is clearly correlated with the azimuth (χ) of the plane of motion.

Finally, Fig. 7 shows that the phase of forearm yaw (α) relative to angular elevation of the arm (θ) is well correlated with the slant (σ) of the ellipse. For the sake of clarity, only data for the three experiments which included trials in which the planar elevation of wrist motion (Ψ) differed significantly from zero are shown. The two variables are highly correlated ($r = -0.977$, $n = 60$), independently of the planar elevation.

CONCLUSION

The results presented in Figs 4–7 extend our previous observations[18] which were restricted to wrist motions close to the vertical plane, and they show that: (1) independently of the plane of wrist motion (at least for $|\Psi| < 45°$), the modulation in forearm (β) and arm (θ) elevation is 180° out of phase, (2) the azimuth (χ) of the motion is given by the phase difference between the two yaw angles (η, α), and (3) the slant is given by the phase of forearm yaw angle (α) relative to the angular elevations.

Note that given a phase difference of 180° between the two angular elevations, the two other relations are implicit in the way the angular orientation of the arm and the plane of motion have been defined. Indeed, this is equivalent to reducing the number of degrees of freedom of the arm from four to three, thus simplifying the problem by providing a unique solution. A second simplification suggested by the data is the existence of linear relationships (points 2 and 3 above). The question now is: are these relationships valid only over the limited domain of motions explored experimentally or are they valid for a wide range of combinations of motions at the shoulder and elbow joints? Such a generalization is a prerequisite for the utilization of the rules expressed by these relationships in an algorithm for the coordination of arm motion in all possible planes. We addressed this point by the use of simulations.

SIMULATIONS

We assumed pure sinusoidal motion in the orientation angles, i.e.

$$\begin{aligned} \theta(t) &= \theta_0 + \theta_1 \cos pt \\ \eta(t) &= \eta_0 + \eta_1 \cos(pt - \delta_1) \\ \beta(t) &= \beta_0 + \beta_1 \cos(pt - \delta_2) \\ \alpha(t) &= \alpha_0 + \alpha_1 \cos(pt - \delta_3). \end{aligned} \tag{15}$$

Random combinations of values for the mean and amplitude for each of the angles were chosen over the following range:

$$\begin{array}{ll} \theta_0: 20° \text{ to } 65° & \theta_1: 5° \text{ to } 30° \\ \eta_0: 10° \text{ to } 50° & \eta_1: 5° \text{ to } 30° \\ \beta_0: 50° \text{ to } 95° & \beta_1: 5° \text{ to } 40° \\ \alpha_0: -50° \text{ to } 10° & \alpha_1: 5° \text{ to } 35°. \end{array}$$

The values chosen encompassed the range of values found experimentally. For 100 such random combinations, the motion of the wrist was calculated

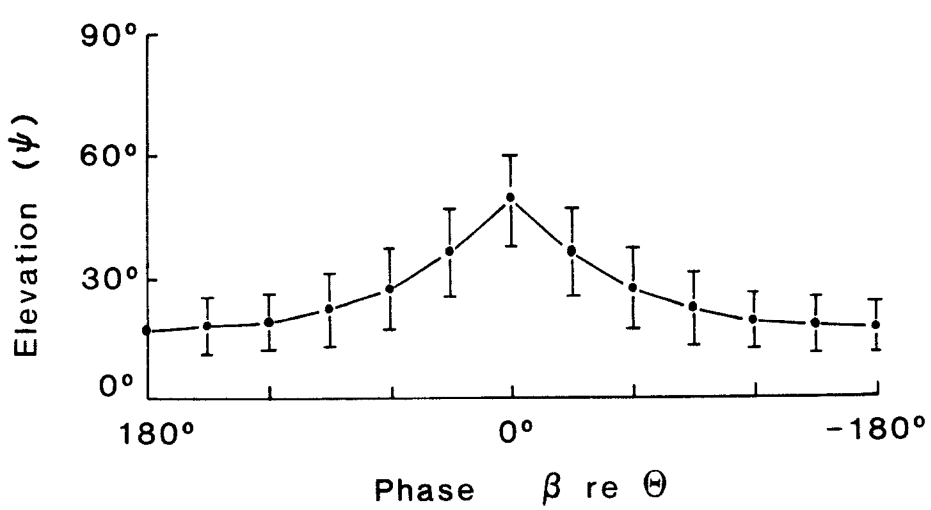

Fig. 8. Simulation results: variation of planar elevation (Ψ) with the phase difference between angular elevation of arm (θ) and forearm (β). The points denote the mean and standard deviation of $|\Psi|$ for random combinations of sinusoidal angular motion of the arm and forearm. All combinations of phase relations of the yaw angles are included in these results.

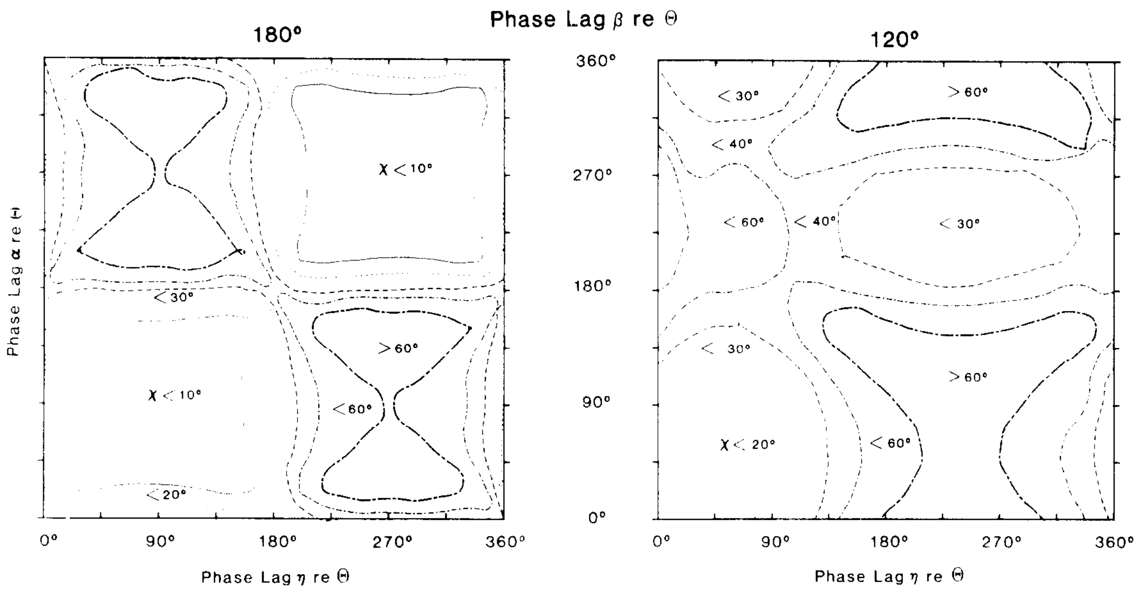

Fig. 9. Simulation results: dependence of azimuth (χ) on the phase difference between the two yaw angles η and α. The traces show contour lines of constant mean values of azimuth for random combinations of sinusoidal angular motion of the arm and forearm; on the left, β lags θ by 180° and by 120° on the right. In the former case, motion in a plane close to the frontal plane ($\chi = 0$) results when η and α are in phase and on average, the motion is close to the sagittal plane when the two yaw angles are 180° out of phase, as was found to hold true for experimental data (Fig. 6). No such correlation exists when the phase difference between the two angular elevations differs substantially from 180° (right panel).

according to (14), assuming the length of the arm and forearm to be equal. The plane of motion (χ, Ψ) and the slant were then computed as before. The phases δ_1, δ_2 and δ_3 were each varied in increments of 15°. For each of the 24^3 combinations of phases, the mean and standard deviation of χ, Ψ and σ were calculated. The results of these simulations are presented in Figs 8–11.

Figure 8 shows that on average the planar elevation of wrist motion (Ψ) is least when β and θ are 180° out of phase ($|\Psi| = 17°$) and is greatest when they are in phase ($|\Psi| = 49°$). In principle then the planar elevation could be regulated by changing the phase relation between β and θ. (Recall that the experimental data indicate that this is not the case.)

Figure 9 illustrates the manner in which the azimuth χ of wrist motion depends on the phases of the yaw angles η and α relative to arm elevation θ. The results of the simulation are plotted as equipotential lines for χ; on the left β and θ are 180° out of phase while β lags θ by 120° on the right. In the former case, in the upper right and lower left quadrants the azimuth of the plane of motion is close to zero (as for motion in the frontal plane). Thus, in agreement with experimental data, the azimuth is close to zero when α and η are in phase (i.e. along the main diagonal of the plot) and χ approaches 90° when the modulation in these two angles is 180° out of phase. This can also be seen in Fig. 10 where we have plotted the mean and standard deviation of χ as a function of the phase lag of η relative to θ for a phase lag of α relative to θ of 60° (top) and 240° (bottom). The second plot of Fig. 9 shows that the simple relationship between azimuth and the phase of α relative to η holds only when the two angular elevations β and θ are 180° out of phase.

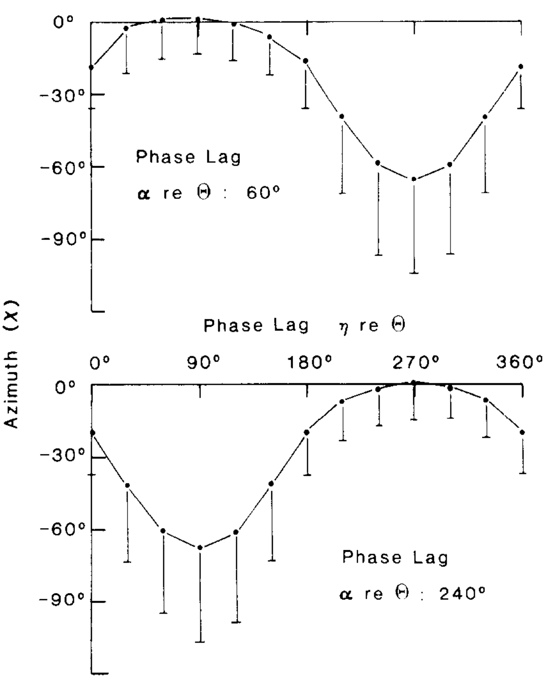

Fig. 10. Simulation results: mean and standard deviation of azimuth (χ) of wrist motion as a function of the phase of arm yaw (η) relative to arm angular elevation (θ). Forearm yaw (α) lagged θ by 60° (top) and by 240° (bottom). In both cases, β and θ were 180° out of phase.

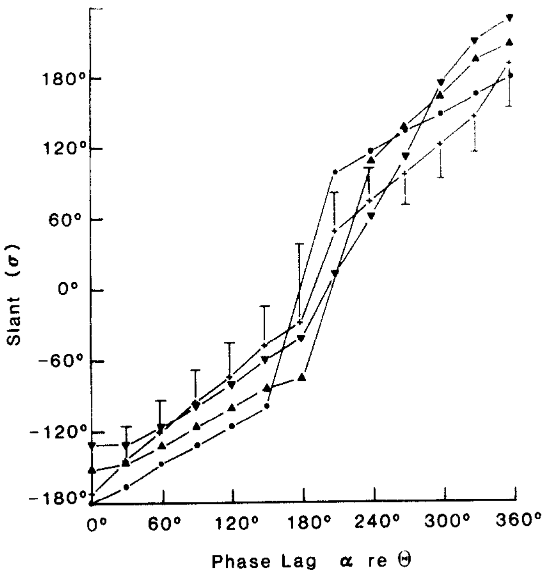

Fig. 11. Simulation results: correlation of slant (σ) with the phase difference between forearm yaw (α) and arm angular elevation (θ). β and θ were 180° out of phase and the different symbols denote the mean values for different lags of arm yaw (η) relative to arm elevation (●, 0°; ▲, 60°; ▼, 120°; +, 180°).

Finally, Fig. 11 shows the dependence of slant (σ) on the phase of forearm yaw α relative to θ. Each of the symbols shows the results for a given lag of η relative to θ; β and θ were 180° out of phase. The results of this simulation also confirm the experimental data presented in Fig. 7; slant shows little dependence on upper arm yaw η. Once again, this conclusion was valid only when β and θ were close to 180° out of phase, the closer the modulation in the two angular elevations was to being in phase, the stronger was the dependence of slant on the phase of η as well as on α.

One can thus conclude that the rules expressed by the relationships in Figs 6 and 7 are generally valid (provided that the modulations of the angular elevations of the arm and forearm are 180° out of phase) and they permit a simple way of achieving a motion at the wrist with a given value of azimuth and slant. The question remains: how is the planar elevation Ψ of the wrist specified? On the basis of calculations presented in the Appendix, it appears possible to solve this problem by regulating the means and amplitudes of the modulation of the angular elevations of the arm and forearm ($\theta_0, \theta_1, \beta_0, \beta_1$).

DISCUSSION

From the data presented above a set of simple rules has emerged whereby elliptical wrist motion can be generated in any arbitrary plane of free, three-dimensional space. Moreover, the general validity of these rules found strong support in the results of simulations. One of these rules is that the phase difference between the modulation of the angular elevation of the arm and forearm is constrained to be approximately 180. As already stated, this constraint in one of the parameters required to define the motion of the limb also permits us to specify in a unique way the angular motion at the shoulder and elbow joints since it reduces the number of degrees of freedom of the arm and forearm to the same number of parameters necessary to describe the motion of the wrist in space. Furthermore, this constraint was also shown previously to be useful in minimizing the distortion of the vertical component of the wrist motion.[18] Finally, because of this constraint and given the choice of coordinate systems utilized in the algorithm to describe wrist trajectories and joint angular motions, a further simplification is achieved in the sense that the coordinate transformation between the intrinsic and extrinsic parameters which define both slant and azimuth becomes of necessity a linear one. (Recall that the phase of the forearm yaw defines the slant of the figure while the azimuth is given by the phase relation between the yaw angles of the arm and forearm.) Four other parameters, at the most, appear to be needed to determine planar elevation (see Appendix) while it is most likely that the eight parameters which describe the mean and amplitude of the modulation of the joint angles (in the intrinsic coordinate frame) are used to specify the size of the figure to be drawn and its location in space. This latter aspect of the problem, however, does require further investigation, as does the question whether there is some simple means to specify the torques required to produce the angular motions and how they can be partitioned among the participating muscles.

The question now is: is the proposed algorithm the one which is actually utilized by the nervous system? Since a direct demonstration or refutation is impossible, one is confined to argue this subject on the basis of indirect evidence. While we have been unable so far to find substantive arguments which speak against the point of view we are pursuing, the following set of arguments is presented here in support of our position: (1) the algorithm has general validity and can readily account in a simple way for phenomena as complex as "motor equivalence", (2) there are consistent and predictable distortions in the motion of the wrist when subjects are asked to draw circles and ellipses in certain planes of space and these distortions are reproduced by the algorithm in simulation experiments.

Since the second point is the best evidence in favor of our point of view, it needs to be pursued in some detail. First of all, we have shown previously[18] that when subjects are asked to draw a circle in a sagittal plane there is a flattening of the portion of the trajectory which is closer to the subject. This characteristic distortion is reproduced in simulations. In that case, the angular motion (in terms of angles defined in the intrinsic coordinate system) is strictly sinusoidal and therefore distortion in the angular motions does not contribute to the distortion the wrist trajectory.

To reinforce this observation, we present in Fig. 12 typical examples, from two subjects, for ellipses drawn in oblique planes (see the actual trajectories of the wrist drawn in perspective at the left of the figure). Wrist motion was then rotated into the principal plane (13) and the plots in the upper row (on the right of Fig. 12A and B) show an edge-wise and head-on view of the motion in that plane. In both trials, note the flattening of the proximal portion of the wrist trajectory (on the left in the planar projection in Fig. 12A and on the right in Fig. 12B). The fundamental component of the modulation in each of the orientation angles was computed by Fourier analysis and the traces labeled "simulation" show the wrist trajectory predicted by the algorithm assuming pure sinusoidal angular motion. The asymmetry in the proximal and distal portions of the movement is evident in both cases.

Another question is: can the experimental results be used to infer whether the movements we studied are planned and organized in terms of kinematic variables of the joints (intrinsic coordinates) or of the endpoint (extrinsic coordinates)? At first glance the answer may seem obvious since the task required the production of a specified motion of the endpoint. More generally, however, a number of investigators have used the existence of invariant relations among extrinsic or intrinsic parameters to try to answer the question posed above.[2,12,17] If one follows this line of reasoning, the data we have described are compatible with both possibilities since invariant relations exist among extrinsic parameters (namely the relation between curvature and speed) and among intrinsic parameters (namely the phase relation between the two angular elevations). Therefore, arguments based on the existence of invariances *vis-à-vis* this particular question can be of limited utility.

This last point we wish to consider deals with the possibility of using the proposed algorithm to account for any arbitrary movement executed in all possible planes of space. Here we begin by noting that the relationship between angular velocity and curvature was shown to be generally valid for all curved trajectories[9,21,22] and that this relationship is satisfied

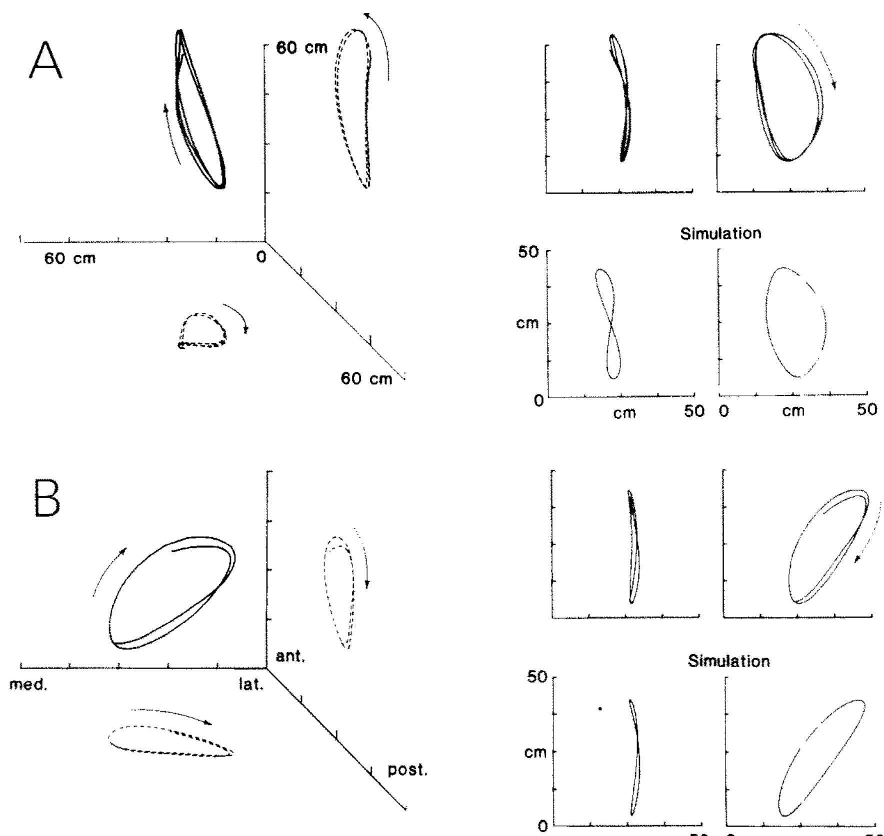

Fig. 12. Distortion of wrist motion: experimental data and simulation. On the left, perspective representation of wrist motion in three-dimensional space for two trials in which subjects were asked to draw an ellipse in an oblique plane in free space. In the top row on the right of (A) and (B) are shown an edge-wise and head-on view of the wrist motion and below, a simulation of the experimental data assuming undistorted sinusoidal angular motion of the arm and forearm. Note the flattening of proximal portion of the wrist trajectory which is reproduced by the simulations.

identically if the motion at the wrist is sinusoidal (see Introduction). Secondly, there is strong evidence that complex trajectories, such as handwriting, are made of unit segments,[20,22] and segmentation of handwriting was one of the basic assumptions of Hollerbach's simulation studies.[8] Starting from these premises there is a way in which one can extend the use of the proposed algorithm to all apparently continuous, complex movements if the following assumptions are made: (1) these movements do in fact consist of unit segments, and (2) each segment consists of an arc which is generated by using the rules of the proposed algorithm.

If so, the relationship between curvature and angular velocity would be satisfied within each segment and the torsion, that is the rate at which the plane of motion changes, would be zero except at the points of junction between segments. This latter prediction is in agreement with some experimental observations of Morasso[12] and was found to be true in *ad hoc* designed experiments to be reported. Note also that the validity of the two assumptions as well as the adequacy of the algorithm to generate complex movements in arbitrary planes of free space can be submitted to experimental verification.

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APPENDIX

In this Appendix we take up the question how the planar elevation of wrist motion can be specified in terms of parameters related to the orientation angles. Some insight into this problem can be obtained by examining the manner in which the normal to the plane of motion **b** depends explicitly on the orientation angles. Substituting (15) into (14), differentiating to obtain wrist velocity and acceleration and calculating **b** from the vector cross-product between velocity and acceleration, one obtains

$$\begin{aligned} b_x &\simeq p^3[\theta_1 s\theta_0 + \beta_1 s\beta_0] \\ &\quad \times [\eta_1 s\theta_0 c\eta_0 s\delta_1 + \alpha_1 s\beta_0 c\alpha_0 s\delta_3] \\ b_y &\simeq p^3[\theta_1 s\theta_0 + \beta_1 s\beta_0] \\ &\quad \times [\eta_1 s\theta_0 s\eta_0 s\delta_1 + \alpha_1 s\beta_0 s\alpha_0 s\delta_3] \\ b_z &\simeq p^3[\eta_1 s\theta_0 s\delta_1(\theta_1 c\theta_0 - \beta_1 c\beta_0 c\gamma_0) \\ &\quad + \alpha_1 s\beta_0 s\delta_3(\theta_1 c\theta_0 c\gamma_0 - \beta_1 c\beta_0) \\ &\quad + \eta_1\alpha_1 s\theta_0 s\beta_0 s\gamma_0 s(\delta_1 - \delta_3)], \end{aligned} \quad \text{(A1)}$$

where

$$\gamma_0 = \eta_0 - \alpha_0$$
$$s\theta_0 = \sin\theta_0 \quad c\theta_0 = \cos\theta_0$$

and the other parameters are defined in (15). Higher order terms, which vary with time, have been neglected in (A1).

From the definition of azimuth (χ) and planar elevation (Ψ) given in (12) we obtain

$$\tan\chi = \frac{\eta_1 s\theta_0 s\eta_0 s\delta_1 + \alpha_1 s\beta_0 s\alpha_0 s\delta_3}{\eta_1 s\theta_0 c\eta_0 s\delta_1 + \alpha_1 s\beta_0 c\alpha_0 s\delta_3} \tag{A2}$$

and

$$\begin{aligned}\tan\Psi = {} & -b_z/[\theta_1 s\theta_0 + \beta_1 s\beta_0] \\ & \times [\eta_1^2 s^2\theta_0 s^2\delta_1 + \alpha_1^2 s^2\beta_0 s^2\delta_3 \\ & + 2\eta_1\alpha_1 s\theta_0 s\beta_0 c\gamma_0 s\delta_1 s\delta_3]^{1/2}.\end{aligned} \tag{A3}$$

Regarding the expression for azimuth χ, one can make the following observation: The mean value η_0 of arm yaw is generally positive, while that of forearm yaw α_0 is usually negative. Since η_1, α_1, $s\beta_0$ and $s\theta_0$ are all positive, the two terms in the numerator in (A2) will tend to cancel when $s\delta_1$ and $s\delta_3$ (the phases of η and α relative to θ) have the same sign and χ will be close to 0°. When $s\delta_1$ and $s\delta_3$ have the opposite sign, the terms in the denominator will tend to cancel and χ will be close to 90°, in agreement with the simulation results presented in Fig. 9.

The expression for planar elevation Ψ is more complicated. However, considering the numerator in (A3), one can see that Ψ will be small if $\theta_1 c\theta_0$ and $\beta_1 c\beta_0$ have the same sign and cancel each other. (β_0 was usually less than 90° when wrist motion was close to the vertical plane.) One way to increase the size of the numerator and to decrease the denominator is to make β_1 small, β_0 approximately equal to 90° (i.e. the forearm horizontal) and to decrease θ_0 (i.e. to bring the upper arm closer to the vertical). In fact, this is what our subjects did most often when they were asked to draw ellipses in the horizontal plane. While we have not pursued this problem further, based on (A1) and (A3) it does appear feasible to regulate the planar elevation of wrist motion by varying the mean and amplitude of the modulation of the angular elevation of arm and forearm (i.e. θ_0, θ_1, β_0, β_1) and maintaining a fixed phase difference of 180° between these two angular motions.

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