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Analysis of passive motion characteristics of the ankle joint complex using dual Euler angle parameters

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Abstract

Objective. To apply the dual Euler angles method to investigate the passive motion characteristics of the human ankle joint complex.

Design. Three-dimensional kinematic data of the ankle joint complex was collected from 10 knee-below foot cadaver specimens.

Background. Besides the Euler angles and screw axis methods, the dual Euler angles method has been proposed as an alternative approach to quantify general spatial human joint motion. The dual Euler angles method provides a way to combine rotational and translational joint motions and to interpret motions in Cartesian coordinate systems, which can avoid the problems caused by the use of the joint coordinate system due to non-orthogonality.

Methods. A non-metal experimental setup was fabricated to generate motion in foot cadaver specimens. The kinematic data during passive dorsiflexion–plantarflexion was measured using an electromagnetic tracking device.

Results. The kinematic coupling characteristics and the respective contribution of the ankle joint and the subtalar joint to the gross motion of the foot with respect to the shank were analyzed based on dual Euler angle parameters. The results obtained in this study are generally in agreement with the observations reported previously.

Conclusions. The dual Euler angles method is suitable for analyzing the motion characteristics of the ankle joint complex. The motion at the ankle joint complex involves rotations about and translations along three axes.

Relevance

Our finding using the dual Euler angle methods allows precise identification of the respective contributions of the subtalar joint and the ankle joint during the motion of ankle joint complex as a whole. © 2003 Elsevier Ltd. All rights reserved.

Keywords: Ankle joint complex; Kinematics; Dual Euler angles; Flock of Birds

1. Introduction

The ankle joint complex consists of the ankle (talocrural) joint and the subtalar (talocalcaneal) joint. The gross motion between the foot and the shank is the result of the motions at the ankle joint complex. Because the ankle joint complex is crucial to human locomotion, accurate knowledge on the kinematics of these joints is essential for the proper diagnosis and treatment of injuries and diseases in this region, and for the design of effective and reliable prosthetic devices. analyze the kinematic characteristics of the ankle joint complex in vitro (Engsberg, 1987; Siegler et al., 1988; Stähelin et al., 1997; Leardini et al., 1999) or in vivo (Lundberg, 1989; Buczek and Cavanagh, 1990; Keppel et al., 1990; Kitaoka et al., 1997). In the previous studies, the Euler angles and screw axis methods are two widely used methods to represent ankle joint complex motions quantitatively (Chao, 1980; Tupling and Pierrynowski, 1987; Ramakrishnan and Kadaba, 1991). Each method has its advantages and disadvantages. The screw axis method is not comparable with clinical motion description and does not facilitate communication between engineers and clinicians, though it can describe full six-degree-of-freedom joint motions. Euler angles

Numerous investigations have been carried out to

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can only describe the rotation of a segment and an additional three-dimensional position vector is required to describe the translation. The position vector is referred with respect to the coordinate system of the fixed segment while the Euler angles are usually referred with respect to the coordinate system of the moving segment. The difference of the reference systems for rotation and translation does not facilitate the interpretation of the parameters. As a variation of the Euler angles method, Grood and Suntay (1983) proposed a non-orthogonal joint coordinate system (JCS) to avoid some of the difficulties inherent in the use of Euler angles. Though JCS combines rotation and translation, it is not orthogonal. Non-orthogonality presents a serious problem when joint forces and moments need to be determined (Zatsiorsky, 1998).

As an alternative method for quantifying general spatial human joint motions, the dual Euler angles method has been proposed and applied to study the gross motion between the foot and the shank in vivo by Ying and Kim (2002) recently. In this method, the gross motion of the foot relative to the shank is represented by three screw motions through the coordinate axes of the Cartesian coordinate system fixed in the foot. In this way, the rotation and translation of the foot are combined and interpreted with respect to the same Cartesian coordinate system. Moreover, the dual Euler angles method has also an advantage over JCS because of its orthogonality.

Because tracking the motion of the talus using noninvasive in vivo techniques is not possible, the respective contributions of the ankle joint and the subtalar joint to the gross motion between the foot and the shank cannot be determined. Studies by Siegler et al. (1988) have indicated that the gross motion between the foot and the shank is the result of motion at both the ankle joint and the subtalar joint. In this study, based on the kinematic data collected from 10 foot cadaver specimens the respective motions at the ankle joint and the subtalar joint and their contribution to the gross motion produced between the foot and the shank were analyzed using dual Euler angle parameters.

2. Methods

2.1. Measurement device and experimental setup

The passive motions at the ankle joint complex were measured using the 'Flock of Birds' (FOB) electromagnetic tracking system (Ascension Technology Inc., Burlington, Vermont, USA). The tracking system, consisting of four receivers and one standard range transmitter, was set at its default configuration (103 Hz, AC wide filter on, DC low pass filter on) during the measurements. Three of the four receivers were used to measure the position and orientation of the bones and one was fixed on a stylus to digitize the position of points of interest. The accuracy of the measurement system has previously been reported by various authors (Bottlang et al., 1998; Bull et al., 1998; Meskers et al., 1999). In the authors' previous work, the accuracy of dual angles obtained from the FOB system was also evaluated as less than 1° and 1 mm for rotations and translations respectively (Ying and Kim, 2002).

To install the receivers into the bones rigidly, receiver fixtures as shown in Fig. 1 were fabricated. Each of receiver fixtures consists of an acrylic square plate, which was used for attachment of a receiver, and a plastic screw, which was used for insertion into a bone. The square plate and the screw were glued together tightly.

An experimental setup made of non-metal materials was used to generate passive motions to shank/foot cadaver specimens, as shown in Fig. 2. This is similar to the design of Stähelin et al. (1997). The foot plate, which is secured on the supporting bracket via a vertical screw in the bracket, is able to rotate around a horizontal axis in the supporting bracket. The supporting bracket can be secured on the ground plate in any direction by a vertical screw in the ground plate. By changing the direction of the foot plate with respect to its supporting bracket and the direction of the supporting bracket with respect to the ground plate, the foot plate can be placed at any orientation with respect to the ground plate to introduce different angular movements to the foot. The shank rod passes through a hole in the horizontal beam, which is in the anterior-posterior direction. The diameter of the hole is larger than that of the shank rod so that the shank rod can move and rotate in the hole. The position of the shank rod in the horizontal plane can be adjusted by moving the anterior-posterior beam in the



Fig. 1. Fixture for installing a receiver into a bone (unit: mm).



Fig. 2. Experimental setup for in vitro experiments on foot/shank specimens. 1: Vertical stands; 2: beams supporting shank rod; 3: shank rod; 4: foot plate; 5: screw securing foot plate on supporting bracket; 6: horizontal axis of the foot plate; 7: supporting bracket; 8: screw securing supporting bracket on ground plate; 9: ground plate.

horizontal plane. Vertically, the distance of the horizontal beams to the ground plate can also be adjusted to adapt to the heights of cadaver specimens.

2.2. Experiment procedure

The passive dorsiflexion-plantarflexion was measured without considering weight and muscle forces. Ten below-knee amputation cadaver specimens were tested.

Prior to the experiment, a specimen was thawed for 24 h at room temperature. At the time of the testing, the specimen was prepared as described below. A hole with 9 mm diameter, a little smaller than the diameter of the lower part of the shank rod, was drilled along the medullary channel to allow insertion of the shank rod. The specimen was dissected carefully to remove soft tissues around the ankle joint complex while the ligamentous system remained intact. The soft tissues at the head of the fibula and the tibia tuberosity were also removed. Four landmarks for definition of the tibiafibula anatomical coordinate system were identified and marked for digitization—the distal apex of the medial and lateral malleolus (MM, LM), the apex head of the fibula (HF), and the prominence of the tibia tuberosity (TT). The screws of the receiver fixtures were inserted into the tibia, the talus, and the calcaneus respectively: the one into the distal part of the tibia was inserted anteriorly and medially; the one into the talus at the antermedial aspect of the neck, and the one into the calcaneus at the lateral side of the heel.

The experimental setup was adjusted as follows. The bracket supporting the foot plate was aligned and secured on the ground plate with the horizontal axis in the medial-lateral direction. The foot plate was then secured on the supporting bracket with its longitudinal axis in the anterior-posterior direction. After the setup was adjusted to the appropriate position, the specimen with three receivers attached was mounted to the setup. The foot was fixed on the foot plate in the anterior-posterior direction and the ankle axis was approximately parallel to the horizontal axis. The shank rod was then inserted into the medullary channel. The position of the anterior-posterior beam in the horizontal plane was adjusted to align the shank rod along with the longitudinal axis of the tibia in vertical position, that is, perpendicular to the foot. This position was considered as the neutral position, with respect to which the motion of the joints would be described. The foot plate was placed horizontally, so as to have the foot in a plantigrade and neutral position.

At the neutral position, each of the four landmarks was digitized. At this joint configuration, the position and orientation of the receivers attached to the bones were also recorded. After collecting the required data at the neutral position, while the foot plate was rotated manually and slowly from maximum dorsiflexion of the specimen to maximum plantarflexion, the position and orientation of the receivers were recorded continuously.

2.3. Data analysis

To describe the passive motions at the ankle joint complex, the anatomical coordinate system of the tibia as proposed by Cappozzo et al. (1995) is used, which is based on the four landmarks on the shank—MM, LM, HF, and TT. The coordinate system of the tibia is defined as

- Origin is at the midpoint of the line joining MM and LM.
- *Y*-axis is orthogonal to the quasi-frontal plane defined by MM, LM, and HF.
- Z-axis is orthogonal to the quasi-sagittal plane defined by the Y-axis and TT.
- X-axis is the cross-product of the Y- and Z-axis.

At the neutral position, the coordinate systems of the talus and the calcaneus coincide with that of the tibia in both origins and the directions of axes. Fig. 3 schematically shows the coordinate systems used for analyzing passive motions of the ankle joint complex.

The relative motion of the calcaneus with respect to the tibia, that is, the gross motion of the foot with respect to the shank, the relative motion of the talus with respect to the tibia, that is, the motion at the ankle joint, and the relative motion of the calcaneus with respect to the talus, that is, the motion at the subtalar joint, were obtained by using the dual Euler angles method. The



Fig. 3. Coordinate systems for analyzing passive motions of ankle joint complex (X-Y-Z): coordinate system of the tibia; $x_t-y_t-z_t$: coordinate system of the talus; $x_c-y_c-z_c$: coordinate system of the calcaneus).

sequence of screw motions is selected as first through the *z*-axis, then through the *y'*-axis, and finally through the *x''*-axis of the moving coordinate systems. According to the definition of the coordinate systems along with the sequence of screw motions adopted in this study, the screw motion through the *z*-axis can be considered as the flexion–extension and lateral–medial shift of the bones. Similarly the screw motion through the *y'*-axis reflects the inversion–eversion and anteroposterior drawer of the bones. Finally the screw motion through the *x''*-axis can be interpreted as the abduction–adduction rotation and distraction–compression of the bones.

Though dual Euler angles can be obtained based on point coordinates (Ying and Kim, 2002), the method suitable for using raw data collected from the FOB is provided as follows. First, using the data measured by the FOB system, at any joint configuration, the position vector d_b and rotation matrix $[R_b]$ of a bone relative to the global coordinate system fixed in the transmitter are computed as

$$[R_{b}] = [R_{s_b}^{N}]^{-1}[R_{b}^{N}][R_{s_b}]$$

$$d_{b} = d_{s_b} - [R_{b}][R_{b}^{N}]^{-1}(d_{s_b}^{N} - d_{b}^{N})$$
(1)

where $[R_b^N]$ and d_b^N are the rotation matrix and position vector of the bone with respect to the global coordinate system when the joint complex is at the neutral position; $[R_{s_b}^N]$ and $d_{s_b}^N$ are the rotation matrix and position vector of the receiver on the bone with respect to the global coordinate system when the joint complex is at the neutral position; $[R_{s_b}]$ and d_{s_b} are the rotation matrix and position vector of the receiver on the bone with respect to the global coordinate system when the joint complex is at any joint position.

Let $[R_T]$ and d_T denote the rotation matrix and position vector of the tibia with respect to the global coordinate system. And $[R_{Ta}]$ and d_{Ta} represent the rotation matrix and position vector of the talus with respect to the global coordinate system respectively. At any joint position, the dual-number transformation matrix of the talus with respect to the tibia $[{}^T\widehat{R}_{Ta}]$ is given as

$${}^{\mathrm{T}}\widehat{R}_{\mathrm{Ta}}] = [{}^{\mathrm{T}}R_{\mathrm{Ta}}] + \varepsilon [{}^{\mathrm{T}}\widehat{S}_{\mathrm{Ta}}]$$
$$= ([R_{\mathrm{T}}]^{-1}[R_{\mathrm{Ta}}]) + \varepsilon ([D_{\mathrm{Ta}}][R_{\mathrm{T}}]^{-1}[R_{\mathrm{Ta}}])$$
(2)

where $[{}^{T}R_{Ta}]$ and $[{}^{T}S_{Ta}]$ are the primary and dual parts of the dual-number transformation matrix of the talus relative to the tibia; $[R_{T}]$ and $[R_{Ta}]$ are the rotation matrix of the tibia and the talus with respect to the global coordinate system respectively; $[D_{Ta}] =$

 $\begin{bmatrix} 0 & -d_3 & d_2 \\ d_3 & 0 & -d_1 \\ -d_2 & d_1 & 0 \end{bmatrix}$ is a skew symmetric matrix given by $\{d_1 \ d_2 \ d_3\}^{\mathrm{T}} = [R_{\mathrm{T}}]^{-1}(d_{\mathrm{Ta}} - d_{\mathrm{T}}).$

At the same joint position, the dual-number transformation matrix of the calcaneus with respect to the tibia $[{}^{\mathrm{T}}\widehat{R}_{\mathrm{C}}]$ can be calculated similarly. Subsequently, at any joint configuration, the dual-number transformation matrix of the calcaneus with respect to the talus $[{}^{\mathrm{Ta}}\widehat{R}_{\mathrm{C}}]$ was computed according to the following equation

$$[^{\mathrm{Ta}}\widehat{R}_{\mathrm{C}}] = [\widehat{R}_{\mathrm{Ta}}]^{-1}[\widehat{R}_{\mathrm{C}}] \tag{3}$$

Once the dual-number transformation matrices were obtained, the dual Euler angles were computed according to the relationship between the dual-number transformation matrix and dual Euler angles (Ying and Kim, 2002). Before computing dual-number transformation matrices, the raw data obtained from the FOB were smoothed using the dynamic programming and generalized cross-validation method to reduce the noise as originally proposed by Dohrmann et al. (1988).

3. Results

(deg)

Figs. 4 and 5 show the relative motion of the calcaneus and the talus with respect to the tibia during the dorsiflexion–plantarflexion. The relative motion of the calcaneus with respect to the talus is shown in Fig. 6. In the figures, the dual Euler angle parameters are plotted against the primary motion, that is, the flexion–extension angle of the foot relative to the shank, which is the

(nah)

rotation angle about the z-axis of the calcaneus with respect to the tibia.

To analyze the kinematic coupling characteristics and the respective contributions of the ankle and subtalar joints to the gross motion of the foot, the average dual Euler angles of the 10 specimens at the maximum range of the dorsiflexion–plantarflexion were obtained. The results indicate that during the dorsiflexion–plantarflexion, at maximum plantarflexion of the foot, 29.2° of plantarflexion of the calcaneus with respect to the tibia is associated with 27° of plantarflexion at the ankle joint and with 2.1° of plantarflexion at the subtalar joint. At maximum of dorsiflexion of the foot, 18.6° of dorsiflexion of the calcaneus relative to the tibia is associated with 19.1° of dorsiflexion at the ankle joint and with 0.6° plantarflexion at the subtalar joint.

While the foot moving from the neutral position to maximum plantarflexion, the plantarflexion of the foot is coupled with about 3.7° of inversion and 5.8° of adduction. Besides rotations, the foot also moves in the lateral direction about 0.7 mm, in the anterior direction about 7.5 mm, and in the proximal direction about 4.4 mm. At the ankle joint, besides the plantarflexion of about 27°, there exists about 1.9° of inversion, 3.2° of adduction, 0.3 mm of lateral shift, 6.2 mm of anterior drawer, and 5.3 mm of compression simultaneously. At the subtalar joint, besides 2.1° of plantarflexion, the calcaneus also undergoes about 1.9° of inversion, 2.5° of adduction, 0.9 mm of lateral shift, 1.4 mm of anterior drawer, and 1 mm of distal translation.

hib specimen1 to the tibia rolative to the specimen2 specimen3 anitala⁻ specimen4 about v-axis of the calcaneus specimen5 specimen6 of the specimen7 specimen8 about specimen9 angle otation angle specimen10 10 1000 ibia to the tibia along x-axis of the calcaneus relative to the tibia 10 to the t elative relative along z-axis of the calcaneus The second second second second of the 10 -20 10 20 30 rotation angle about z-axis of the calcaneus relative to the tibia (deg) rotation angle about z-a

Fig. 4. Relative motion of the calcaneus with respect to the tibia during the dorsiflexion-plantarflexion.



Fig. 5. Relative motion of the talus with respect to the tibia during the dorsiflexion-plantarflexion.



Fig. 6. Relative motion of the calcaneus with respect to the talus during the dorsiflexion-plantarflexion.

While the foot moving from the neutral position to maximum dorsiflexion, the dorsiflexion of the foot is coupled with about 2.5° of eversion and about 4.5° of abduction. Meanwhile, the foot also moves in the medial

direction about 0.7 mm, in the posterior direction about 4.9 mm, and in the distal direction about 1.2 mm. At the ankle joint, with respect to the tibia, besides the dorsi-flexion of about 19.1° , the talus also everts about 2° ,

4. Discussion

relatively small.

In the past, lots of effort has been devoted to analyzing the motion characteristics of the ankle joint and the subtalar joint and their contributions to the gross motion between the foot and the shank. The results obtained by researchers such as Sammarco et al. (1973), Engsberg (1987), Siegler et al. (1988), and Lundberg et al. (1993) have questioned the view that the ankle and subtalar joints were uniaxial, ideal hinge joints, held by Hicks (1953), Isman and Inman (1969), and Inman (1976). From Figs. 4–6 it can be seen that during the dorsiflexion-plantarflexion, both the ankle joint and the subtalar joint rotate about and translate along the three axes simultaneously. The kinematic results from this in vitro study indicate that both the ankle joint and the subtalar joint have six-degree-of-freedom and their motions demonstrate multiaxial motion characteristics, which is in agreement with what reported by Engsberg et al. (1987).

rotation and translation occur as well, the values are

Manter (1941) modeled the subtalar joint as a spiral of Archimedes. The results from this study show that the pitches of the screw motions through the three axes vary during the movements and cannot be fitted linearly, which means that it may not be accurate enough to model the subtalar joint as a screw with a fixed pitch and rotation axis when detailed motion characteristics of the joint is required.

The results also indicate that the movements of dorsiflexion-plantarflexion of the foot result from the combination of both the motions at the ankle joint and the subtalar joint. However, during the dorsiflexionplantarflexion, the motion of the calcaneus with respect to the tibia occurs mainly at the ankle joint with little motion at the subtalar joint, which is similar to those observed by Siegler et al. (1988) and Leardini et al. (1999). There is also a significant anteroposterior translation of the talus during this movement, indicating a significant rolling motion, and a lesser amount of vertical motion also. In summary the result supports previous descriptions of the ankle-subtalar complex. The ankle joint is primarily a hinge joint, but also has a significant amount of translational motion. It is easy to understand in this context why the early constrained prostheses failed. For example, plantarflexion is associated with the contingent anterior translation in the normal ankle. With the use of a rigid hinged prosthesis, plantarflexion would be associated with an anteriorly directed force due to the surrounding ligaments, predicting an early failure.

In this study, the in vitro measurements on foot cadaver specimens were performed under conditions without weight and muscle forces applied. The motions of the foot with respect to the shank were also produced by applying moments without forces. Further investigation on the motions of the ankle joint complex responding to other loading conditions such as applying forces in the axial, anterior–posterior, and medial– lateral direction is still needed.

The results based on the dual Euler angle parameters indicates kinematic characteristics of the ankle joint complex consistent with the literature. Compared with Euler angles and screw axis methods, this method provides an alternative approach for describing general spatial human joint motions by decomposing the motions into the the axes of orthogonal Cartesian coordinate systems, which will benefit kinetic studies. The similarity and relationship of dual-number transformation matrices and rotation matrices provide a convenient way for obtaining dual angles when the FOB is used as measurement system.

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