

The accuracy of joint surface models constructed from data obtained with an electromagnetic tracking device

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Abstract

Electromagnetic tracking devices are widely used in biomechanics. In this article a method is evaluated to construct models of articular surfaces using an electromagnetic tracking device. First, the accuracy of the space tracker was examined and optimised. Then, from several joint surfaces random points were measured and eighth degree polynomials were fitted to these measurements. To check if the fit converged well, plots of cross sections of the model with corresponding data points were examined. The accuracy of the models was determined by comparing them with computed tomography data and by reproducibility tests. All the fits converged well to the data. The root mean square (RMS) error of the models varied from 0.07 to 0.18 mm, and was proportional to the size and complexity of the surface. This was mainly due to systematic errors made by the space tracker, which were also proportional to the size and complexity of the surface. © 2000 Published Elsevier Science Ltd. All rights reserved.

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1. Introduction

The general approach to model articular surfaces is to measure the position of a series of surface points, and to construct a mathematical function that matches these points (e.g. Huiskes et al., 1985; Ghosh and Poirier, 1987; Hirokawa, 1991). The instruments used for the measurements were mostly optical (e.g. Ateshian et al., 1994; Blankevoort and Huiskes, 1996) or mechanical (e.g. Scherrer and Hillberry, 1979). An accuracy of less than 0.09 mm (95% confidence level) has been reported for optical measurements (Ateshian et al., 1991). Mechanical instruments are available in a wide range of precisions. Scherrer and Hillberry (1979) reported a precision of 0.01 mm. Electromagnetic instruments are not costly, widely used in biomechanics, and can sample the surface points data fast. They are, however, seldom used to reconstruct articular surfaces (Hefzy and Yang, 1993). This is probably due to their poor precision, and the lack of a method to compensate for this inaccuracy. Generally a RMS error of 1.5 mm or worse is reported by the

specifications (with a standard normal error distribution a RMS error of 1.5 mm corresponds to a 95% confidence level of 3 mm), but this can be improved (An et al., 1988; Zoghi et al., 1992; Luo et al., 1996; Milne et al., 1996);

The mathematical models can be subdivided into interpolating and approximating models. Interpolating models connect the data points with smooth functions. Examples are Coons' blended patches (Scherrer and Hillberry, 1979) and basic splines (Ateshian, 1993). Extra calculations are required when they are used with noisy data. Approximating models start with a general function, containing several constants. For every surface the constants are determined by fitting the function to the data points (Ateshian et al., 1991). A problem of this method is that beforehand it is unknown if a general function will converge well enough. This may lead to systematic errors such as smoothing of small undulations and sharp edges. Recently, Van Ruijven et al. (1999) developed a method to fit polynomial functions with random surface points. By this method, data is automatically filtered and the accuracy of their models can be improved by increasing the number of data points. This makes the method very suitable for high-density measurements (i.e. the distances between the data points is smaller than the noise), as can easily be obtained with an electromagnetic tracking device.

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In the present study the accuracy of that method applied to several articular surfaces was determined. Data points were measured with an electromagnetic tracking device. First, the precision of the tracking device was analyzed in more detail. Then a number of articular surfaces was measured, and models were fitted to these measurements. Finally, the precision of the models was evaluated by comparing one of the models with a micro-computed tomography (CT) scan of a surface and by analyzing the reproducibility of the method.

2. Material and methods

2.1. Accuracy of the tracking device

The 3SPACE® FASTRAK™ System (Polhemus Inc., USA) was used to measure all surfaces. This instrument measures the three-dimensional position of the tip of a stylus and transfers these co-ordinates to the computer. The surfaces were measured by moving the tip of this stylus over the surface while continuously recording its position with a frequency of 30 Hz.

The accuracy of the instrument can be enhanced by limiting the measurement space (An et al., 1988; Luo et al., 1996; Milne et al., 1996; Bull and Amis, 1997). To find the optimal spatial volume for the measurements, a number of tests was done with a Plexiglas cylinder and a Plexiglas spherical cavity. The size of the resulting volume was $50 \times 100 \times 100 \text{ mm}^3$, which was in agreement with the region used by Luo et al. (1996). Furthermore, the offset of the tip of the stylus along the axis of the stylus was calibrated. This was done by fitting cylinders and spheres with variable radii through the data points. The mean difference between the fitted radii and the real radii was used to correct the offset of the stylus. The precision of the instrument for the measurement of surfaces in the optimal domain was determined by measuring the cylinder and the spherical cavity at 14 and 8 different positions, respectively. Mathematical models of these shapes with the previously mentioned dimensions were fitted to the data points, and finally the distances between the data points and the fitted surfaces were calculated. The RMS of these distances was used as an estimate for the precision of the system when measuring surfaces.

2.2. Construction of articular surface models

The surfaces of the tibiofemoral joint (femur: condylus medialis, condylus lateralis, facies patellaris; tibia: condylus medialis, condylus lateralis) and the shoulder joint (cavitas glenoidalis, caput humeri) obtained from a human cadaver were used for the testing. In addition, the caput mandibulae of a dried skull was used. The articular surfaces of the femur and the tibia were measured and modelled separately. In order to know the positions of

these different surfaces relative to each other, a set of reference points was applied to the bones. Before the joint was opened, the positions of these points were measured relative to each other. Later the surfaces were measured relative to these reference points. In this way the different articular surfaces of a joint could be positioned relative to each other. The number of random data points ranged from 3000 to 11,000, depending on the size of the articular surface. The scanning of a surface varied from 2 to 10 min.

The method described by Van Ruijven et al. (1999) was used to construct models of the surfaces. In this method a surface s is modelled by eighth-degree polynomial functions

$$S = \left(\sum_{i,j=0}^{i+j \leq 8} a_{ij} u^i v^j, \sum_{i,j=0}^{i+j \leq 8} b_{ij} u^i v^j, \sum_{i,j=0}^{i+j \leq 8} c_{ij} u^i v^j \right).$$

Here u and v are the parameters of the surface, and a_{ij} , b_{ij} and c_{ij} the constants that determine the shape of the surface. The constants are determined iteratively with a least-squares fit. The fitting error was defined as the RMS of the distances between the data points and the surface. These distances were not calculated exactly, but estimated with a special algorithm. This algorithm has a tendency to overestimate the distances. To check the convergence of the fit, plots of cross sections of the model were visually compared with data points from the cross sections.

2.3. Precision of the surface models

The caput mandibulae was also measured with micro-CT (Rüegsegger et al., 1996) with a voxel size of $34 \times 34 \times 34 \mu\text{m}^3$. From this scan the cranialmost part, including the articular area, was selected using an oblique cutting plane, after which the surface voxels were extracted. For each voxel with five or less neighbouring voxels (surface voxels) the three-dimensional position was calculated, and the set of voxels was positioned such that it matched the polynomial surface optimally. For this purpose, a transformation consisting of the three rotation angles and the three translation distances was calculated such that the difference between surface voxels and the polynomial surface was minimal. This difference was defined as the mean of the distances of all the individual surface voxels to the polynomial surface.

Reproducibility tests were done on the surfaces of the caput mandibulae and the cavitas glenoidalis. The caput mandibulae was measured twice at one position, and a few days later twice at another position. The cavitas glenoidalis was measured at three different positions. The surface models derived from these measurements were compared in pairs. The difference between two models was calculated by extracting approximately 2000 points

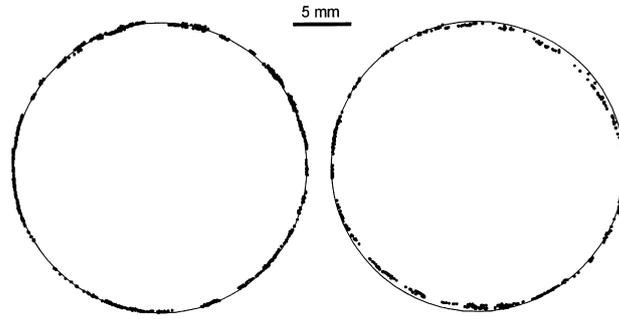


Fig. 1. A rectangular view on two cross sections of a fitted cylinder (continuous line) and the data points in this cross section (dots). The right panel illustrates the worst example of systematic deviations of the data points from the cylindrical shape, the left panel the best example.

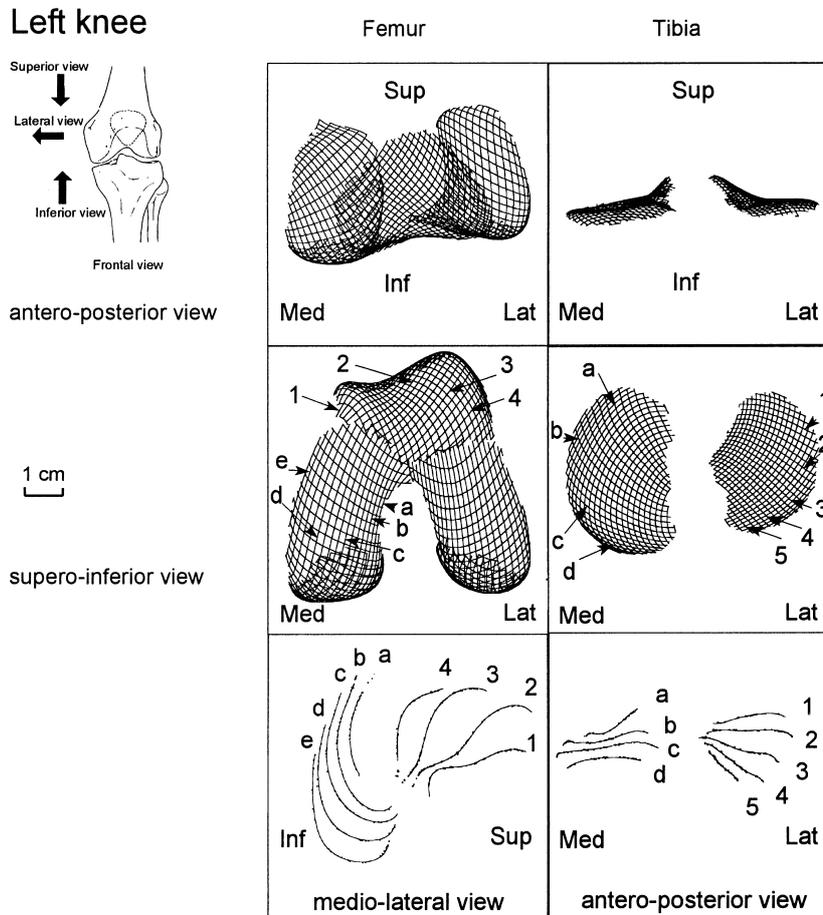


Fig. 2. The models of the surfaces of the knee joint seen from different directions. The color of all surfaces was set to transparent. The cross sections in the bottom panel together with many others were visually inspected to check the quality of the fit. In the two middle panels the positions of the cross sections are indicated ((Inf) inferior; (Sup) superior; (Med) medial; (Lat) lateral).

from the first model, and calculating the RMS distance from these points to the second model with the method described for the CT data. Statistically subtraction of two surfaces with an error of σ gives zero vectors, with an error of $\sigma\sqrt{2}$. Therefore we divided the RMS of the distances with $\sqrt{2}$ to obtain an estimation of the error of the model. The reproducibility tests will not reveal any convergence problems; these problems only depend on

the shape of the surface and not on the location where the surface was measured.

3. Results

The measurements of the cylinder and spherical cavity had a precision of 0.16 and 0.08 mm, respectively. The

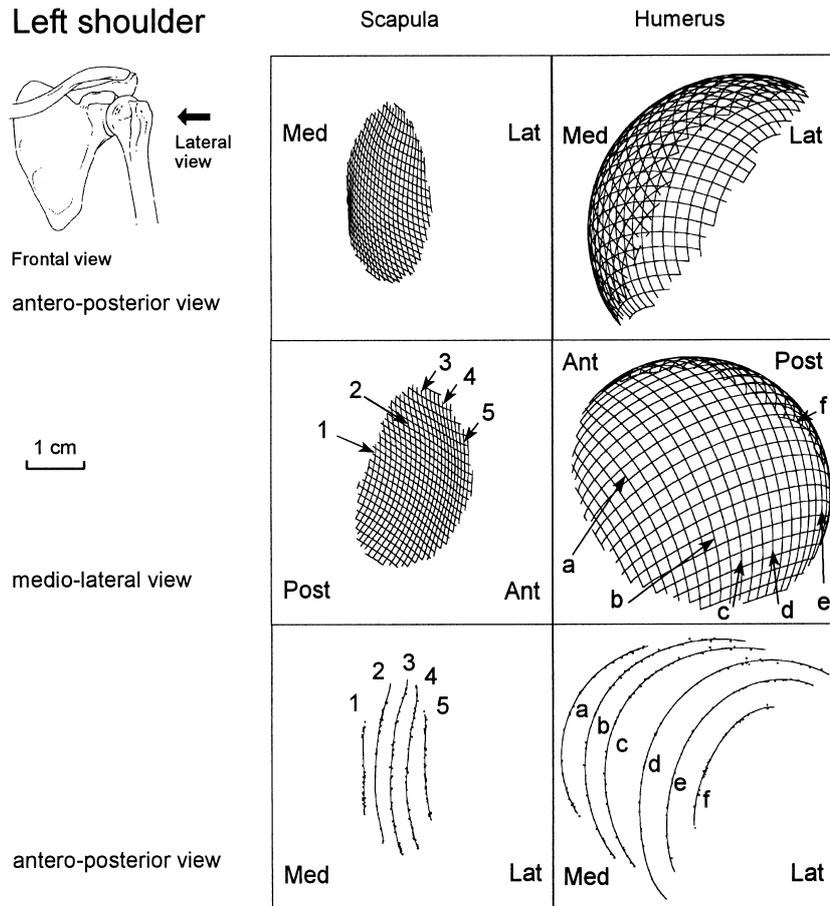


Fig. 3. The models of the surfaces of the shoulder joint are shown. In the bottom 2 panels cross sections of the model are shown together with the data points ((Med) medial; (Lat) lateral; (Post) posterior; (Ant) anterior).

visual comparison of the data points with the fitted surfaces revealed that at some positions in the measurement space the systematic errors in the measurements of the cylinder were larger than the random errors (Fig. 1). No systematic errors were found for the spherical cavity.

In Fig. 2 the surfaces of the knee joint including a few cross sections together with the corresponding data points are shown. The average fitting errors of the femoral surfaces were 0.26 mm (condylus lateralis), 0.19 mm (condylus medialis), and 0.12 mm (facies patellaris), and of the tibial surfaces 0.15 mm (condylus lateralis), and 0.12 mm (condylus medialis). The cross sections in Fig. 2, as well as many others analysed, showed good convergence of the fit. The models derived from the first measurement of the shoulder joint are shown in Fig. 3. The fit error of the caput humeri was 0.17 mm, and the average fit error of the cavitas glenoidalis was 0.20 mm. Again good convergence of the fit was observed in the cross sections. The reproducibility tests showed that the differences between the models of the cavitas glenoidalis were 0.06, 0.09 and 0.08 mm for the three pairs, respectively.

Fig. 4 shows the results of the first measurement of the caput mandibulae. The fit errors were 0.13, 0.18, 0.11 and

0.13 mm for the four measurements, respectively, and again the cross sections showed good convergence. The differences between the models of the caput mandibulae measured at the same position were 0.07 and 0.08 mm for the two positions, respectively. The mean difference of the four combinations measured at different positions was 0.07 mm, indicating an error of 0.05 mm. The differences between the four models and the CT scan were 0.07, 0.07, 0.06 and 0.08 mm, respectively. Also shown in Fig. 4 are cross sections of the surface model with points extracted from the CT scan. The largest difference was observed in the cross section marked "e"; this cross section, however, included a part of the surface area which did not belong to the articulating area. A similar artefact was found with the other three models of the caput mandibulae.

4. Discussion

4.1. Precision of the tracking device

The error of the space tracker consists of two components: a systematic error and a random error. The

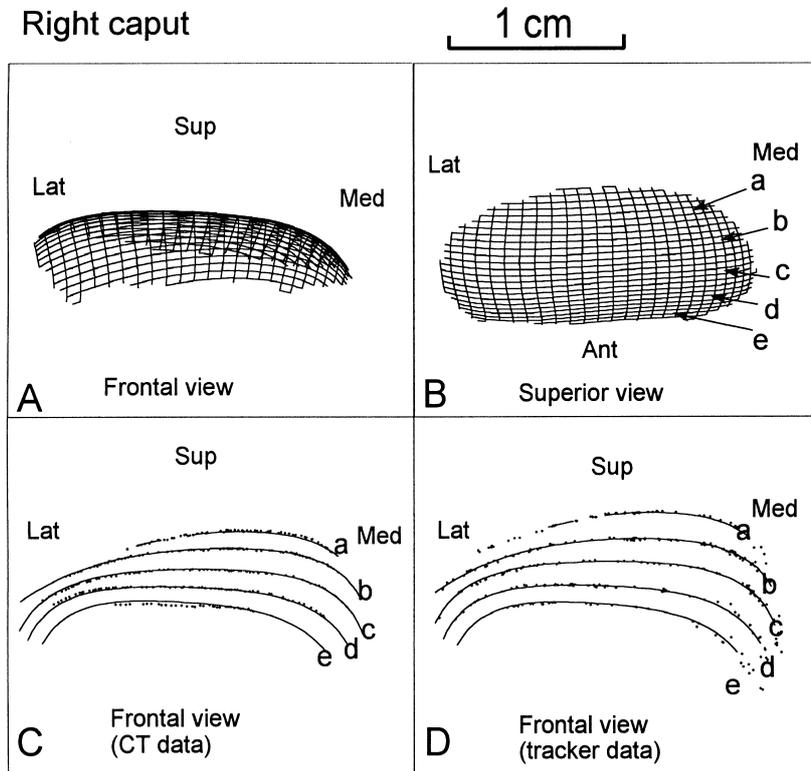


Fig. 4. A and B show the surface of the caput mandibulae from two directions. C and D show cross sections of the surface as indicated in 4B. In C these cross sections are accompanied with the surface points extracted from the CT scan. In D they are accompanied with the data points measured with the space tracker ((Sup) superior; (Lat) lateral; (Med) medial; (Ant) anterior).

systematic error appeared to be proportional to the size of the surface (especially the range of angles needed to measure a surface). In that case the cylinder reflects the worst case (it fits hardly in the selected region and the stylus has to be rotated over 360°) and the spherical cavity the best case. Assuming that for the spherical cavity the systematic errors can be neglected, then the random error can be calculated to be 0.08 mm, and the systematic error to be maximally 0.14 mm. So the total error can be estimated to range from 0.08 mm to 0.16 mm. A table with corrections for the systematic error will improve the precision of the space tracker considerably.

4.2. Precision of the models

The precision of the surface models is determined by the goodness of fit, and errors of the tracking device. To check the fit, the models were visually compared with the data points. With this comparison the maximal difference was estimated to be 0.08 mm (the random error of the data points). So the average smoothing is less than 0.08 mm. The random error of the tracking device is filtered by the usage of a least-squares fit, and its contribution to the model precision is substantially less than

0.08 mm (Van Ruijven et al., 1999). The systematic error of the tracking device ranged from almost 0.00 mm for small surfaces to 0.14 mm for large surfaces. So the total error in the models can be estimated to range from much less than 0.11 ($= \sqrt{0.08^2 + 0.08^2}$) mm for small surfaces to about 0.18 ($= \sqrt{0.14^2 + 0.08^2 + 0.08^2}$) mm for large surfaces. These estimations are in good agreement with the results presented. With reference to the CT data the precision of the models of the caput mandibulae ranged from 0.06 to 0.08 mm. The repetition tests gave a precision of 0.05 mm. For the cavitas glenoidalis the repetition tests showed the same result.

Until now only Scherrer and Hillberry (1979) did an extensive analysis of the precision of their surface models. They measured the scapula of a dog and interpolated the data with Coons' bicubic patches. The precision of their measurements was 0.01 mm while their model had a precision of 0.05 mm (RMS). So in fact their method was less accurate than their measurements. The main advantage of electromagnetic tracking devices is that they can collect surface points with a high frequency. In the present study a method was described that eliminates the main drawback of these devices, i.e. their poor precision. It was shown that the models, which were fitted through the random measurements, can have a higher accuracy than the measurements themselves.

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