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Computer Simulation Tool for Predicting Sound Propagation in Air-Filled Tubes with Acoustic Impedance Discontinuities

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***Abstract*— A computer tool, based on an acoustic transmission line model, was developed for modeling and predicting sound propagation and reflections in cascaded tube segments. This subroutine considered the number of interconnected tubes, their dimensions and wall properties, as well as medium properties to create a network of cascaded transmission line model segments, from which the impulse response of the network was estimated. Acoustic propagation was examined in air-filled cascaded tube networks and model predictions were compared to measured acoustic pulse responses. The model was able to accurately predict the location and morphology of reflections. The developed code proved to be a useful design tool for applications such as the guidance of catheters through compliant air-filled biological conduits.**

I. INTRODUCTION

ACOUSTIC pulse reflectometry is a non-invasive measurement technique used mostly for determining the internal dimensions of ducts of varying cross-sectional areas. This method was originally developed by seismologists to calculate impedance differences between layers in the Earth's crust and for the location of the epicenter of earthquakes [1]. Acoustic reflectometry entails the generation of short duration acoustic pulses that travel axially in a duct. When the acoustic pulse encounters an impedance discontinuity, a portion of the pulse is transmitted past the discontinuity and a portion is reflected back towards the acoustic source. Acoustic reflections are then measured and analyzed to estimate the location of impedance discontinuities within the systems under investigation. Acoustic reflectometry has utility in a number of different fields, including measurement of the bores of musical instruments and the detection of leaks and fractures in pipes that transport liquids over large distances [2,3]. Biomedical applications of acoustic reflectometry include the guidance and monitoring of endotracheal tubes [4-5].

The objective of this research was to develop a MATLAB-based simulation tool to predict acoustic propagation and reflections in serially connected tube networks. An acoustic transmission line model was used to predict the distortion that occurred with axial propagation and the reflections that arose at impedance discontinuities. Pulse propagation was simulated and compared to the data obtained from an experimental acoustic reflectometry measurement system to assess the ability of the model to forecast attenuation, propagation speeds, and reflections in interconnected air-filled tubes.

II. BACKGROUND

A. Acoustic Transmission Line Model

The differential equations that govern planar sound propagation in a duct are the same as those that describe electrical propagation along an electrical transmission line, where acoustic pressure and volume velocity are analogous to voltage and electrical current, respectively. Therefore, the sound propagation and distortion in lossy compliant tubes can be predicted using an acoustical circuit analog [6]. An acoustic transmission line model (TLM) can be used to predict axial propagation in a lossy tube that has complex impedances. This model is based on Rayleigh's formulations for wave propagation in cylindrical ducts [7].

The medium properties are incorporated into the acoustic circuit as displayed in Fig. 1. The acoustic inertance, L_a, and acoustic compliance, C_a, represent the mass and the stiffness of the fluid, respectively. Acoustic resistance, R_a, represents viscous friction and acoustic conductance, G_a, represent the thermal losses at the tube-fluid interface [8].

Transverse wall motion in response to acoustic excitation of the fluid is accounted for via a shunt resistance-inertance-compliance (R-L-C) impedance. The wall resistance, R_w, represents the viscous losses that occur with radial wall motion; the inertance, L_w, and compliance, C_w, represent the mass and stiffness of the tube wall, respectively.

Using the transmission line model, planar acoustic propagation in any non-ideal system can be modeled in terms of the tube and fluid material properties. The characteristic impedance $Z_0(j\omega)$ for each segment as a function of the angular frequency (ω) is defined by:

$$Z_0(j\omega) = \sqrt{\frac{Z_a(j\omega)}{Y_a(j\omega) + Y_w(j\omega)}} \tag{1}$$

where $Z_a(j\omega)$ and $Y_a(j\omega)$ are the longitudinal impedance and radial admittance of the fluid, respectively, defined by:

$$Z_a(j\omega) = R_a + j\omega L_a \tag{2}$$

$$Y_a(j\omega) = G_a + j\omega L_a \tag{3}$$

The transverse wall admittance, $Y_w(j\omega)$, is defined in terms of

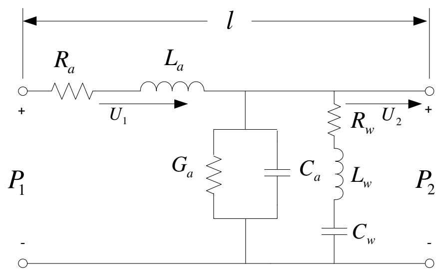

Fig.1. Transmission line model segment of length l, for non-rigid wall.

the wall impedance, $Z_\omega(j\omega)$, as:

$$Z_w(j\omega) = R_w + j\left(\omega L\omega - \frac{1}{\omega C_w}\right) \quad (4)$$

$$Y_w(j\omega) = \frac{1}{Z_w(j\omega)} \quad (5)$$

The impedances and admittances defined by (2) – (5) can be further utilized to describe how a propagating wave is distorted with axial propagation via the propagation coefficient, $\gamma(j\omega)$:

$$\gamma(j\omega) = \sqrt{Z_a(j\omega)(Y_a(j\omega) + Y_w(j\omega))} = \alpha(\omega) + j\beta(\omega) \quad (6)$$

where $\alpha(\omega)$ is the attenuation factor, which quantifies the rate of exponential decay that occurs with the propagation, and the phase coefficient, $\beta(\omega)$, is used to predict dispersion. The phase coefficient is used to calculate phase velocity, $V_p(\omega)$, which is the speed at which each frequency component of a wave propagates through the medium.

$$V_p = \frac{\omega}{\beta(\omega)} \quad (7)$$

Acoustic impedance discontinuities and the reflections that arise from them are modeled using cascaded transmission line model segments. The response of cascaded network to an acoustic excitation is predicted in terms of each TLM segment's input impedance, $Z_{in}(j\omega)$:

$$Z_{in}(j\omega) = \frac{Z_{out}(j\omega) + Z_0(j\omega)\cdot\tanh(\gamma(j\omega)\cdot l)}{1 + \dfrac{Z_{out}(j\omega)}{Z_0(j\omega)}\cdot\tanh(\gamma(j\omega)\cdot l)} \quad (8)$$

where $Z_{out}(j\omega)$ is the load impedance on a model segment. The equivalent impedance of a set of interconnected TLM segments is determined by iteratively calculating the input impedance of each segment. The resultant input impedance of the entire tube network is applied to (9) to predict the refection coefficient, $R(j\omega)$, of the interconnected tubes.

$$R(j\omega) = \frac{Z_{in}(j\omega) - Z_0(j\omega)}{Z_{in}(j\omega) + Z_0(j\omega)} \quad (9)$$

The acoustic TLM is most accurate at frequencies where only the planar mode propagates axially through the fluid. To ensure that only the planar mode is propagating, the fluid should be excited at frequencies less than the first nonplanar cuton frequency (f_{max}), defined by:

$$f_{\max} = \frac{c}{1.707D}(Hz) \quad (10)$$

where D is the tube diameter and c is the free field sound speed of the propagating medium. Also, the acoustic circuit model is valid if the segment length, l, is significantly less than the smallest acoustic wavelength of interest, which is determined by the frequency limitation imposed by the nonplanar cuton frequency limit

$$l << \frac{c}{f_{\max}}(Hz) \quad (11)$$

III. SIMULATIONS AND EXPERIMENTAL METHODOLOGY

A. Simulations

A MATLAB subroutine was developed that utilizes cascaded TLM segments to predict axial propagation and distortion in interconnected tubes with varying cross-sectional areas, wall material properties, and propagating media. The user is prompted to enter the number interconnected tubes, the dimensions of each tube, and the propagating media. The tool calculates f_{max} using (10). Each tube is then divided into an integer number of segments such that l is shorter than the maximum length requirement as defined by (11).

The equivalent input impedance of the entire tube network was determined by iteratively calculating the input impedance of each TLM segment. The input impedance of the final segment (m) was calculated first using (8). For the segment immediately proximal to the final segment, (m-1), the load impedance was set equal to the input impedance of the distal segment and the input impedance was again calculated using (8). This process was repeated until the input impedance of the first TLM segment, which corresponded to the equivalent impedance of the entire network, was calculated. The reflection coefficient, $R(j\omega)$ was subsequently calculated using the input impedance of the entire network, and substituting in (9).

Acoustic pulse propagation was simulated in order to examine the ability of the cascaded model to predict reflections that would arise at impedance discontinuities. Hanning pulses were designed with a user-defined duration, amplitude, and delay to the onset of the pulse. The reflection coefficient calculated using (9) was equivalent to the impulse response of the system. Therefore, the time domain impulse response $h(t)$ was determined by obtaining the inverse Fourier transform of the predicted reflection coefficient:

$$h(t) = \mathrm{Re}\left\{fft^{-1}\left[R(j\omega)\right]\right\} \quad (12)$$

The pulse response, $y(t)$, of the system was determined by convolution of the impulse response with the user-defined input pulse:

$$y(t) = x(t) * h(t) \quad (13)$$

where $x(t)$ is the input pulse. The resultant pulse response displayed the reflections that were expected to arise at impedance discontinuities in the simulated tube network.

B. Experimental Setup

The experimental setup consisted of a loudspeaker (Morel MDT 30) that was coupled to a rigid polyethylene tube through a plastic funnel. The polyethylene source tube was approximated as a rigid tube for modeling purposes. As shown in Figure 2, a microphone (Knowles EM-3046) was mounted such that its pressure sensing port was flush with the inner wall of the source tube, at a distance of 3.5 cm from the end of the tube. This microphone recorded the incident

TABLE I

TUBE MATERIAL PROPERTIES AND DIMENSIONS [9].

	LATEX$_1$ (TUBE I)	LATEX$_2$ (TUBE II)
ρ_w (g/cm^3)	1.4	1.4
η_w (dyne·s/cm^2)	2.54 x10^3	2.54 x10^3
E (dyne/cm^2)	16x10^9	16x10^9
d (cm)	0.647	0.647
h (cm)	0.119	0.317

pulses and reflections returning from the tubular object under test, which was coupled to the far end of the source tube. The source tube was long enough so that reflections arising from the proximal end of the tube did not overlap with the reflections of interest (those arising from the cascaded tubes).

An inverse filter technique was implemented to generate an acoustic Hanning pulse. Acoustic pulses of 150 μs in duration were transmitted through a long source tube utilized to probe the system under investigation.

Six different configurations were examined. The secondary tube of interest for each configuration consisted of a latex rubber tube, whose material properties were measured prior to the acoustic measurements [9]. Two latex tubes that had the same inner diameters (d) but differing wall thicknesses (h) were connected to the source tube to evaluate the effects of changing the impedance of the second tube on acoustic reflections (see Table I for tube dimensions and properties). The relevant physical properties of air can be found in [8]. The source tube was initially coupled to long (130 cm) latex tubes (see Fig. 3.a). Additionally, 4 cm and 12 cm segments that had the same wall thicknesses and inner diameters as Tube I were cascaded with the source tube and connected to a 130 cm length of polyethylene tube with the same dimensions as the source tube (see Fig. 3.b). The length of the tubes was selected in order to simulate an infinite tube length, therefore no additional reflections were expected.

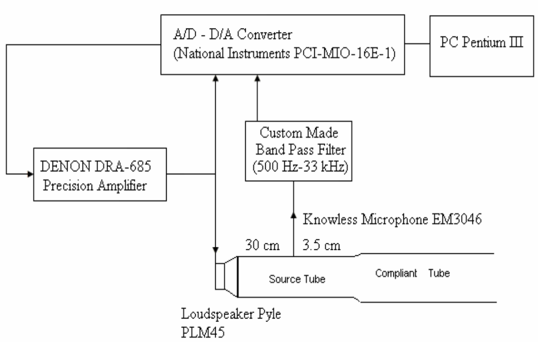

Fig. 2. Experimental setup utilized to perform the acoustic measurements.

The results presented were recorded over 16 trials and averaged in the time domain to improve the signal-to-noise ratio by approximately a factor of 4. The time delay (t_d), polarity, and amplitude (A) of these reflections were compared to the model predictions.

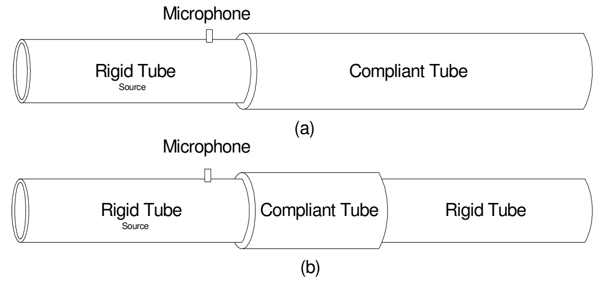

Fig. 3. Experimental setup utilized to perform the acoustic measurements: (a) The source tube is cascaded with the compliant tube. (b) 4 cm and 12 cm segments of the compliant tube are placed between long rigid tubes.

TABLE II

EXPERIMENTAL AND MODEL-PREDICTED TIME DELAYS AND REFLECTED PULSE AMPLITUDES

	Exp. t_d (μs)	Model t_d (μs)	Exp. A (V/V)	Model A (V/V)
Tube I	219	222	-.361	-.279
Tube II	217	220	-.354	-.277

IV. RESULTS AND DISCUSSION

The normalized pulse measurements and the model predictions obtained using the MATLAB subroutines are shown in Figure 4.

The latex tubes had lower acoustic impedance than the source tube; therefore a negative polarity reflection was expected to arise at the interface between both tubes. Experimentally, acoustic reflections were evident at some time after the incident pulses for both Tube I and II. The experimental and model predicted time delays and normalized reflected pulse amplitudes are listed in Table II. For both tubes, there was strong agreement between the predicted and experimental time delays, as predicted delays were within 1.4% of experimental values. The amplitudes, as seen in Figure 4, present great similarity to one another. Additionally, the model accurately predicted the polarities of the reflections that occurred at the decrease in impedance between the source and latex tubes. In both cases the predicted reflections followed the trend of occurring faster as the wall thickness increases.

To examine reflections in tubes with localized impedance discontinuities, pulses were simulated and measured with 4 cm and 12 cm long latex tube segments connected to the source tube and to another long rigid polyethylene tube (see Figure 3.b). In addition to the reflections that were expected to arise at the interface between the source and latex tubes, a second reflection was expected due to the incident pulse encountering the impedance discontinuity between the latex segment and the second polyethylene tube. A secondary reflection was expected to be detected at some time after the first reflection. The measured and model-predicted pulse responses for the 4 cm and 12 cm latex segments are displayed in Figure 5.

The time delays for the first reflections were the same as those obtained for the first experiment. The delay between the second reflection and the incident pulse was 437 μs for the 4 cm segment and 883 μs for the 12 cm segment of latex tube. The model showed delays of 435 μs and 920 μs for each tube, respectively. The errors between experimental and model-predicted delays were 4.2% and 0.46%, again indicating strong agreement between predicted and experimental time delays. As shown in Figure 5 the amplitude prediction for the first reflection was 0.244, whereas the measured value was 0.179. For the 12cm segment, the amplitude prediction was 0.139 and the measured value was 0.0995.

These values demonstrate the model's reliability for predicting reflections due to localized acoustic impedance discontinuities. Also, the model was able to account for the distortion due to propagation through the latex tubes. Notice that in the second reflections the predicted wave form exhibited the same type of distortion as the measurements, indicating that the model accurately predicted attenuation and dispersion in the latex segments.

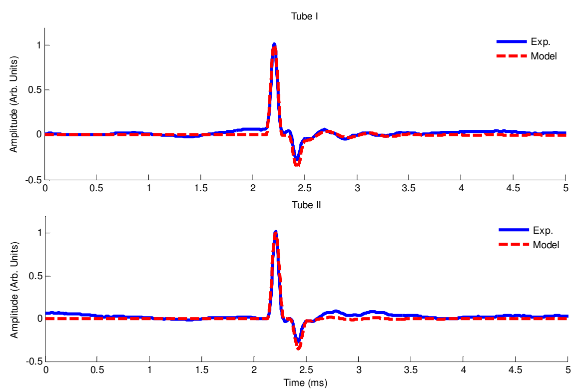

Fig. 4. Model Prediction and experimental data for Tube I (upper plot) and Tube II (lower plot) cascaded with the source tube.

V. CONCLUSIONS

Acoustic wave propagation in tubes with well-defined impedance discontinuities was predicted and studied using an acoustic transmission line model. With this model, the effect of any parameter variation could be easily determined in terms of its effects on the overall acoustic response of a system consisting of cascaded tube segments of various compositions and dimensions. These features are very important since the acoustic transmission model may be applied to suggest a frequency range for the use of acoustic pulse reflectometry in different tubes or media.

Based on the known dimensions of the tubes and locations of impedance discontinuities, the experimental reflections were well represented by the model-predicted reflections. The model accurately predicted the polarity, amplitudes, and time delays of reflections that occurred on interconnected air-filled tubes. All the time delays measurements stayed within a 22 percent error from model predictions. The maximum percent error between the predicted amplitudes and those measured experimentally was 46%.

Although the overall percent errors for both time delays and amplitudes were somewhat high, this tool can be used to help understand and predict the acoustic response of a series

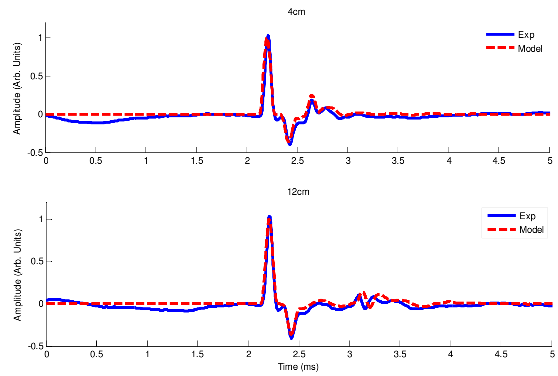

Fig. 5. Model prediction and experimental measurement comparison for 4cm (upper plot) and 12cm Tube I segments.

of air-filled interconnected tube segments. These experimental results indicate that this computer model could aid in the interpretation of sound reflections that result from complex systems, since it accounts for contractions, expansions and wall material variation. Also, the developed code proved to be a useful design tool for applications such as catheter guidance systems for air-filled biological conduits.

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