MATHEMATICAL MODELS OF VOCAL TRACT WITH DISTRIBUTED SOURCES

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ABSTRACT

Frequency domain models for the vocal tract with distributed sources are introduced. Pressure and volume velocity distributed sources are modeled within incremental lossy cylindrical pipes in a manner similar to that of modeling electrical components. Equations for transmission matrices of uniform-area vocal tract sections with distributed sources are derived. Transfer functions for pressure and volume velocity at various segments of the vocal tract are computed. Comparison of distributed source models with point source models reveal several improvements on the traditional lumped source modeling method and shows that the effects of the finite impedence constriction and back cavity cannot be adequately modeled using point sources. The distributed source vocal tract framework is important for building articulatory and acoustic models for fricative sounds and other phoneme categories. This paper provides such a mathematical framework for the first time.

1. INTRODUCTION

Turbulence noise generated in the production of fricatives has traditionally being modeled as a point pressure source some distance anterior to the constriction [1]. Other representations have included monopole volume velocity source, dipole flow source, and multiple point pressure sources [2].

Experimental studies indicate that the noise source is in fact distributed over some distance anterior to the constriction [3]. Lack of a mathematical framework for distributed sources has prevented from the realization of such a configuration. This paper develops a mathematical model for distributed pressure and flow sources in several stages. Section 2 derives distributed models for an incremental section of the vocal tract. Section 3 presents transfer functions for point source systems. Section 4 discusses the transfer functions for distributed sources. Section 5 compares distributed modeling to point source modeling for a vocal tract configuration of a fricative /s/. It is shown that point source models are inadequate in representing an accurate accoustic model.

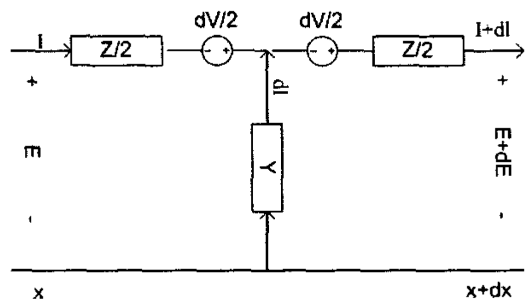

Fig. 1. Incremental section for distributed pressure.

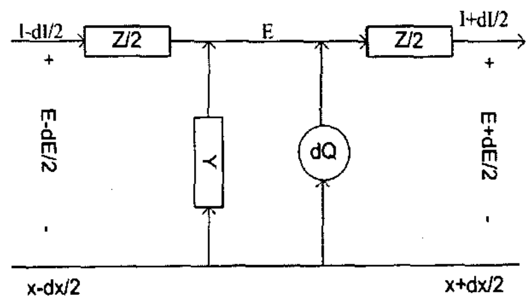

Fig. 2. Incremental section for distributed flow.

2. INCREMENTAL MODELS

The vocal tract is considered a contiguous lossy tube consisting of air surrounded by tissues such as the tongue, cheeks, pharyngeal wall, hard palate, and terminated by the glottis and radiation load at the mouth or nostrils. This three dimensional system is represented using one-dimensional acoustic wave equations for frequencies below 4000 Hz. The tube may be modeled as a concatenation of incremental lossy sections of uniform properties.

Figures 1 and 2 illustrate the proposed incremental section models in the frequency domain with distributed pressure source and flow source respectively. The acoustic pressure and volume velocity are $E(x,s)$ and $I(x,s)$ at distance x from the glottis and complex frequency s.

The passive properties of the incremental section are incorporated in the cross impedance per unit length, $Z(x,s)$, and shunt admittance per unit length, $Y(x,s)$ [4]. The propogation constant, γ, characteristic impedance, Z_0, and characteristic admittance, Y_0, are given by $\gamma = \sqrt{ZY}, Z_0 = \sqrt{Z/Y}, Y_0 = \sqrt{Y/Z}$.

We propose a frequency domain model for an incremental section with distributed pressure source and flow source in the next two subsections.

2.1. Pressure Source

The distributed pressure source is modeled as a pressure increase of $dV(x,s)$ across the incremental section of length dx, as illustrated in Figure 1. The differential equations are given below, where $V(x,s)$ is pressure per unit length:

$$\begin{aligned} dV(x,s) &= V(x,s)dx \\ \frac{d}{dx}\begin{bmatrix} E \\ I \end{bmatrix} &= \begin{bmatrix} 0 & -Z \\ -Y & 0 \end{bmatrix}\begin{bmatrix} E \\ I \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} V(x,s) \end{aligned}$$

The solution to the equation for a uniform (non-incremental) section of length l, uniform distribution of pressure source $V(l,s) = V_0(s)/l$, input pressure and volume velocity, E_1 and I_1, and output pressure and volume velocity E_2 and I_2 is as follows.

$$\begin{bmatrix} E_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} V_0$$

$p = \cosh(\gamma l)$, $q = -Z_0 \sinh(\gamma l)$, $r = -Y_0 \sinh(\gamma l)$, $s = \cosh(\gamma l)$, $e = \sinh(\gamma l)/(\gamma l)$, and $f = Y_0\{1 - \cosh(\gamma l)\}/(\gamma l)$.

2.2. Flow Source

The distributed flow source is modeled as an injection of volume velocity $dQ(x,s) = Q(x,s)dx$ into the incremental section of length dx, as illustrated in Figure 2. The differential equations are the same as that of the pressure source in the previous section, with $Q(x,s)$ replacing $V(x,s)$, and a source multiplier of $[0\ 1]^T$ instead of $[1\ 0]^T$

The input/output relations for a uniform section of length l, and uniformly distributed volume velocity source $Q(l,s) = Q_0(s)/l$, are similar to those for the distributed pressure source in the previous subsection, with the same p, q, r, s; Q_0 replacing V_0; and $e = Z_0\{1 - \cosh(\gamma l)\}/(\gamma l)$, $f = \sinh(\gamma l)/(\gamma l)$.

3. POINT SOURCE SYSTEM FUNCTION

We present the transfer function equations for the lumped (point) source models represented by the system coniguration of Figure 3, to provide comparison with the results of the distributed models derived in the next section. The diamond represents the part of the system consisting of the point source with an amplitude V_0 for a pressure source and amplitude Q_0 for a volume velocity source. The rectangle represents the passive part of the system governed by the following equation, and represents the relationship for a concatenation of uniform (non-incremental) tubes.

$$\begin{bmatrix} P_{out} \\ U_{out} \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} P_{in} \\ U_{in} \end{bmatrix}$$

3.1. Pressure Source

The transfer functions for a pressure source with infinite output impedence are as follows.

$$\begin{aligned} D &= (d - cZ_b) \\ P_{in}/V_0 &= d/D \\ P_{out}/V_0 &= 1/D \\ U_{in}/V_0 &= -c/D \\ U_{out}/V_0 &= 0/D \\ Z_{in} &= d/(-c) \end{aligned}$$

The system functions for an arbitrary output impedance are given below.

$$\begin{aligned} D &= [(aZ_b - b) - Z_e(cZ_b - d)] \\ P_{in}/V_0 &= (Z_e d - b)/D \\ P_{out}/V_0 &= Z_e/D \\ U_{in}/V_0 &= (a - Z_e c)/D \\ U_{out}/V_0 &= 1/D \\ Z_{in} &= (Z_e d - b)/(a - Z_e c) \end{aligned}$$

3.2. Flow Source

The transfer functions for a volume velocity source with input admittance, Y_b, and $Z_e = \infty$ are described below.

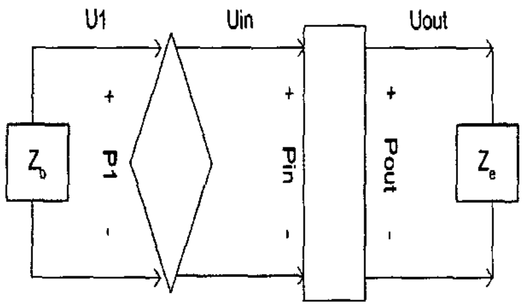

Fig. 3. System model for point and distibuted source.

$$
\begin{aligned}
D &= (Y_b d - c) \\
P_{in}/Q_0 &= d/D \\
P_{out}/Q_0 &= 1/D \\
U_{in}/Q_0 &= -c/D \\
U_{out}/Q_0 &= 0/D \\
Z_{in} &= d/(-c)
\end{aligned}
$$

The equations for a finite output impedance are given by:

$$
\begin{aligned}
D &= [(a - Y_b b) - Z_e(c - Y_b d)] \\
P_{in}/Q_0 &= (Z_e d - b)/D \\
P_{out}/Q_0 &= Z_e/D \\
U_{in}/Q_0 &= (a - Z_e c)/D \\
U_{out}/Q_0 &= 1/D \\
Z_{in} &= (Z_e d - b)/(a - Z_e c)
\end{aligned}
$$

4. DISTRIBUTED SOURCE SYSTEM FUNCTION

The transfer functions for distributed pressure and volume velocity sources are derived in this section by solving circuit equations resulting from the vocal tract system configuration in Figure 3 and utilizing the relations developed in section 2.

4.1. Pressure Source

The formulas for a distributed pressure source system with the load modeled as infinite impedance are:

$$
\begin{aligned}
N &= [(se - fq) + Z_b(fp - re)] \\
D &= [(cq + ds) - Z_b(cp + dr)] \\
U_1/V_0 &= -(ce + df)/D \\
P_1/V_0 &= Z_b(ce + df)/D \\
P_{in}/V_0 &= dN/D \\
U_{in}/V_0 &= -cN/D \\
P_{out}/V_0 &= N/D \\
U_{out}/V_0 &= 0/D \\
Z_{in} &= d/(-c)
\end{aligned}
$$

The equations resulting from a pressure source for the general case are given below:

$$
\begin{aligned}
N &= [(se - fq) + Z_b(fp - re)] \\
D &= [Z_b(ap + br) - (aq + bs)] \\
&\quad -Z_e[Z_b(cp + dr) - (cq + ds)] \\
U_1/V_0 &= [(ae + bf) - Z_e(ce + df)]/D \\
P_1/V_0 &= -Z_b[(ae + bf) - Z_e(ce + df)]/D \\
P_{in}/V_0 &= (Z_e d - b)N/D \\
U_{in}/V_0 &= (a - Z_e c)N/D \\
U_{out}/V_0 &= N/D \\
P_{out}/V_0 &= (Z_e N)/D \\
Z_{in} &= (Z_e d - b)/(a - Z_e c)
\end{aligned}
$$

4.2. Flow Source

The relationships for the infinite load impedance with a distributed flow source are:

$$
\begin{aligned}
N &= [(pf - er) + Y_b(es - qf)] \\
D &= [Y_b(cq + ds) - (cp + dr)] \\
P_1/Q_0 &= (ce + df)/D \\
U_1/Q_0 &= -Y_b(ce + df)/D \\
P_{in}/Q_0 &= dN/D \\
U_{in}/Q_0 &= -cN/D \\
P_{out}/Q_0 &= N/D \\
U_{out}/Q_0 &= 0/D \\
Z_{in} &= d/(-c)
\end{aligned}
$$

The general equations for a distributed volume velocity source are:

$$
\begin{aligned}
N &= [(pf - er) + Y_b(es - qf)] \\
D &= [(ap + br) - Y_b(aq + bs)] \\
&\quad -Z_e[(cp + dr) - Y_b(cq + ds)] \\
P_1/Q_0 &= [Z_e(ce + df) - (ae + bf)]/D \\
U_1/Q_0 &= -Y_b[Z_e(ce + df) - (ae + bf)]/D \\
P_{in}/Q_0 &= (Z_e d - b)N/D \\
U_{in}/Q_0 &= (a - Z_e c)N/D
\end{aligned}
$$

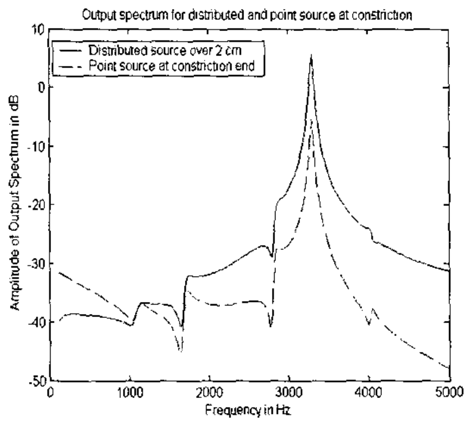

Fig. 4. Output spectrum for distributed and point source at finite impedance constriction.

$$
\begin{aligned}
U_{out}/Q_0 &= N/D \\
P_{out}/Q_0 &= (Z_e N)/D \\
Z_{in} &= (Z_e d - b)/(a - Z_e c)
\end{aligned}
$$

5. EXAMPLE

The vocal tract configuration for the fricative consonant /s/ in the context of the vowel /a/ is used to compare the distributed source model with the point source model. The shape can be approximated by concatenation of three uniform area tubes: back cavity, constriction, and front cavity [5].

The back cavity, residing immediately in front of the glottis has an area of $4.0cm^2$, and length $13.5cm$. The constriction in front of the back cavity has a length of $1cm$ and area of $0.1cm^2$. The front cavity has a length of $2.5cm$ and an area of $1cm^2$. The glottis is modeled as an orifice of length $1cm$ and area $0.1cm^2$, the impedence of the glottis and the constriction are computed using traditional methods [6]. Similarly existing models for the tube passive elements and losses due to wall vibration are used [7].

Figure 4 shows the spectrum of output volume velocity for a pressure source at the constriction. The solid line is for a source distributed uniformly across $2cm$ of the front cavity, and the dashed line is for a point source immediately anterior to the constriction. The point source model predicts the resonances and anti-resonances properly but is inadequate in modeling the spectral shape and low frequency behavior.

Figure 5 illustrates the spectrum with a point pressure source $0.5cm$ anterior to the constriction. The solid line is for a finite impedance constriction with

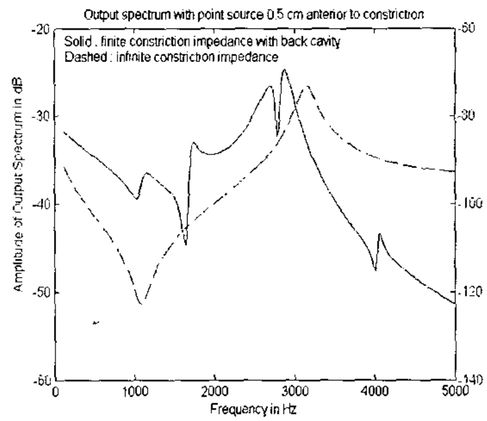

Fig. 5. Output spectrum for point source 0.5 cm anterior to constriction.

a back cavity, and the dashed line is for an infinite impedance constriction. The predictions for the formants / anti-formants and the spectral shape in the finite case are substantially different from the distributed model. The infinite impedance model is inaccurate for low frequencies and the spectral shape, but predicts the resonant frequency properly.

Thus, incorporation of the constriction and back cavity necessitates the use of distributed source models.

6. REFERENCES

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