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An Eigenvalue Based Acoustic Impedance Measurement Technique

A method is developed for measuring acoustic impedance. The method employs a one-dimensional tube or duct with excitation at one end and an unknown acoustic impedance at the termination end. Microphones placed in the tube are then employed to measure the frequency response of the system from which acoustic impedance of the end is calculated. This method uses fixed instrumentation and takes advantage of modern Fast Fourier Transform analyzers. Conventional impedance tube methods have errors resulting from movement of microphones to locate the maxima and minima of the wave pattern in the impedance tube or require phase matched microphones with specific microphone spacing. This technique avoids these problems by calculating the acoustic impedance from measured duct eigenvalues. Laboratory tests of the method are presented to demonstrate its accuracy.

Introduction

Measuring the acoustic impedance of a boundary is important since the acoustic response of any acoustic system is governed by the acoustic impedance of its boundaries. Accurate mathematical models of acoustic systems require accurate measurements of acoustic impedance. The acoustic impedance of boundaries determines the magnitude and frequency of resonant peaks and the spatial distribution of acoustic response.

A variety of acoustic impedance measurement techniques have been developed in the past. The first techniques used an impedance tube and a single microphone (Hall, 1939; Beranek, 1940; Morse and Ingard, 1968; Dickinson and Doak, 1970; Pierce, 1981). They require measurement of maximum and minimum sound pressure levels at an acoustic resonance in an impedance tube and their spatial locations. These locations and magnitudes are then used to calculate the corresponding impedance (ASTM Standard C 384, 1985a). Identifying the location of maximum and minimum sound pressure levels in an impedance tube is normally difficult and requires physical changes in microphone position. Two of the impedance tube measurement methods (Hall, 1939; Beranek, 1940) use approximate formulas for computing impedance which can also lead to impedance measurement error.

A recent acoustic impedance measurement technique utilizes a two microphone system (Seybert and Ross, 1977; Chung and Blaser, 1980a, 1980b). This technique requires two similar, phase calibrated, microphones at some location in the tube with a known distance between them. The acoustic wave response is then mathematically decomposed into its reflected and incident components using a transfer function between

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This paper develops a method for calculating the acoustic impedance based on the eigenvalues of a tube with unknown end impedance. A Fast Fourier analyzer is used to measure complex frequency response from which the eigenvalues of the system are extracted. Acoustic impedance at each resonance is then computed from these eigenvalues. The eigenvalue measurement is independent of microphone position and the location of the response microphone in the tube is arbitrary. The computation of the acoustic impedance from the duct eigenvalues is a closed form solution based on the same plane wave assumptions present in previous methods. The only physical constants required are duct length and the speed of sound in the duct.

System Model

The system model is of a one-dimensional hard-walled duct excited by a pressure input at one end and a partially reflective boundary condition at the other end represented by a complex boundary impedance. The partially reflective condition in the

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Fig. 1 System eigenvalues λ_m , for constant K

duct allows some energy to be dissipated out the end while the rest is reflected back into the system.

The termination end impedance is a ratio between the pressure and the particle velocity at x=L and is expressed as (Seto, 1971; Pierce, 1981; Spiekermann and Radcliffe, 1988)

$$\frac{\partial u}{\partial x}(L, t) = -K\left(\frac{1}{c}\right)\frac{\partial u}{\partial t}(L, t)$$
(1)

where K = complex acoustic impedance of the termination end (dimensionless), u(L, t) = particle displacement at x = L(m), c = wave speed in the duct (m/s), t = time(s), x = spatial location (m) and L = length of the duct (m). Implicit in (1) is the acoustic analogy with electrical systems in which volume velocity is analogous to current and duct pressure is analogous to voltage. The reciprocal acoustic mobility analogy is also sometimes used; and if applied to this system, the parameter K in (1) would be the acoustic admittance. Acoustic impedance K=0+0i corresponds to an ideal fully reflective termination and K = 1 + 0i corresponds to ideal fully absorptive termination. In general, K is a complex value which does not match either of these ideal conditions. The real part of K (acoustic resistance) is associated with nonconservative power dissipation at the end while the imaginary part (acoustic reactance) is associated with conservative inertial and/or compliant characteristics of the end.

The linear second order wave equation modeling particle displacement in a hard-walled, one-dimensional duct is (Seto, 1971; Doak, 1973)

$$\frac{\partial^2 u(x,t)}{\partial t^2} - c^2 \frac{\partial^2 u(x,t)}{\partial x^2} = -\frac{\partial}{\partial x} \left[\frac{\delta(x) P_e(t)}{\rho} \right]$$
(2)

where u(x,t) = particle displacement (m), $\rho = \text{density of the medium (kg/m³)}$, $P_e(t) = \text{pressure excitation at point } x=0$ (N/m²), and $\delta(x) = \text{the Dirac delta function}$. The wave equation assumes an adiabatic system, no mean flow in the duct, uniform duct cross section and negligible air viscosity effects. The hard-wall assumption yields a system with dissipation only at the termination end; the losses at the duct walls due to heat transfer, viscosity, and vibration are negligible. The one-di-

mensional assumption requires the diameter of the duct to be small compared to the wavelength of sound which yields plane wave response. The one-dimensional assumption is usually valid when f < 0.586(c/d) where f is the frequency (Hertz) and d is the diameter of the tube (m) (Annual Book of ASTM Standards, 1985a; 1985b).

The duct end at x = 0 is modeled as a totally reflective, open end. This boundary condition is (Seto, 1971; Hull et al., 1990)

$$\frac{\partial u}{\partial x}(0, t) = 0.$$
(3)

This corresponds to an open duct end. Equation (3) along with the right hand side of (2) model the speaker as a pressure source at x=0. Although speakers are sometimes modeled as velocity sources, the eigenvalues and eigenvectors from the experiment discussed later in the paper correspond to the speaker modeled as a pressure source. Implicit in (3) is the assumption the source impedance is negligible. If the source impedance is not small, it can be incorporated into the model (Swanson, 1988). The acoustic pressure of the system is related to the spatial gradient of the particle displacement by (Seto, 1971)

$$P(x, t) = -\rho c^2 \frac{\partial u}{\partial x} (x, t).$$
(4)

The above four equations represent a mathematical model of a long, thin duct with a speaker at one end and a partially reflective termination end.

Separation of Variables

The eigenvalues of the model are found by applying separation of variables to (1) and (3) and the homogeneous version of (2). Separation of variables assumes each term of the series solution is a product of a function in the spatial domain multiplied by a function in the time domain:

$$v(x, t) = X(x)T(t).$$
⁽⁵⁾

Substituting (5) into the homogeneous version of (2) produces two independent ordinary differential equations, each with complex valued separation constant Λ , namely

$$\frac{d^2 X(x)}{dx^2} - \Lambda^2 X(x) = 0$$
 (6)

and

$$\frac{d^2 T(t)}{dt^2} - \Lambda^2 c^2 T(t) = 0.$$
 (7)

The separation constant $\Lambda = 0$ is a special case where X(x) = T(t) = 1 to satisfy (1) and (3). Although $\Lambda = 0$ is a separation constant of the system, it does not contribute to the pressure field in the duct, therefore it is ignored for further computational purposes (Hull et al., 1990; MacCluer et al., 1990). The spatial ordinary differential equation (6) is solved for $\Lambda \neq 0$ using the boundary condition (3) yielding

$$X(x) = e^{\Lambda x} + e^{-\Lambda x}.$$
 (8)

The time dependent ordinary differential equation yields the following general solution

$$T(t) = Ae^{\Lambda ct} + Be^{-\Lambda ct}.$$
(9)

Applying boundary condition (1) to (8) and (9) yields B=0 and the separation constant

$$\Lambda_n = \frac{1}{2L} \log_e \left(\frac{1-K}{1+K} \right) - \frac{n\pi i}{L}, \quad n = 0, \ \pm 1, \ \pm 2, \ \dots \ .$$
(10)

The system eigenvalues λ_n are equal to the separation constant multiplied by the wave speed c ($\lambda_n = c\Lambda_n$). An eigenvalue plot is shown in Fig. 1. These eigenvalues are each functions of

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acoustic impedance, K. The inverse function will allow impedance, K, to be computed from measured eigenvalues.

Acoustic Impedance Computation

The acoustic impedance K of the end can be determined at each duct resonance from the eigenvalue at that resonance. This computation assumes the eigenvalues of the system are known. Measuring these duct system eigenvalues is discussed in the next section. Directly solving for K in terms of λ is very difficult, therefore an intermediate variable β is introduced to simplify the acoustic impedance computation. The variable β_n is related to the *n*th eigenvalue λ_n by

$$Re(\lambda_n) + iIm(\lambda_n) = \frac{c}{2L} \log_e[Re(\beta_n) + iIm(\beta_n)] - \frac{n\pi ci}{L}$$
(11)

where Re() denotes the real part, Im() denotes the imaginary part, and the subscript "n" denotes the nth term. Equation (11) is now broken into two parts, one equating the real coefficients and the other equating the imaginary coefficients. The complex logarithm on the right hand side is rewritten as

$$og_e[Re(\beta_n) + iIm(\beta_n)] = log_e|\beta_n| + i \ arg(\beta_n)$$
(12)

where $|\beta_n|$ is the magnitude of β_n and $\arg(\beta_n)$ is the argument of β_n , i.e., the arctangent of $[Im(\beta_n)/Re(\beta_n)]$.

The intermediate variable β_n is now solved for in terms of the real and imaginary parts of the eigenvalues. The real part of β_n is

$$Re(\beta_n) = \pm \left[\frac{exp\left(\frac{4LRe(\Lambda_n)}{c}\right)}{1 + \tan^2\left(\frac{2Ld_n}{c}\right)} \right]^{\frac{1}{2}}$$
(13a)

where
$$d_n = Im(\lambda_n) - \frac{nc\pi}{L}$$
.

The sign of $Re(\beta_n)$ in (13a) is determined by

$$sgn[Re(\beta_n)] = \begin{cases} +1 \text{ if } 0 \le |\Delta| \le 0.25\\ -1 \text{ if } 0.25 < |\Delta| \le 0.50 \end{cases}$$
(13b)
where $\Delta = \frac{d_n}{\sqrt{c\pi}}$.

 $\left(\frac{1}{L}\right)$

If the value of Δ is less than -0.5 or greater than 0.5, the eigenvalue index *n* is incorrect and corresponds to an eigenvalue other than the *n*th one. The value, *n*, must then be changed to produce a Δ between -0.5 and 0.5 which will correspond to the correct eigenvalue index. Once $Re(\beta_n)$ is found, $Im(\beta_n)$ is found by the equation

Table 1 Measured duct eigenvalues for the foam end

Eigenvalue (n)	Re(λ _n) mean, x	Re(λ _n) std. dev., s	Im(λ _n) mean, x (Hertz)	Im(λ _n) std. dev., s (Hertz)
1	-4.955	0.033	57.89	0.045
2	-5.204	0.124	116.3	0.055
3	-5.713	0.091	174.7	0.152
4	-5.755	0.131	233.8	0.259
5	-4.231	0.122	291.7	0.164
6	-5.414	0.117	350.5	0.192

Table 2 Calculated acoustic impedance for the foam end

Eigenvalue (n)	Re(K _n)	Im(K _n)
1	0.260	0.032
2	0.273	0.037
3	0.298	0.042
4	0.300	0.014
5	0.224	0.046
6	0.283	0.031
mean, x	0.273	0.034
std. dev., s	0.028	0.011

$$Im(\beta_n) = Re(\beta_n) \tan\left(\frac{2Ld_n}{c}\right)$$
(14)

where $Re(\beta_n)$ is given in (13).

The term (1-K)/(1+K) is now equated to the intermediate variable β_n using (10) and (11) as

$$Re(\beta_n) + iIm(\beta_n) = \frac{1 - Re(K_n) - iIm(K_n)}{1 + Re(K_n) + iIm(K_n)}$$
(15)

where $Re(K_n)$ is the real part of K and $Im(K_n)$ is the imaginary part of K for the *n*th eigenvalue. Breaking (15) into two equations, and solving for K_n as a function of β_n yields the acoustic impedance as

$$Re(K_n) = \frac{1 - [Re(\beta_n)]^2 - [Im(\beta_n)]^2}{[Re(\beta_n) + 1]^2 + [Im(\beta_n)]^2}$$
(16)

$$Im(K_n) = \frac{-2Im(\beta_n)}{[Re(\beta_n) + 1]^2 + [Im(\beta_n)]^2}$$
(17)

Acoustic impedance measurement K_n represents the acoustic impedance at the *n*th resonant frequency.

Experiment

The viability of the above acoustic impedance method was investigated through laboratory tests. The test used a 0.0762 m (3 in) circular PVC schedule 40 duct that was 2.94 m (9.65 ft) long driven by a 0.254 m (10 in) diameter speaker (Realistic 40-1331B). The impedance of a piece of 30 mm thick packing foam inserted in the termination end was tested. The packing foam will be shown to have acoustic impedance which is nearly constant with frequency (Table 2), allowing for frequency response comparison to known theory. Speaker input pressure was measured in the exit plane of the input speaker with a Bruel and Kjaer Type 4166 half inch microphone (input reference microphone) attached to a Hewlett-Packard 5423A digital signal analyzer. The response of the tube was measured at various locations with another Bruel and Kjaer Type 4166 half inch microphone (response measurement microphone) at-

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Fig. 3 Frequency response of duct with foam end at x = 0.792 m

tached to the signal analyzer (Fig. 2). Both microphones were calibrated using a Bruel and Kjaer Type 4230 Sound Level Calibrator.

The impedance measurement technique developed here does not require phase matched microphones nor does it require compensation for phase mismatched microphones. Phase mismatch in the microphones is neglected since the measurements are made at a duct resonant frequency, i.e. the measurements are made when the system phase angles are changing rapidly through 180 degrees. Microphones operating under 500 Hertz rarely have phase error greater than 5 degrees (Bruel and Kjaer, 1982). Steady state eigenvalue measurements are amplitude dominated. The distance between the microphones is not critical, since the duct eigenvalues are independent of measurement location. This is unlike previous methods (Seybert and Ross, 1977; Chung and Blaser, 1980a, 1980b) where microphone spacing is a required parameter in the analysis and phase matched microphones (or a compensation function) are necessary because wave propagation across the microphones is detected. Errors in the method developed here are only a function of errors associated with measuring the eigenvalues of the duct, the duct length, and the speed of sound. The computation of acoustic impedance from duct eigenvalues is a closed form solution. The method uses the input microphone as an amplitude reference and the excitation speaker does not require a flat response around the frequency of interest, because the response is normalized by the pressure input reference when the Fast Fourier Transform is computed.

The 5423A Structural Dynamics Analyzer used here is capable of providing a number of real time analyses including determining the transfer function (frequency response) of a system and calculating the corresponding eigenvalues. The 5423A Structural Dynamics Analyzer does this by curve fitting a single mode vibration model (two first order states) to the experimental data using the following equation (Hewlett-Packard, 1979)

Table 3 Measured duct eigenvalues for the capped end

Eigenvalue (n)	$\operatorname{Re}(\lambda_n)$	Im(λ _n) (Hertz)
1	-0.723	29.1
2	-0.847	88.4
3	-1.010	147.2
4	-1.279	206.0
5	-1.419	264.6
6	-0.914	324.1
7	-1.038	381.9

Table 4 Calculated acoustic impedance for the capped end

Eigenvalue (n)	Re(K _n)	Im(K _n)
1	24.37	-5.88
2	15.06	-10.22
3	11.06	-9.03
4	8.54	-7.16
5	8.07	-6.38
6	3.32	-7.47
7	6.81	-8.68

$$H(\omega) = \frac{Re(A_n) + iIm(A_n)}{i\omega - Re(\lambda_n) - iIm(\lambda_n)} + \frac{Re(A_n) - iIm(A_n)}{i\omega - Re(\lambda_n) + iIm(\lambda_n)} + B_1\omega + B_0$$
(18)

where $H(\omega)$ = the transfer function,

A = the system residue, and

 B_1 and B_0 = compensation constants for overlapping modes.

Included in the single mode vibration model is compensation for other modes which may be overlapping at that particular frequency. During the curve fitting process, the real and imaginary parts of the eigenvalues are calculated. It is beyond the scope of this paper to describe this process; however, there exist additional alternative methods to extract modal parameters from the transfer function of a system (Hewlett-Packard, 1979; Structural Dynamics Research Corporation, 1983). Eigenvalue extraction is a common function of commercial Fast Fourier analyzers.

The first part of the experiment measured the transfer function of the duct with the foam end impedance. The 5423A Structural Dynamics Analyzer does this by sending a random noise signal to the speaker and then computing the ratio of the Fast Fourier Transforms of the input and response signals. Once the transfer function was known, the eigenvalues of the duct were found using the curve fitting process discussed in (18). From the eigenvalues, the acoustic impedance of the foam was determined using equations (11)-(17).

The mean and standard deviation of the measured eigenvalues of the system with the foam end impedance are shown in Table 1. These values are derived from five independent sets of measurements at x = 0.792 m (2.60 ft) through x = 1.42 m (4.67 ft) at 0.157 m (0.52 ft) increments. Each individual eigenvalue was measured from a transfer function composed of 20 averaged Fast Fourier transforms. The calculated acoustic impedance of the foam is shown in Table 2. In this case, the real part of the acoustic impedance dominates the response. Figure 3 shows the measured frequency response at x = 0.792 m (2.60 ft) compared to the theoretical frequency response for K = 0.273 + 0.034i at the same point (Spiekermann and

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Fig. 4 Frequency response of duct with closed end at x = 0.792 m

Radcliffe, 1988; Hull et al., 1990). The value of P/P_0 is the ratio of the response to the input and the measured responses are marked by X's while the theoretical response is denoted by a solid line. The impedance, K, used in the theoretical response is the average of the six individual acoustic impedance measurements taken at different locations along the duct. Figure 3 demonstrates that the theoretical model using the measured acoustic impedance of a material can accurately predict duct response. There is a high degree of accuracy in both the magnitude and the phase angles.

The acoustic impedances of the above experiment were calculated by increasing the magnitudes of both the real and imaginary parts of the measured eigenvalues by one, two, and three standard deviations from their mean values. After these changes, the magnitude of the calculated impedance K only changed by an average of 1.7 percent, 3.2 percent, and 5.0 percent, respectively. This shows the high stability of the measurement technique, its resistance to error propagation, and the accuracy of acoustic impedances determined using it.

The experiment was repeated for a capped end. The results are shown in Tables 3 and 4. An ideal closed end would have an impedance of infinity; however, the real material used here has some absorption. The large impedances shown in Table 4 indicate this trend and the variation of impedance with frequency in this case. Figure 4 shows the measured frequency response compared to a theoretical frequency response calculated using the measured impedances. The theoretical frequency response was produced by assembling a state space model which used the measured acoustic impedances at each eigenvalue (Hull et al., 1990). As in Fig. 3, there is a high degree of accuracy in both the magnitude and phase angles.

It is important to monitor the value of Δ when testing extremely reflective ends. It is possible for eigenvalue computation errors to yield a Δ greater than 0.50 with the correct index *n* for large impedances. From the above measurements, a value of coefficient, $\Delta = .51$ was calculated for the one of the eigenvalues. Because the measurements were only accurate to two significant figures, the coefficient was rounded to $\Delta = 0.5$. For most materials the reflectivity is not large enough for this to be a concern.

Conclusions

Calculation of the acoustic impedance of a duct end from experimentally obtained impedance tube eigenvalues is developed here. These eigenvalues are easily determined from a measured tube transfer function by commercially available Fast Fourier analyzers. This method has the advantage of stationary microphone positioning at any location in the impedance tube. The computational step from eigenvalue to acoustic impedance is a closed form solution. Errors in measured impedance can arise only from errors in measured system eigenvalues, duct length, and the speed of sound. Experimental results show that duct response can be accurately predicted from measured impedances and demonstrate the method is both accurate and insensitive to measurement errors. Future work will quantify and compare that accuracy and error sensitivity with previous standard methods.

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