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ACOUSTICAL ADMITTANCE OF CYLINDRICAL CAVITIES

A recent letter to this journal from Ballagh [1] explained how the present method of correcting the acoustical admittance of a cavity for heat conduction to the cavity wall can be replaced by one which allows for the interaction between heat conduction and wave motion in the cavity. It was shown that in the pressure calibration of one-inch microphones by the reciprocity method [2], heat conduction has a significant effect over the whole of the audible frequency range and not just at low frequencies as previously assumed. At NPL we have been using a theory similar to this in work to extend the reciprocity method to the calibration of half-inch microphones, and would like to present some evidence in support of Ballagh. We would also like to explain a further modification to the method proposed by Ballagh and that recommended in IEC 327, which takes into account the impedance of the driver.

Gerber [3] showed that the effect of heat conduction depends on the acoustical impedance of the driver and that wrong assumptions have been made about this, both in the formula quoted in IEC 327 (originally derived by Biagi and Cook [4]) and in that given by Ballagh (which was originally derived by Ballentine [5]).

To explain why the impedance of the driver affects the thermodynamics consider two cases: one where a cavity is driven by a constant displacement source (e.g., a mechanically-operated piston), and the second where a cavity is driven by a constant sound pressure source (e.g., a resonant loudspeaker). If in the first case the cavity is suddenly compressed to a new volume, and the gas temperature is consequently raised, that volume will remain constant while the temperature falls due to the heat exchange with the walls. If in the second case the cavity is suddenly compressed, as the gas cools the pressure will fall and the source will tend to move further inwards to sustain the pressure. The inward motion of the source will raise the temperature of the gas, so the gas in the second case will cool at a lower rate than the gas in the first case. In terms of the mathematical model the first case requires the solution of an equation where the temperature is given in terms of the volume of the cavity, and the second requires the solution of an equation where the temperature is given in terms of the pressure in the cavity. For a driver of known impedance, the two solutions can be mixed to find the overall effect. Without appreciating these subtleties, both Ballentine and Biagi and Cook set up their models in terms of the pressure in the cavity and so implicitly assumed that the microphone had zero acoustic impedance. In reality the acoustic impedance of microphones calibrated by reciprocity is usually very high. For a Brüel & Kjær microphone type 4160, the heat conduction correction calculated by using its exact impedance differs so little from the correction calculated on the assumption of infinite impedance that the effect on a calibration in the IEC 3 cm^3 coupler is less than 0·001 dB at frequencies above 63 Hz.

There are some misprints in Gerber's paper so the correct form of the heat conduction correction assuming an infinite impedance source is reproduced here. The correction at low acoustic frequencies is given by $V_e/V_g = k/[1+(k-1)E_v]$, where V_e is the effective volume of the cavity, V_g is the geometric volume of the cavity, and k is the ratio of specific heats in the gas. Also $E_v = 1 - S + D_1 S^2 + D_2 3\sqrt{\pi} S^3/4$, $S^2 = -\mathrm{i}/2\pi Y$ and $Y = fL^2/\Delta k$, where f is the frequency, L is the ratio of the volume of the cavity to its surface areas and Δ is the thermal diffusivity of the gas. The values of D_1 and D_2 depend on the geometry of the cavity. For a finite cylinder, $D_1 = (\pi R^2 + 8R)/\pi(2R+1)^2$ and

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$D_2 = (R^3 - 6R^2)/3\sqrt{\pi}(2R+1)^3$, where R is the ratio of the length to the diameter of the cylinder. For an infinite narrow box, (i.e., a cavity where only two parallel faces are considered) D_1 and D_2 are zero. The result for the finite cylinder is accurate to 0·01% for $\frac{1}{8} < R < 8$ and $Y > 5$; for values outside this range the full double infinite sum solution given in reference [3] should be used.

When this set of equations is used, instead of those derived by Ballentine or Biagi and Cook, the effect on the sensitivity of a microphone calibrated in the 3 cm^3 coupler is as shown in Figure 1. It can be seen that introducing Gerber's theory to treat correctly the impedance of the driver produces as large a modification to the accepted heat conduction correction at 63 Hz as does Ballagh's treatment of the high frequency effects at 8 kHz. The problem is how to combine these two new features of the theory to produce a consistent approach across the whole frequency range. The way we have chosen to do this is to let the theory described by Ballagh account for the effects of heat conduction into the side walls of the cylindrical coupler, but treat the end faces of the coupler as if they formed an infinite narrow box containing the volume of the cavity. The above equations (with $D_1 = 0 = D_2$ and L being half the length of the coupler) give a fractional increase in the total volume of the cavity. Half this extra volume is attributed to each microphone and included in the terminating impedances of the transmission line. The outcome of this approach is also shown in Figure 1; it should be noted that the agreement between this approach and the simple application of Gerber's theory to the whole of the cylindrical cavity is better than 0·0002 db below 500 Hz, for the 3 cm^3 coupler.

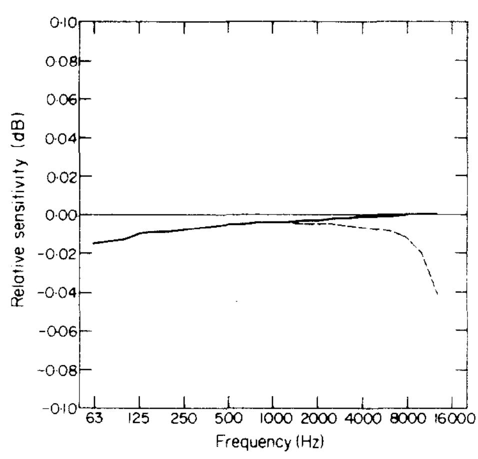

Figure 1. The effect on the apparent sensitivity of a microphone calibrated in the 3 cm^3 coupler, relative to that calculated by using the heat conduction correction in IEC 327. ——, According to Gerber's theory only; - - - -, according to Gerber and Ballagh.

We started to consider the problem of the interaction between heat conduction and wave motion in a cavity because we were experimenting with three plane-wave couplers designed to calibrate half-inch microphones. The couplers have a diameter of 9·3 mm and lengths ranging from 5·2 mm to 8·3 mm. Initial microphone calibrations revealed discrepancies of more than 0·1 dB at 16 kHz and any attempt to correct for radial wave motion in the cavity increased the discrepancy. Applying the above theory reduced the discrepancy to 0·07 dB and corrections for radial wave motion in the cavity then further reduced the discrepancy to about 0·05 dB.

Whilst the method recommended here does not have the simplicity of the methods used by Ballentine or Ballagh, it is more accurate. At NPL we quote an uncertainty of $\pm 0{\cdot}03$ dB (at an estimated confidence level of 95%) on the low frequency sensitivity of one-inch standard condenser microphones, so the change of nearly $0{\cdot}02$ dB in the heat conduction correction brought about by introducing Gerber's theory is highly significant.

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