Sound Transmission in a Duct With an Array of Lined Resonators

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A simple method is presented for describing the sound transmission in a duct containing an array of lined resonators. The impedance of a resonator element is calculated, including the effect of the lining and the duct is modeled as a one-dimensional waveguide with lumped impedances. An expression for the TL is derived from considerations of pressure and mass flow continuity along the duct. Experimental data compared to numerical computations show that the method developed here describes satisfactorily the performance of the duct and allows useful parametric analyses which can lead to improved design.

1 Introduction

There are many applications where the conventional types of dissipative mufflers cannot be used. These applications usually involve hot gaseous flow containing carbon or other particles tending to close the pores of the sound absorbing lining or to cause thermal cracking of the lining. Furthermore, the performance of dissipative mufflers at low frequencies is poor.

Mufflers consisting of a rigid wall flow duct in which a series of resonators is formed by partitioning a side wall have been proved very useful in many applications involving hot gaseous flow of carbon or oil particles providing adequate attenuation ranging from as low as 50 Hz up to 500 Hz. The main advantages of this type of "resonator-mufflers" are their low cost of construction and maintenance, their high durability and their good performance at low frequencies. By using resonators with different dimensions a "resonator-muffler" can be "tuned" to perform over a predetermined frequency range.

Ingard and Pridimore-Brown (1951) investigated experimentally the effect of the length of the backing cavity in a lined duct configuration to find the optimum resonator configuration. They have shown that an increase in the bandwidth of attenuation could be obtained when the partitions were spaced at one-half wavelength at the Helmholtz resonance frequencies. In their experiments they used a series of identical resonators. Cummings et al. (1988) also studied the sound propagation in a duct lined with an array of resonators considering the nonlinear interaction between the acoustic field and the resonators.

It is the aim of the present paper to develop a simple theory that describes the performance of mufflers incorporating lined resonator-elements taking into account the main parameters involved. Our approach is similar to that described by Sullivan (1979) for modelling perforated tube mufflers. The model is based on the assumptions that only plane wave propagation is allowed and that the flow velocities are small, so that flow effects can be neglected. Based on these assumptions, the impedance of a resonator-element is calculated and the duct is treated like a one-dimensional wave-guide with lumped impedances at discrete points. An expression for the TL is obtained and the main parameters involved, i.e., the dimensions of the resonators and their openings, the flow resistance of the lining, and the overall geometry of the duct are systematically investigated.

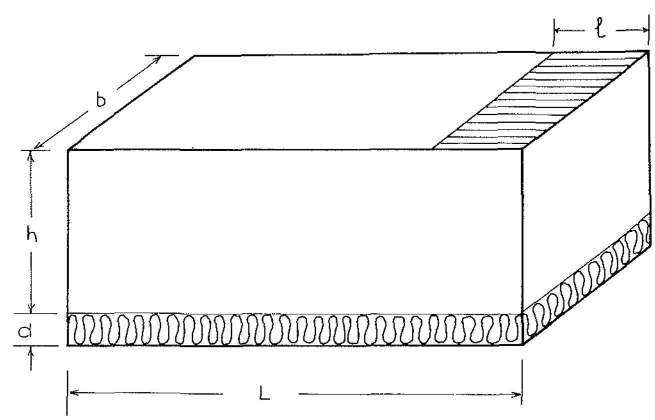

Fig. 1 Configuration of duct

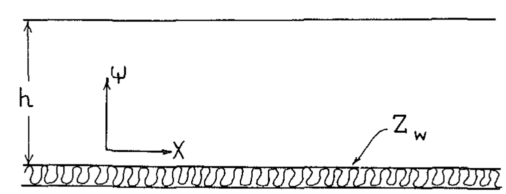

Fig. 2 Geometry of a resonator element

Finally, experimental results from measurements conducted on model mufflers are compared to numerical calculations. It is shown that the simple theory developed describes satisfactorily the performance of the mufflers, at least for design purposes, and allows useful parametric analyses to find the optimum resonator configuration which can lead to improved design.

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2 Theory

The problem here is the propagation of low frequency sound in a straight duct in which a wall is partitioned so that an array of different Helmholtz resonators is formed.

All resonators are lined with usual rock wool absorptive material (Fig. 1).

We will assume that only one-dimensional plane wave propagation is allowed and the flow velocity is low so that it can be neglected. First, an expression for the impedance of resonator-element must be derived, including the effect of the lining. Then, by treating the duct as a one-dimensional waveguide with lumped impedances in discrete positions, an expression for the transmission loss is obtained. These aspects are discussed in the following sections.

2.1 Propagation Constant. We start by analyzing the resonator-element shown in Fig. 2. To take into account the additional losses due to the lining we assume a complex propagation constant K_x within the resonator. To calculate K_x one needs to know the wall impedance Z_w of the lined wall.

It is well known that the case of a plane wave incident on a locally reacting lining of uniform thickness d backed by a rigid wall, the impedance encountered by the plane wave is given by

$$Z_w = -jY_w \cot(K_w d), \tag{1}$$

where Y_w and K_w are the complex characteristic impedance and propagation constant of the absorptive lining, respectively.

For fiber-based porous sound absorbing material often used in silencers, K_w and Y_w are given by Munjal (1987)

$$\frac{K_w}{K_0} = (x)^{1/2}\left\{1 - j\frac{\xi}{\omega\rho_0 x}\right\}^{1/2} \tag{2}$$

$$\frac{Y_w}{Y_0} = \frac{1}{\sigma}\frac{K_w}{K_0}, \tag{3}$$

where ρ_0 is the air density, ξ is the flow resistance of the unit thickness of the porous bulk material, σ is the porosity and x is the structural factor of the material.

Considering each resonator element as an infinite duct of height h lined on one side (Fig. 3) the wavenumber is given as a solution of the well known transcendental equation

$$-j\sqrt{E}\,\tan\sqrt{E} = \beta \tag{4}$$

with $E = (K_0 h)^2 - (K_x h)^2$ and $\beta = K_0 h \rho_0 c_0 \dfrac{1}{Z_w}$.

The real and imaginary parts of the first root of the transcendental equation may be obtained by nomograms or numerically. A very reasonable and convenient approximation can be obtained by writing (Mechel, 1987)

$$E = \frac{105 + 45j\beta \pm \sqrt{11025 + 5250j\beta - 1605\beta^2}}{20 \pm 2j\beta}, \tag{5}$$

$$\text{with } \beta = K_0 h \rho_0 c_0 \frac{1}{Z_w} = jh\frac{K_0^2}{K_w}\frac{\sin(K_w d)}{\cos(K_w d)}. \tag{6}$$

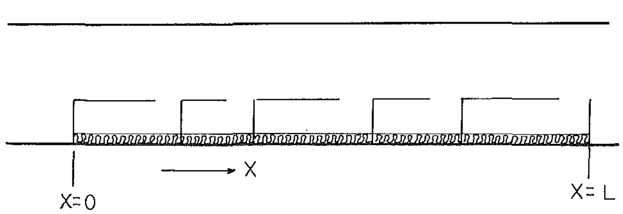

Fig. 3 Resonator element as infinite duct

Equations (5) and (6) give two complex values for K_x. Of particular importance is the one that gives lower attenuation.

2.2 Impedance of a Resonator-Element. The impedance of the resonator-element shown in Fig. 2 consists of the neck inertance and the cavity compliance with damping in series. The compliance can be expressed in terms of K_x as

$$Z_s = \frac{p_s}{u_s} = -j\rho_0 c_0 \frac{K_x}{K_0}\frac{\cos K_x(L-l)}{\sin K_x(L-l)}, \tag{7}$$

where p_s and u_s are the acoustic pressure and volume velocity in the resonator, L is the length of the resonator and l is the length of the opening.

The inertance of the neck, i.e., the opening can be expressed as

$$Z_M = \frac{F_M}{u_S} = \frac{P_M S_0}{u_M} = j\omega m_s, \tag{8}$$

where F_M is the driving force, u_M is the volume velocity at the neck, and S_0 is the area of the opening. Hence, m_s is the effective oscillatory mass taking into account the end correction. In other words, the opening is considered circular with an equivalent radius $\sqrt{S_0/\pi}$. By adding the impedances given by equations (7) and (8) the combined impedance of the resonator can be obtained. However, by adding the impedances the geometry must be taken into account. The area of the opening is $S_0 = lb$, while that of the duct cross section $S = hb$. Assuming plane wave propagation, the continuity of pressure yields

$$p_s = p_M \tag{9}$$

while the continuity of mass flux requires

$$S_0 u_M = S u_s. \tag{10}$$

Thus, one can write

$$\frac{p_s}{u_s} = \frac{p_M S}{u_M S_0} = \frac{F_M}{u_M}\cdot\frac{S}{S_0^2} \tag{11}$$

or alternatively

$$\frac{F_M}{u_M} = \frac{p_s}{u_s}\cdot\frac{S_0^2}{S}. \tag{12}$$

Using equations (9) through (12) the total impedance of the resonator can be expressed as

$$Z_t = j\omega m_s + \frac{S_0^2}{S}\left[-j\rho_0 c_0 \frac{K_x}{K_0}\frac{\cos K_x(L-l)}{\sin K_x(L-l)}\right]. \tag{13}$$

2.3 Transmission Loss. The idealized model of the duct is shown in Fig. 4. Since every resonator can be represented by an equivalent impedance Z_t given by equation (13), the duct can be modeled as a one-dimensional waveguide with lumped impedances at discrete points, i.e., the middle of each opening. The distance X is measured from the surface Z_1 (the opening

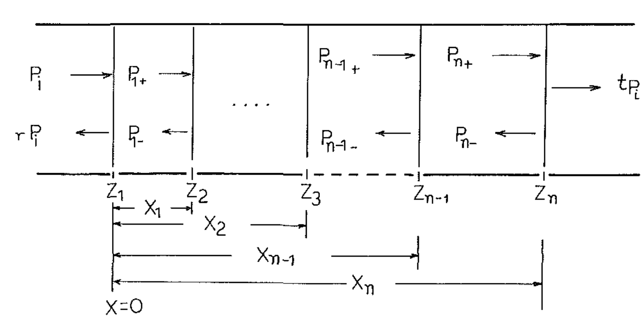

Fig. 4 One-dimensional idealized model of duct

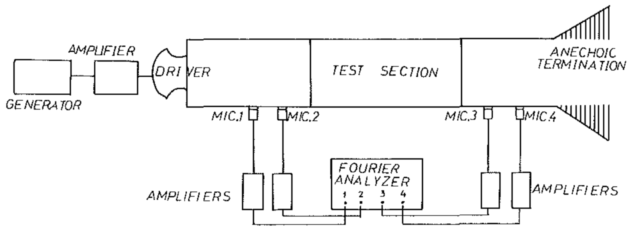

Fig. 5 Experimental arrangement

of the first resonator) in the direction of the incident wave. The equations necessary for determining the acoustic pressures and velocities along the duct are derived from considerations of continuity of pressure and mass flow.

The continuity of pressure at any cross section through an impedance position, say Z_n, requires

$$p_n = p_{n+1}, \tag{14}$$

where p_n, p_{n+1} are the acoustic pressures before and after the considered cross section. Furthermore, the continuity of pressure at any point X between two resonators, say $n-1$, n requires

$$p(X) = p_{n+}e^{-jK_0(X-X_{n-1})} + p_{n-}e^{-jK_0(X_n-X)} \tag{15}$$

where p_{n+}, p_{n-} are the complex amplitudes of incident and reflected waves at the nth resonator.

The continuity of the mass flow on the nth resonator opening yields

$$S_F u_n(X_n) = S_0 u_{rn}(X_n) + S_F u_{n+1}(X_n), \tag{16}$$

where $u_n(X_n)$, $u_{n+1}(X_n)$ are the volume velocities before and after the opening, respectively, $u_{rn}(X_n)$ is the velocity at the opening and S_F in the free cross section of the duct.

Finally, the volume velocity at any point X between two resonators n, $n-1$ can be expressed as

$$u_n(X)\rho_0 c_0 = p_{n+}e^{-jK_0(X-X_{n-1})} - p_{n-}e^{-jK_0(X_n-X_{n-1})}. \tag{17}$$

By defining the incident pressure by p_i and introducing a reflection coefficient r, equations (14) and (16) give at $X=0$

$$p_i + rp_i = p_1(0) = p_{1+} + p_{1-}e^{-jK_0X_1} \tag{18}$$

and

$$\rho_0 c_0(u_i + u_r) = p_1(0)\,\frac{\rho_0 c_0}{Z_1}\cdot\frac{S_{01}^2}{S_F} + u_1(0)\rho_0 c_0 \tag{19}$$

where u_i, u_r are the velocities of the incident and reflected waves respectively, Z_1 is the impedance for the first resonator and S_{01} is the area of its opening.

Taking into account that

$$(u_i + u_r) = p_i(1-r)/\rho_0 c_0 \tag{20}$$

equation (19) can also be written as

$$p_i(1-r) = p_1(0)\,\frac{\rho_0 c_0}{Z_1}\cdot\frac{S_{01}^2}{S_F} + p_{1+} + p_{1-}e^{-jK_0X_1}. \tag{21}$$

Similarly, introducing a transmission coefficient t for the transmitted wave, one can write for the end position X_n, namely after the last resonator

$$p_n(X_n) = tp_i \tag{22}$$

and

$$p_{n+}e^{-jK_0(X_n - X_{n-1})} - p_{n-} = p_i t\,\frac{\rho_0 c_0}{Z_n}\,\frac{S_{0n}^2}{S_F} + tp_i. \tag{23}$$

Using the aforementioned procedure, one can easily write in general for a system consisting of n resonators a system of $2n+2$ equations for $2n+2$ unknows, namely the complex amplitudes p_{n+}, p_{n-} and the coefficients r and t. These equations can be solved straightforwardly for t and the transmission loss can be expressed as

$$TL = 20\,\log(1/t). \tag{24}$$

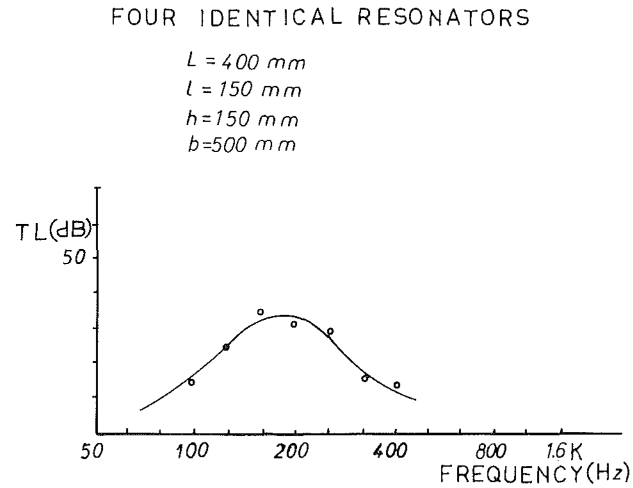

Fig. 6 Transmission loss of a duct with four identical resonators. Predicted ———, measured 0.
L (length) = 400 mm, b (width) = 500 mm
h (height) = 150 mm, l (opening) = 150 mm
ξ = 40.000 Ns/m^4.

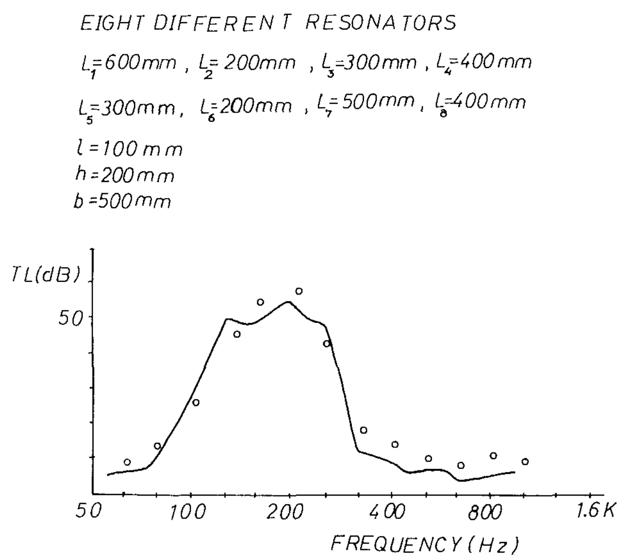

Fig. 7 Transmission loss of a duct with eight resonators of different length. Predicted ———, measured 0.
b (width) = 500 mm, h (height) = 20 mm
l (opening) = 100 mm, ξ = 40.000 Ns/m^4
for all resonators.

3 Measurements

The experimental arrangement consisted of a square section duct (anechoically terminated) with double 2+2 mm walls having internal dimensions 500 mm × 500 mm and a length of 4 m. On the bottom of the duct a series of Helmholtz resonators were constructed using L-shaped steel panels. All cavities were 100 mm deep and 500 mm wide. The length of the resonators as well as the length of their openings were systematically varied. All resonators were lined with a 50 mm thick mineral wool layer.

An acoustic driver supplied the acoustic signal fed to the

Length(mm)	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8
1	600	500	400	500	300	200	400	300
2	600	200	300	400	300	200	500	400
3	600	400	300	600	300	200	400	200

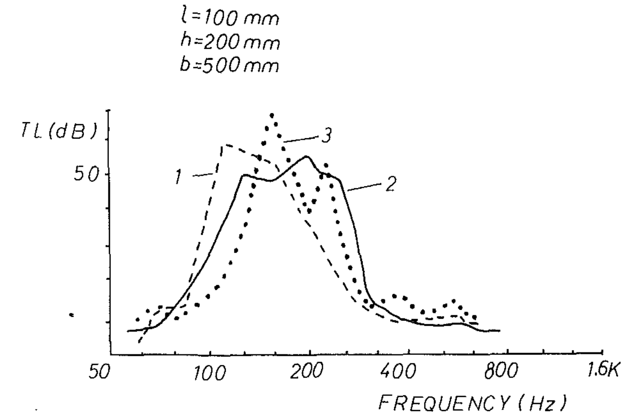

Fig. 8 Effect of the ordering of the resonators on the transmission loss

source from a signal generator and a power amplifier. The measurements were made using the transfer function method (Chung et al., 1980). The formula used for determining the transmission loss was

$$TL = 20 \log \left| \frac{H_r - H_{12}^u}{H_r - H_{12}^d} \right| - 20 \log |H_t|, \qquad (25)$$

where H_{12}^u and H_{12}^d are the transfer functions measured at the upstream and downstream directions, respectively, $H_t = |S_{dd}/S_{uu}|^{1/2}$ with S_{uu}, S_{dd} the autospectra at the upstream and downstream measurement locations. Finally, $H_r = e^{jk_0\lambda}$, where λ is the microphone spacing. To measure the transfer functions four 1/4 in. condenser microphones, two at either side, were used the outputs of which were fed to a four-channel analyzer. The experimental arrangement is shown in Fig. 5.

4 Results and Comparison With Theory

The results of this part of the investigation are presented in the form of curves of transmission loss in decibels plotted against frequency in cycles per second. The theoretical curves were calculated using the procedure described previously.

Typical results for two different resonator combinations compared with theoretical predictions are shown in Figs. 6 and 7. The first case concerns a duct with four identical resonators while the second one a configuration of eight different resonators. It can be seen that the agreement between predicted and measured results is good over the entire frequency range of interest. Both cases indicate that the method developed is sufficiently accurate to give confidence in its use at least for design purposes.

To investigate the effect of the main parameters involved on the transmission loss, a series of parametric studies were made, where the length of the resonators, the length of the openings, and the flow resistance of the lining were systematically varied.

(*1*) *Effect of the Sequence of the Resonators.*
The effect of varying the sequence of the eight resonators used in our investigation is shown in Fig. 8. The length of the opening for all resonators was kept constant equal to 100 mm. This figure shows clearly that the ordering of the resonators has a considerable effect on the results. As would be expected the longer resonator determines the low limit while the shorter one the upper limit of the frequency range over which the system is active. There is a certain combination that provides

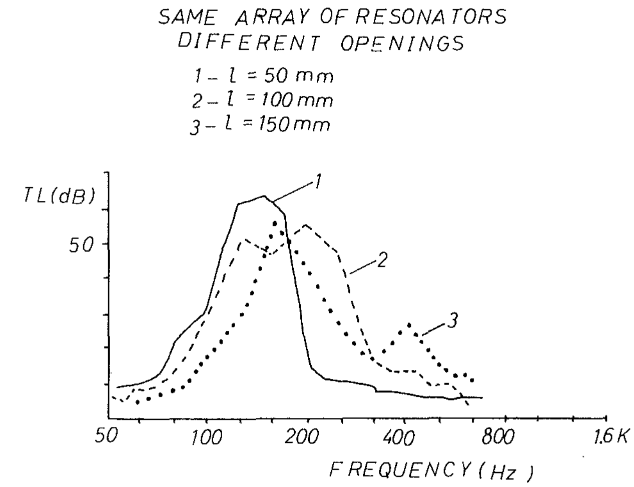

Fig. 9 Effect of the length of the openings of the resonators on the transmission loss

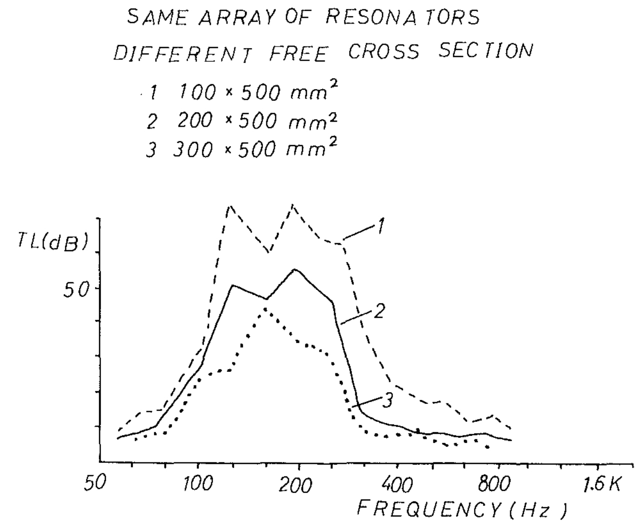

Fig. 10 Effect of the free cross section of the duct on the transmission loss

the optimal *TL* over the entire frequency range without any depressions in the *TL* curve. It is a well known principle of room acoustics that a random combination of Helmholtz resonators is not effective as long as overlap of the effective absorbing area of two adjacent resonators occurs. It seems that the determination of the ordering of the resonators that leads to optimal performance is the most difficult aspect.

(*2*) *Effect of the Opening Length.*
The effect of varying the length of the opening of the resonators is shown in Fig. 9. The sequence and the length of the resonators were kept constant, while the length of the openings were varied from 50 mm to 150 mm in three steps. It can be seen that the length of the opening effects mainly the frequency range over which the silencer is active since it determines the "Quality Factor" Q of the resonators. It seems that for the resonators used in our investigation a length of 100 mm provides the optimal results.

(*3*) *Effect of the Flow Resistance.*
To investigate the effect of the flow resistance of the lining on the *TL* a series of measurements were made using three different values of ξ, i.e., 40.000, 60.000, and 80.000 Ns/m^4. It was seen that the variation of the flow resistance of the lining, within the aforementioned limits, has no significant effect on the *TL*.

(*4*) *Effect of the Free Cross Section.* The results of changing the cross section of the duct are shown in Fig. 10. These are predicted results obtained by using the theoretical model developed. These curves demonstrate the importance of the flow velocity in the duct. The cross section, however, of a silencer in a practical application is determined by limitations imposed by the velocity and pressure required.

5 Conclusions

A simple theory for modeling silencers incorporating Helmholtz resonators has been presented. It is based on a procedure in which each resonator is considered as an impedance, so that the whole duct is modeled as a one-dimensional waveguide with lumped impedances.

Comparisons of calculated curves with experimental results has shown that it is possible to predict accurately the transmission loss of a silencer lined with an array of Helmholtz resonators. Furthermore, the model permits quick parametric analyses leading to optimal design.

Conclusions on the effect of varying some of the main parameters involved are as follows:

(*1*) The ordering of the resonators is important for the transmission loss. The position of the longer and shorter elements determines the combination which provides optimal attenuation.

(*2*) The length of the openings for a given combination of resonators determines the width of the frequency range over which the silencer is active.

(*3*) The variation of the flow resistance of the lining, within the values exhibited by the usual fiber-based absorptive materials, has no significant effect on the transmission loss.

Based on the aforementioned conclusions, the following design guide lines could be proposed for this type of silencers:

(*1*) Determination of the frequency range over which the silencer should be active by considering the noise spectrum of the source. Thus, the length of the longer and shorter resonator-elements can be fixed.

(*2*) Determination of the cross section and the overall dimensions of the silencer considering the limitations imposed by the flow velocity and pressure required for the specific practical application.

(*3*) Order the resonators in pairs starting with the longest and shortest elements forming the first pair, then the next longer and shorter elements forming the second pair and so on.

(*4*) After the fixation of the main parameters involved, use a computer program based on the developed procedure in order to check the performance of the silencer.

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