A Method for Measuring Source Impedance and Tube Attenuation*

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If the active face, or acoustic output terminal, of a sinusoidal sound source moves as a plane piston, then the source can be characterized by a blocked pressure and an acoustic output impedance. If this piston is coupled to a microphone by means of a closed air column, the pressure at the microphone depends on the acoustic impedance of the microphone, on the impedance of the source, and on the air column. An expression for this pressure as a function of the length of the air column is developed, and data are presented which show how source impedance, tube attenuation and other quantities may be obtained.

THE GENERAL EXPRESSION

THE following definitions are used.

Z_0 is characteristic impedance of air column (in mechanical ohms).
Z_m is mechanical impedance of microphone.
Z_s is mechanical impedance of sound source.
Z_a is load impedance on source.
$Z_m/Z_0=\coth(\psi_m+j\phi_m)$.
$Z_s/Z_0=\coth(\psi_s+j\phi_s)$.
l is length of air column.
$(\alpha+j\beta)$ is propagation constant for air column.
p_l is pressure amplitude at microphone for arbitrary length of air column.
p_s is pressure amplitude at source.
p_B is pressure amplitude at source when piston is blocked.

Over a wide frequency range, sound in a tube can be treated as a combination of two plane waves travelling in opposite directions. They have the same phase velocity and attenuation coefficient. In Fig. 1, one may consider that one wave originates at the sound source and that the other is reflected at the microphone. It can be shown[1] that in such a case, the pressure amplitude, at a distance l from the termination is:

$$p_s=p_l\frac{\cosh(\psi_m+j\phi_m+\alpha l+j\beta l)}{\cosh(\psi_m+j\phi_m)}. \tag{1}$$

By using the fact that the particle velocity is proportional to the pressure gradient, one can show that the mechanical impedance presented by an air column of length l terminated by the microphone is:

$$Z_A=Z_0\coth(\psi_m+j\phi_m+\alpha l+j\beta l).$$

It is true that the pressure at the source differs from the blocked pressure by the factor:

$$p_s=p_B\frac{Z_A/Z_0}{Z_A/Z_0+Z_s/Z_0}$$

or

$$p_s=p_B\frac{\coth(\psi_m+j\phi_m+\alpha l+j\beta l)}{\coth(\psi_m+j\phi_m+\alpha l+j\beta l)+\coth(\psi_s+j\phi_s)}. \tag{2}$$

Between (1) and (2) we eliminate the pressure at the source

$$p_l\frac{\cosh(\psi_m+j\phi_m+\alpha l+j\beta l)}{\cosh(\psi_m+j\phi_m)}=p_B\frac{\dfrac{\cosh(\psi_m+j\phi_m+\alpha l+j\beta l)}{\sinh(\psi_m+j\phi_m+\alpha l+j\beta l)}}{\dfrac{\cosh(\psi_m+j\phi_m+\alpha l+j\beta l)}{\sinh(\psi_m+j\phi_m+\alpha l+j\beta l)}+\dfrac{\cosh(\psi_s+j\phi_s)}{\sinh(\psi_s+j\phi_s)}}$$

$$\frac{p_l}{p_B}=\frac{\sinh(\psi_s+j\phi_s)\cosh(\psi_m+j\phi_m)}{\cosh(\psi_s+j\phi_s)\sinh(\psi_m+j\phi_m+\alpha l+j\beta l)+\sinh(\psi_s+j\phi_s)\cosh(\psi_m+j\phi_m+\alpha l+j\beta l)}$$

$$=\frac{\sinh(\psi_s+j\phi_s)\cosh(\psi_m+j\phi_m)}{\sinh[(\psi_s+\psi_m+\alpha l)+j(\phi_s+\phi_m+\beta l)]}.$$

To avoid measuring phase, expand into real and imaginary parts and deal with square of magnitude.

$$\frac{p_l^2}{p_B^2}=\frac{(\cosh^2\psi_s-\cos^2\phi_s)(\cosh^2\psi_m-\sin^2\phi_m)}{\cosh^2(\psi_s+\psi_m+\alpha l)-\cos^2(\phi_s+\phi_m+\beta l)}.$$

If microphone has finite impedance, pressure at zero length is not the blocked pressure, but is rather given by:

$$\frac{p_0^2}{p_B^2}=\frac{(\cosh^2\psi_s-\cos^2\phi_s)(\cosh^2\psi_m-\sin^2\phi_m)}{\cosh^2(\psi_s+\psi_m)-\cos^2(\phi_s+\phi_m)}.$$

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[1] P. M. Morse, *Vibration and Sound* (McGraw-Hill Book Company, Inc., 1948), p. 239.

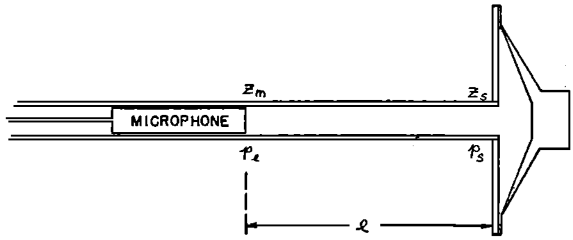

FIG. 1. Schematic view of apparatus.

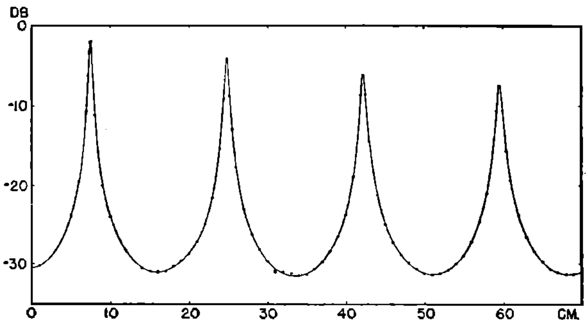

FIG. 2. Observed pressure as a function of length for an open-end tube.

The pressure for arbitrary length can then be specified relative to the measured pressure for zero length of air column.

$$\frac{p_l^2}{p_0^2}=\frac{\cosh^2(\psi_s+\psi_m)-\cos^2(\phi_s+\phi_m)}{\cosh^2(\psi_s+\psi_m+\alpha l)-\cos^2(\phi_s+\phi_m+\beta l)}. \quad (3)$$

This is a workable general expression.

It is a very good assumption that $\cosh^2(\psi_s+\psi_m+\alpha l)$ varies slowly enough so that pressure maxima occur when

$$\cos^2(\phi_s+\phi_m+\beta l)=1$$

and minima occur when

$$\cos^2(\phi_s+\phi_m+\beta l)=0.$$

That is:

$$\phi_s+\phi_m+\beta l_{\max}=n\pi \quad n=0,\,1,\,2,\text{ etc.} \quad (4)$$

$$\phi_s+\phi_m+\beta l_{\min}=[(2n+1)/2]\pi \quad n=0,\,1,\,2,\text{ etc.} \quad (5)$$

If we put these results into (3) and leave l continuous, so as to get a continuous curve passing through all maxima (or minima), we have:

$$\frac{p l_{\max}^2}{p_0^2}=\frac{\cosh^2(\psi_s+\psi_m)-\cos^2(\phi_s+\phi_m)}{\sinh^2(\psi_s+\psi_m+\alpha l)}. \quad (6)$$

$$\frac{p l_{\min}^2}{p_0^2}=\frac{\cosh^2(\psi_s+\psi_m)-\cos^2(\phi_s+\phi_m)}{\cosh^2(\psi_s+\psi_m+\alpha l)}. \quad (7)$$

Combining (6) and (7) leads to a simple expression relating the ratio of maximum to minimum pressure and some of the parameters describing source impedance and tube attenuation.

$$\frac{p l_{\max}}{p l_{\min}}=\coth(\psi_s+\psi_m+\alpha l). \quad (8)$$

As already noted, the locations of the maxima (or minima) determine the other quantities needed.

There is a similarity between these expressions and those relating to the standing wave method of measuring acoustic impedance. In that case, the source and termination remain fixed and the pressure is measured by a moving microphone. The ratio of maximum to minimum pressure and the locations of the minima depend on the acoustic impedance of the termination and on the propagation constant of the air column, whereas here the source impedance enters also.

AN EXAMPLE

The use of the above relations will be discussed relative to a set of data, assuming that one is interested in as much precision as the method will afford.

In the first place, it may be noted that $(\psi_s+\psi_m)$ and $(\phi_s+\phi_m)$ occur as sums, so that it is impossible by this method to determine source and microphone impedances separately. If either is known, the other can be determined. In these calculations, the well justified

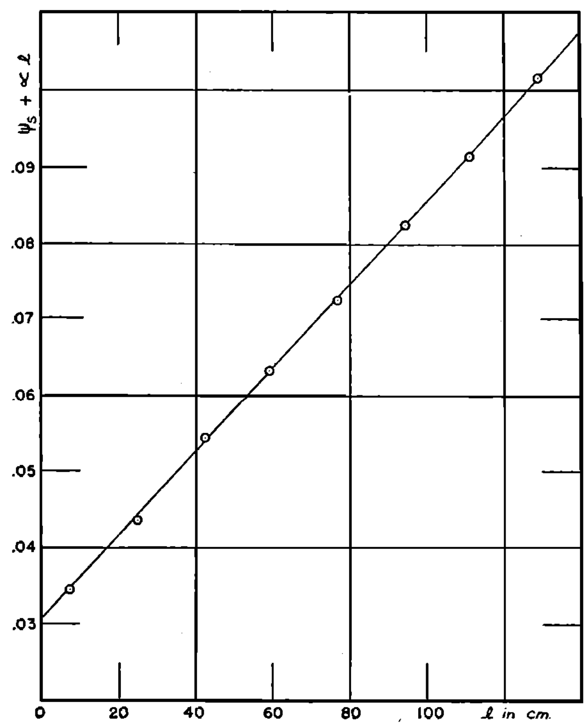

FIG. 3. Plot from which tube attenuation and source parameter are determined.

assumption will be made that the microphone is a rigid termination for the air column, that is $\psi_m = 0$ and $\phi_m = 0$.

The microphone was free to slide in a tube, one end of which was open to the room. The source of sound at an accurately controlled frequency of 1000 c.p.s. was a loudspeaker open to the room at a distance of about two feet from the open end of the tube. The source impedance in this case is essentially the same as the "end correction" impedance for an unflanged pipe. The tube was $1\frac{3}{8}$ in. in inside diameter and had a $\frac{1}{8}$ in. wall thickness.

A plot of microphone output for constant speaker voltage is shown as Fig. 2. By means of (8), each maximum yields a value of $(\psi_s+0+\alpha l)$. Plotting these values against the appropriate distances, one gets a straight line for which the slope yields α and the intercept is ψ_s. Figure 3 is such a plot. Steps leading from Fig. 2 to Fig. 3 are given in Table I.

From Fig. 3, the following values were determined:

$$\psi_s = 0.0308 \text{ and } \alpha = 5.50\times 10^{-4}\ \text{cm}^{-1}.$$

From a similar determination using the locations of the maxima in conjunction with Eq. (4),

$$\phi_s = 1.77 \text{ and } \beta = 0.181\ \text{cm}^{-1}.$$

The impedance of the source is found by computation to be primarily a mass reactance of low magnitude

$$\begin{aligned} Z_s/Z_0 &= \coth(0.0308+j1.77) \\ &= \tanh(0.0308+j0.20) \\ &= 0.03+j0.20. \end{aligned}$$

TABLE I.

l	$20 \log p_{max}/p_{min}$	p_{min}/p_{max}	$\psi_s+\alpha l$
7.4	29.3	0.0343	0.0343
24.9	27.2	0.0436	0.0436
42.25	25.3	0.0543	0.0543
59.6	24.0	0.0631	0.0632
77.0	22.8	0.0724	0.0725
94.4	21.7	0.0823	0.0824
111.7	20.8	0.0914	0.0916
129.0	19.9	0.1015	0.1018

CONCLUSION

The method described represents a sensitive and accurate means of measuring attenuation and velocity of sound in gases or vapors at middle-audio frequencies. With additional information about the microphone, it provides a way of measuring rapidly the acoustic impedance of any sound source which can be coupled to an air column. It is distinctly similar to other methods involving standing waves in air columns, but is sufficiently different that it should be useful in instances where the other methods are not applicable.