

## Operational Amplifiers Part V of VI

### Every Amplifier is Waiting to Oscillate and Every Oscillator is Waiting to Amplify

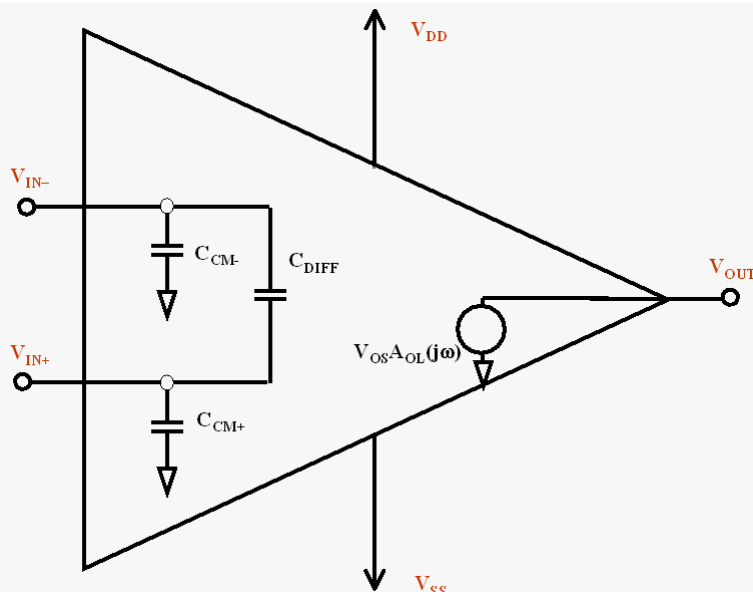
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What is op amp circuit stability, and how do you know when you are on the "hairy edge?" Typically, there is a feedback system around the op amp to limit the variability and reduce the magnitude of the open-loop gain from part to part making the stability of your amplifier circuit dependant on the variability of the resistors in your circuit, not the op amps. Using resistors around your op amp provides circuit "stability." At least you hope that a predictable gain is ensured. But, it is possible to design an amplifier circuit that does quite the opposite.

You can design an amplifier circuit that is extremely unstable to the point of oscillation. In these circuits, the closed gain is somewhat trivial, because an oscillation is "swamping out" your results at the output of the amplifier. In a closed-loop amplifier system, stability can be ensured if you know the phase margin of the amplifier system. In this evaluation, the Bode stability-analysis technique is commonly used. With this technique, the magnitude (in dB) and phase response (in degrees) of both the open-loop response of the amplifier and circuit-feedback factor are included in the Bode plot. This article looks at these concepts and makes suggestions on how to avoid the design of a "singing" circuit when your primary feedback is frequency stability.

### The Internal Basics of the Operational-Amplifier Block Diagram

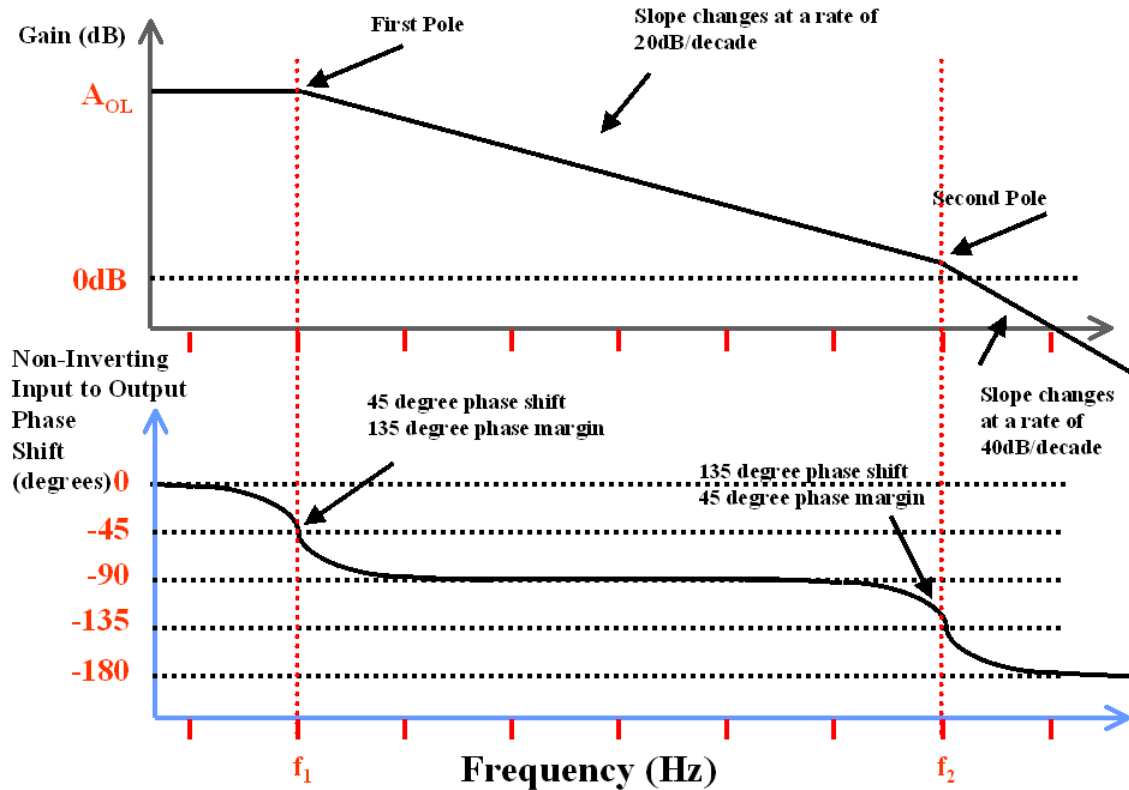


**Fig. 1: The Voltage-Feedback Operational Amplifier Frequency Model**

Before getting started on the frequency analysis of an amplifier circuit, let's review a few amplifier topology concepts. Fig. 1 shows the critical internal op amp elements that you need to be familiar with if you engage in a frequency analysis. This amplifier has five

terminals, as expected, but it also has parasitics, such as input capacitance ( $C_{DIFF}$  and  $C_{CM}$ ) and the frequency dependent open-loop gain includes them to ensure that the interaction of the external input-source parasitics and the feedback parasitics can be taken into account in a frequency evaluation. This model also has the internal open-loop gain over frequency [ $A_{OL}(j\omega)$ ]. These two parameters ensure that the internal parasitics of the output stage are in the analysis.

Fig. 2 shows the frequency response of a typical voltage-feedback amplifier using the Bode-plot method.



**Fig. 2: Gain/Phase Plots Show Frequency Behavior Of Voltage-Feedback Amplifier**

In this simple representation, the plots illustrate the gain (top) and phase (bottom) responses of a typical amplifier modeled with a simple second-order transform function that has two dominant poles. The first pole occurs at lower frequencies, typically between 10 Hz to 1 kHz (depending on the gain-bandwidth product of the amplifier). The second pole resides at higher frequencies. This pole occurs at a higher frequency than the 0 dB gain (dB) frequency. If it is lower, the amplifier is usually unstable in a unity-gain circuit.

The units of the y-axis of the gain plot (Fig. 2) are dB.

Ideally, the open-loop gain of an amplifier is equal to the magnitude of the ratio of the voltage at the output terminal, divided by the difference of the voltages applied between the two input terminals.

$$A_{OL}(\text{dB}) = 20\log [| V_{OUT} \div (V_{IN+} - V_{IN-}) |]$$

It would be nice if this open-loop gain ratio were infinite. But, in reality, the complete frequency response of the open-loop gain,  $AOL(j\omega)$ , is less than ideal at dc and FT attenuates at a rate of 20 dB/decade starting the frequency,  $f_1$ , (Fig. 2, again) where the first pole in the transfer function appears.

Usually the first pole of the open-loop response of an operational amplifier occurs between 1 Hz to 1 kHz. The second pole occurs at a higher frequency,  $f_2$ , nearer to where the open-loop gain-curve crosses 0 dB. The gain response of an amplifier then starts to fall off at 40 dB/decade.

The units of the y-axis of the phase plot (Fig. 2, bottom plot) are degrees ( $^\circ$ ). You can convert degrees to radians with the following formula:

$$\text{Phase in radians} = (\text{Phase in degrees}) * (2\pi \div 360^\circ)$$

Phase in degrees can be translated to phase delay or group delay (seconds) with the following formula:

$$\text{Phase delay} = - (\delta\text{phase} \div \delta f) \div 360^\circ$$

The same x-axis frequency scale aligns across both plots.

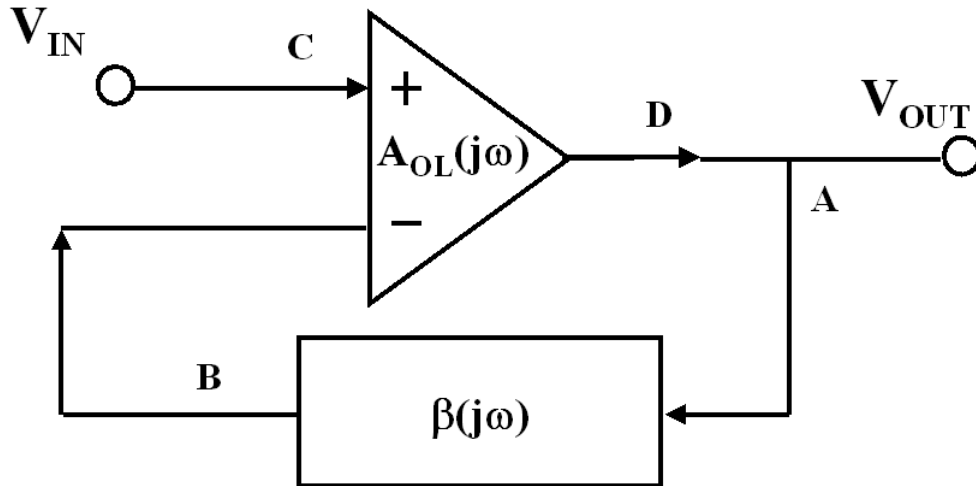
The phase response of an amplifier in this open-loop configuration is also predictable. The phase shift or change from the non-inverting input to the output of the amplifier is zero degrees at dc. Conversely, the phase shift from the inverting input terminal to the output is equal to  $-180^\circ$  at dc.

At one decade ( $1/10 f_1$ ) before the first pole,  $f_1$ , the phase relationship of non-inverting input to output has already started to change, about  $-5.7^\circ$ . At the frequency where the first pole appears in the open-loop gain curve ( $f_1$ ), the phase margin has dropped to  $-45^\circ$ . The phase continues to drop for another decade ( $10 * f_1$ ) where it is  $5.7^\circ$  above its final value of  $-90^\circ$ . These phase-response changes are repeated for the second pole,  $f_2$ .

What is important to understand are the ramifications of changes in this phase relationship with respect to the input and output of the amplifier. One frequency decade past the second pole, the phase shift of the non-inverting input to output is about  $-180^\circ$ . At this same frequency, the phase shift of the inverting input to output is close to zero or about  $-360^\circ$ . With this type of shift,  $V_{IN+}$  is actually inverting the signal to the output. In other words, the rôles of the two inputs have reversed. If the rôle of either of the inputs changes like this, the amplifier will ring as the signal goes from the input to the output in a closed-loop system. The only thing stopping this condition from occurring with the standalone amplifier is that the gain drops below 0 dB. If the open-loop gain of the amplifier drops below 0 dB in a closed-loop system, the feedback is essentially turned "OFF".

## Stability In Closed-Loop Amplifier Systems

Typically, op amps have a feedback network around them. This reduces the variability of the open-loop gain response from part to part. Fig. 3 shows a block diagram of this type of network.



**Fig. 3: Block Of Amplifier With Gain Cell,  $A_{OL}$ , And Feedback Network,  $\beta$**

In Fig. 3,  $\beta(j\omega)$  represents the feedback factor. Due to the fact that the open-loop gain of the amplifier ( $A_{OL}$ ) is relatively large and the feedback factor is relatively small, a fraction of the output voltage is fed back to the inverted input of the amplifier. This configuration sends the output back to the inverting terminal, creating a negative-feedback (NFB) condition. If  $\beta$  were fed back to the non-inverting terminal, this small fraction of the output voltage would be added instead of being subtracted, which would be positive feedback, and the output would eventually saturate.

### Closed-Loop Transfer Function

If you analyze the loop in Fig. 3, you must assume an output voltage exists. This makes the voltage at A equal to  $V_{OUT}(j\omega)$ . The signal passes through the feedback system,  $\beta(j\omega)$ , so that the voltage at B is equal to  $\beta(j\omega)V_{OUT}(j\omega)$ . The voltage, or input voltage, at C is added to the voltage at B; C is equal to  $[V_{IN}(j\omega) - \beta(j\omega)V_{OUT}(j\omega)]$ . With the signal passing through the gain cell,  $A_{OL}(j\omega)$ , the voltage at point D is equal to  $A_{OL}(j\omega)[V_{IN}(j\omega) - \beta(j\omega)V_{OUT}(j\omega)]$ . This voltage is equal to the original node, A, or  $V_{OUT}(j\omega)$ . The formula that describes this complete closed-loop system is:

$$A = D, \text{ or,} \\ V_{OUT}(j\omega) = A_{OL}(j\omega)[V_{IN}(j\omega) - \beta(j\omega)V_{OUT}(j\omega)]$$

By collecting the terms, the manipulated transfer function becomes:

$$V_{OUT}(j\omega) \div V_{IN}(j\omega) = A_{OL}(j\omega) \div [1 + A_{OL}(j\omega)\beta(j\omega)]$$

This equation is essentially equal to the closed-loop gain of the system, or  $A_{CL}(j\omega)$ .

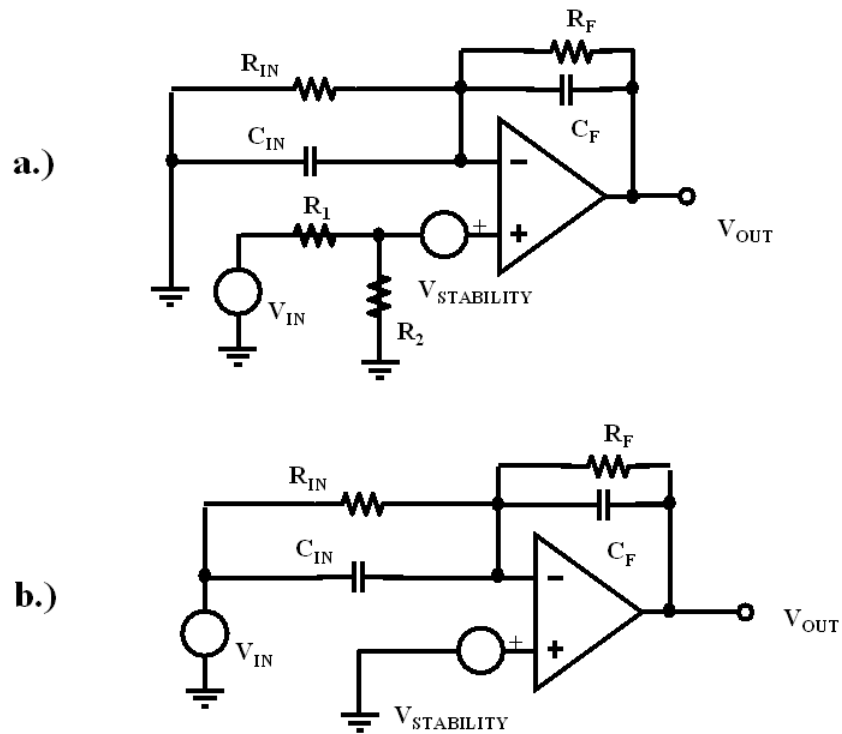
This is a very important result. If the open-loop gain ( $A_{OL}(j\omega)$ ) of the amplifier is allowed to approach infinity, the response of the feedback factor can easily be evaluated as:

$$A_{CL}(j\omega) = 1 \div \beta(j\omega) \text{ -- aka } 1/\beta$$

This formula allows an easy determination of the frequency stability of an amplifier's closed-loop system.

### Calculation of $1/\beta$

The easiest technique to calculate  $1/\beta$  is to place a source directly on the non-inverting input of the amplifier and ignore error contributions from the amplifier. You could argue that this does not give the appropriate circuit closed-loop-gain equation for the actual signal, and this is true. But, you can determine the level of circuit stability.



**Fig. 4: Input Signal In Circuit a.) At Dc Is Gained By  $(R_2/(R_1+R_2))(1+R_F/R_{IN})$ . The Input Signal In Circuit b.) Has Dc Gain of  $-R_F/R_{IN}$ . Neither Match  $1/\beta$**

In Figs. 4a/b,  $V_{STABILITY}$  is a fictitious voltage source equal to 0 V. It is used for the  $1/\beta$  stability analysis. Note that this source is not the actual application-input source.

Assuming that the open-loop gain of the amplifier is infinite, the transfer function of this circuit is equal to:

$$V_{OUT}/V_{STABILITY} = 1/\beta$$

$$1/\beta = 1 + (R_F \parallel C_F)/(R_{IN} \parallel C_1)$$

where,  $C_1 = C_{IN} + C_{CM}$  for Fig. 4a, and  $C_1 = C_{IN} + C_{CM} + C_{DIFF}$  for Fig. 4b

or:

$$1/\beta(j\omega) = (R_{IN}((j\omega)R_F C_F + 1) + R_F((j\omega)R_{IN} C_1 + 1)) / R_{IN}((j\omega)R_F C_F + 1)$$

In the equation above, when  $\omega$  is equal to zero:

$$1/\beta(j\omega) = 1 + R_F/R_{IN}$$

As  $\omega$  approaches infinity:

$$1/\beta(j\omega) = 1 + C_1/C_F$$

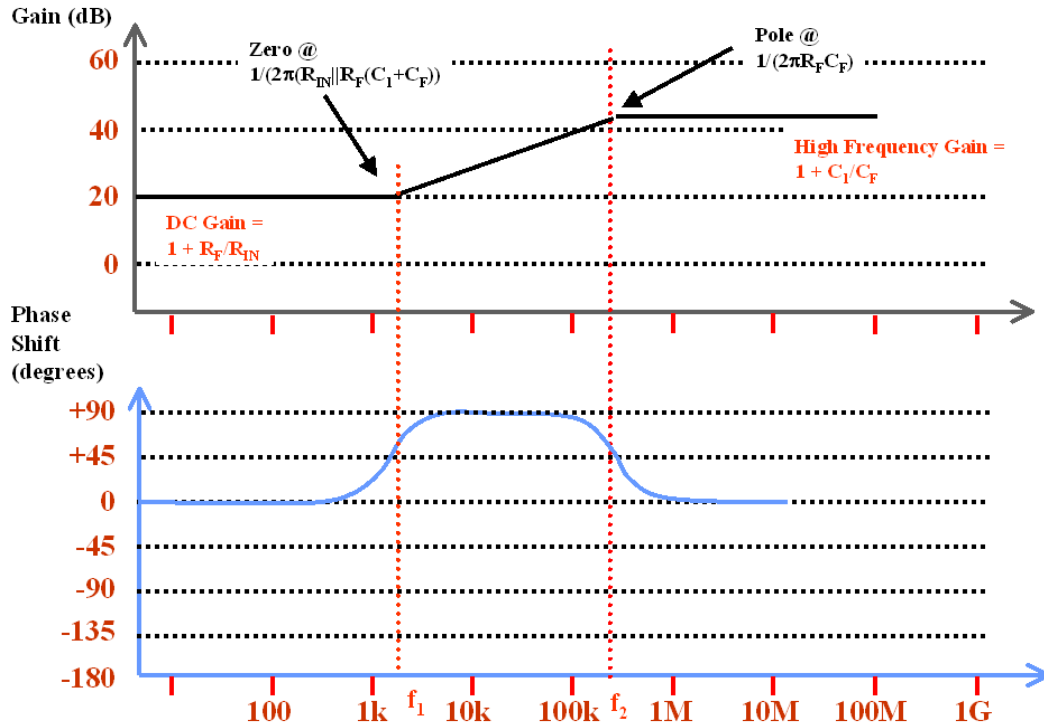
The transfer function has one zero and one pole. The zero is located at:

$$f_z = 1/(2\pi R_{IN} \| R_F [C_1 + C_F])$$

And the pole is at:

$$f_p = 1/(2\pi R_F C_F)$$

The Bode plot of the  $1/\beta(j\omega)$  transfer function of the circuit in Fig. 4a is shown in Fig. 5.



**Fig. 5: Bode Plots Of Inverse Feedback Factor ( $1/\beta$ ) For Fig. 4a**

Once again (Fig. 4b) the input source used for this analysis is not the same as the actual application circuit and amplifier stability is determined in the same manner. The closed-loop transfer function, using  $V_{STABILITY}$ , is equal to:

$$V_{OUT}/V_{STABILITY} = 1/\beta$$

$$1/\beta = 1 + (R_F \| C_F)/(R_{IN} \| C_{IN})$$

or,

$$1/\beta(j\omega) = (R_{IN}((j\omega)R_F C_F + 1) + R_F((j\omega)R_{IN} C_1 + 1)) / R_{IN}((j\omega)R_F C_F + 1)$$

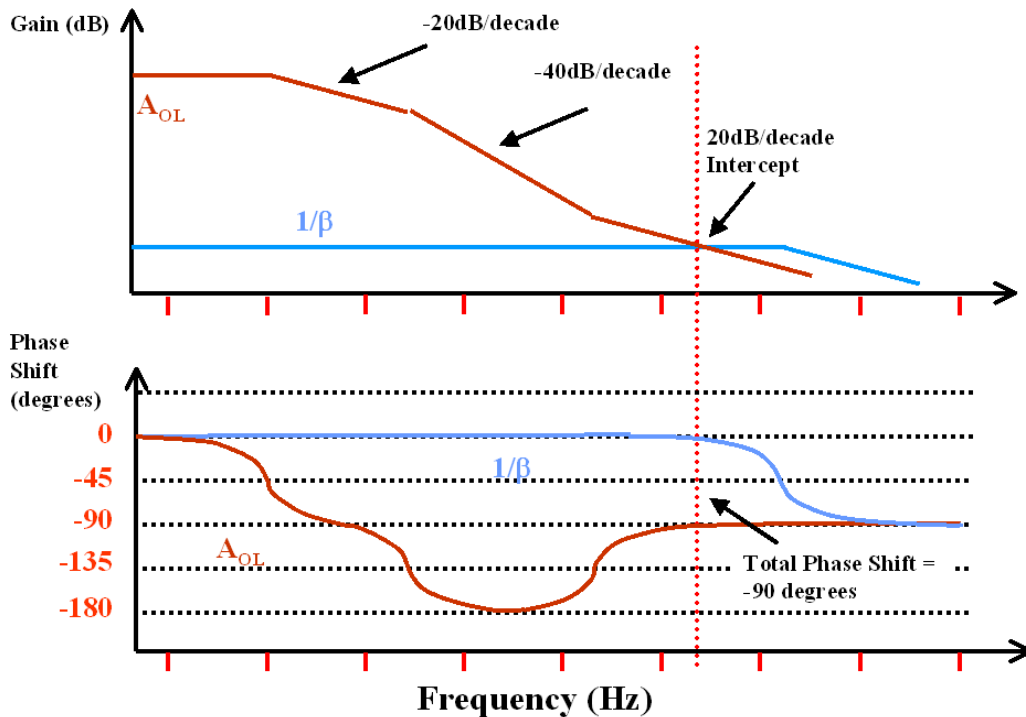
Note: the transfer functions of  $1/\beta$  between Figs. 4a and 4b are identical.

## Determining System Stability

If you know the phase margin, you can determine the stability of the closed-loop amplifier system. In this analysis the Bode stability-analysis technique is commonly used. With this approach, the magnitude (in dB) and phase response of both the open-loop response of the amplifier and circuit feedback factor are included in a Bode plot.

The system closed-loop gain is equal to the lesser (in magnitude) of the two gains. The phase response of the system is equal to the open-loop gain phase shift minus the inverted feedback factor's phase shift.

The stability of the system is defined at the frequency where the open-loop gain of the amplifier intercepts the closed-loop gain response. At this point, the theoretical phase shift of the system should be greater than  $-180^\circ$ . In practice, the system phase shift should be smaller than  $-135^\circ$ . This technique is illustrated in Figs. 6 - 9. The cases presented in Figs. 6 and 7 represent stable systems. Those in Figs. 8 and 9 represent unstable systems.

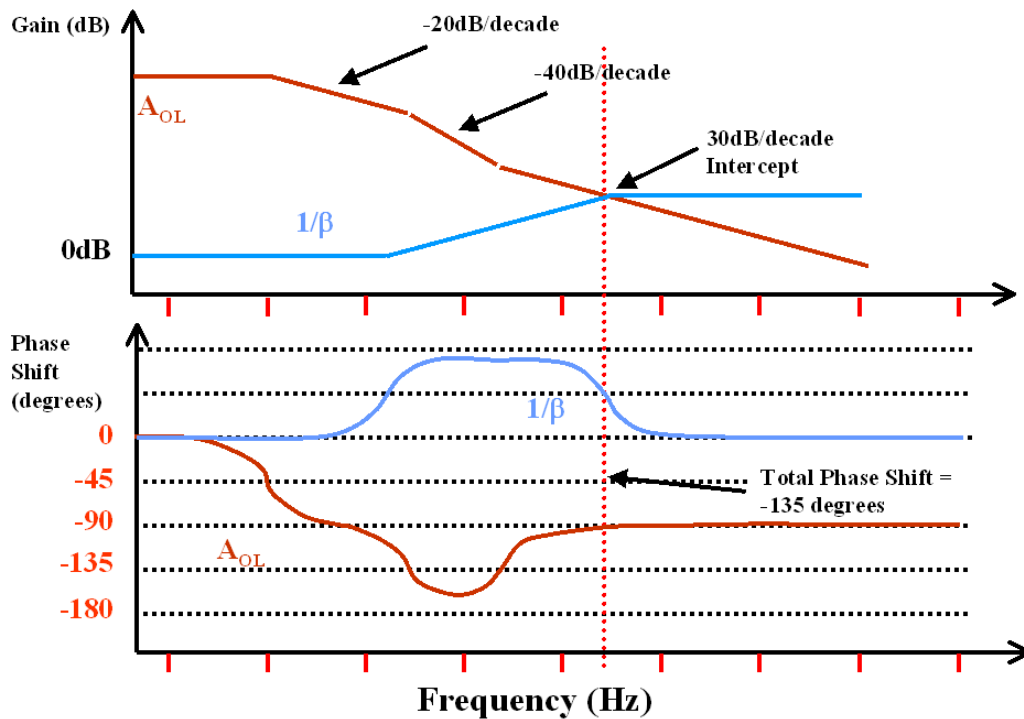


**Fig. 6: Stable System With Phase Shift Of  $-90^\circ$  At Intercept Of  $A_{OL} / 1/\beta$  Curves**

In Fig. 6, the open-loop gain of the amplifier ( $A_{OL}(j\omega)$ ) starts with a 0 dB change in frequency and quickly changes to a  $-20$  dB/decade slope. At the frequency where the first pole occurs, the phase shift is  $-45^\circ$ . At that frequency, one decade above the first pole, the phase shift is approximately  $-90^\circ$ . As the gain slope progresses with frequency, a second pole is introduced, causing the open-loop-gain response to change to  $-40$  dB/decade. Once again, this is accompanied with a phase change. The third incident that occurs in this response is where a zero is introduced and the open-loop gain response returns back to a  $-20$  dB/decade slope.

The  $1/\beta$  curve in this same graph starts with a 0 dB change with frequency. This curve remains flat with increased frequency until the very end of the curve, where a pole occurs and the curve starts to attenuate  $-20$  dB/decade.

The point of interest in Fig. 6 is where the  $A_{OL}(j\omega)$  curve intersects the  $1/\beta$  curve. The  $20$  dB/decade of closure between the two curves suggests the phase margin of the system and in turn predicts the stability. In this situation, the amplifier is contributing a  $-90^\circ$  phase shift and the feedback factor is contributing a  $0^\circ$  phase shift. The stability of the system is determined at this intersection point. The system phase shift is calculated by subtracting the  $1/\beta(j\omega)$  phase shift from the  $A_{OL}(j\omega)$  phase shift. In this case, the system phase shift is  $-90^\circ$ . Theoretically, a system is stable if the phase shift is between zero and  $-180^\circ$ . In practice, you should design to a phase shift of  $-135^\circ$ , or smaller.



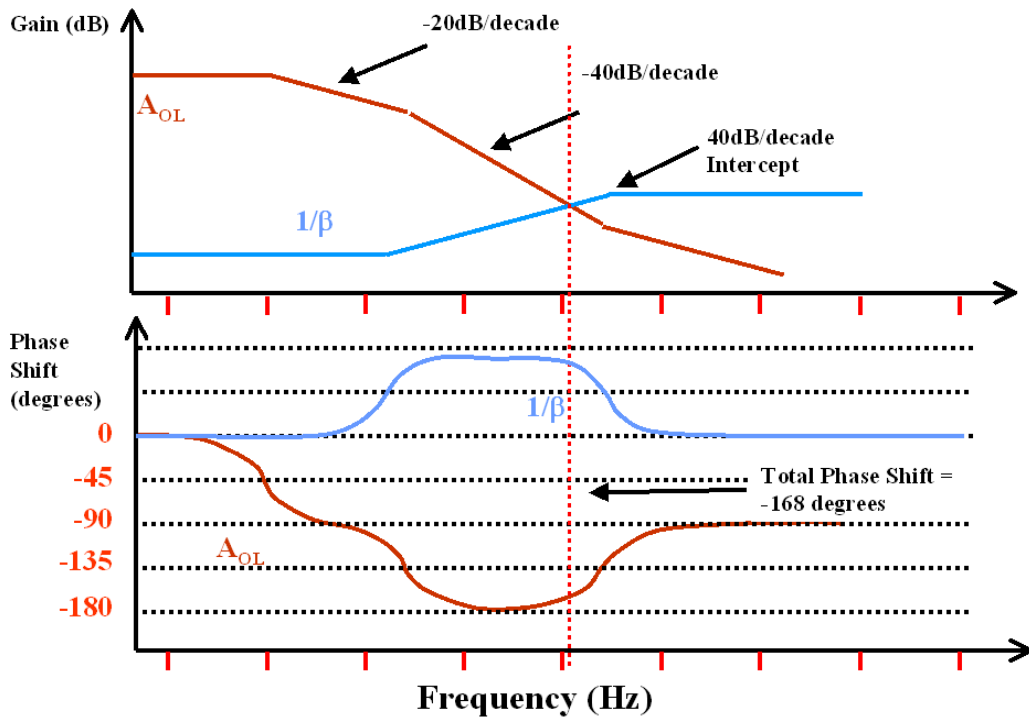
**Fig. 7: Marginally Stable System With  $-135^\circ$  Phase Shift At Intersection**

In the case presented in Fig. 7, the intersection between the  $A_{OL}(j\omega)$  and  $1/\beta(j\omega)$  curves suggests a marginally-stable system.  $A_{OL}(j\omega)$  is changing  $-20$  dB/decade and  $1/\beta(j\omega)$  is changing from  $+20$  dB/decade to  $0$  dB/decade. The phase shift of the  $A_{OL}(j\omega)$  curve is  $-90^\circ$ , that of the  $1/\beta(j\omega)$  curve is  $+45^\circ$ , and the system phase shift is equal to  $-135^\circ$ .

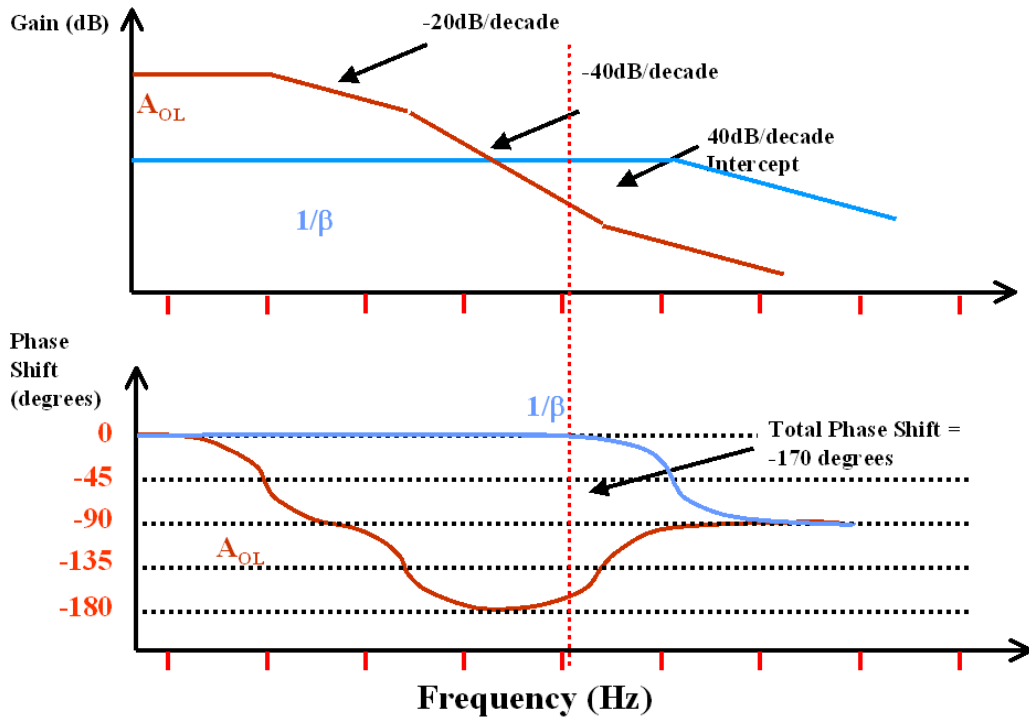
Although this system appears to be stable, ie the phase shift is between zero and  $-180^\circ$ , the circuit implementation will not be as clean as calculations or simulations might imply. Parasitic capacitance and inductance on the board can contribute additional phase errors. Consequently, this system is "marginally stable" with this magnitude of phase shift. This closed-loop circuit has a significant overshoot and ringing with a step response.



In Fig. 8,  $A_{OL}(j\omega)$  is changing at a rate of  $-20$  dB/decade.  $1/\beta(j\omega)$  is changing at a rate of  $+20$  dB/decade. The rate of closure of these two curves is  $40$  dB/decade and the system phase shift is  $-168^\circ$ . The stability of this system is very questionable.



**Fig. 8: With Layout Parasitics In A Practical Circuit This System Is Unstable**



**Fig. 9: This System Is Also Unstable**

In Fig. 9,  $A_{OL}(j\omega)$  is changing at a rate of  $-40$  dB/decade.  $1/\beta(j\omega)$  is changing at a rate of  $0$  dB/decade. The rate of closure in these two curves is  $40$  dB/decade, indicating a phase shift of  $-170^\circ$ . The stability of this system is also questionable.

## Conclusion

At the beginning of this article you were asked, "What is operational-amplifier circuit stability and how do you know when you are on the 'hairy edge?'" There are many definitions of stability in analog such as, unchanging over temperature; unchanging from lot-to-lot; noisy signals, etc. But, an analog circuit becomes critically unstable when the output unintentionally oscillates without excitation. This kind of stability problem stops the progress of circuit design until you can track it down.

You can only evaluate this kind of stability in the frequency domain. A quick paper-and-pencil examination of your circuit readily provides insight into your oscillation problem. The relationship between the open-loop gain of your amplifier and the feedback system, over frequency, quickly identifies the source of the problem. If you use gain and phase Bode plots, you can estimate where these problems reside. If you keep the closed-loop phase shift below  $-135^\circ$  your circuit oscillations do not occur and ringing will be minimized. If you do this work up front with amplifiers, you can avoid those dreadful designs that kick into an unwanted song, aka "The Amplifier Circuit Blues."

