## TECHNICAL NOTE

## VOLTAGE CONTROLLED AMPLIFIER ANALYSIS

### 1.0 Circuit



Figure 1.0-1

### 1.1 Analysis model



Figure 1.1-1
$r_{\text {on }}{ }^{\prime}$ is the controlled FET on resistance with non-zero gate to source voltage, Ohms $r_{\text {on }}$ is the reference FET on resistance with zero gate to source voltage, Ohms
A is the operational amplifier open loop gain,
V/V

### 1.2 Analysis

Define $\alpha$, the attenuation factor from the signal input terminal to the amplifier inverting input, and $\beta$, the attenuation factor from the output terminal to the amplifier inverting input;

$$
\begin{align*}
& \alpha=\frac{\mathrm{R}_{2} \cdot \mathrm{r}_{\text {on }}^{\prime}}{\mathrm{R}_{1} \cdot \mathrm{r}_{\text {on }}^{\prime}+\mathrm{R}_{1} \cdot \mathrm{R}_{2}+\mathrm{R}_{2} \cdot \mathrm{r}_{\text {on }}^{\prime}} \\
& \beta=\frac{\mathrm{R}_{1} \cdot \mathrm{r}_{\text {on }}^{\prime}}{\mathrm{R}_{1} \cdot \mathrm{r}_{\text {on }}^{\prime}+\mathrm{R}_{1} \cdot \mathrm{R}_{2}+\mathrm{R}_{2} \cdot \mathrm{r}_{\text {on }}^{\prime}}
\end{align*}
$$

Define $\sigma$, the attenuation factor from the signal input terminal to the amplifier non-inverting input.

$$
\sigma=\frac{\mathrm{R}_{2} \cdot \mathrm{r}_{\text {on }}}{\mathrm{R}_{1} \cdot \mathrm{r}_{\mathrm{on}}+\mathrm{R}_{1} \cdot \mathrm{R}_{2}+\mathrm{R}_{2} \cdot \mathrm{r}_{\text {on }}}
$$

solving for $\mathrm{V}_{\mathrm{o}}$,

$$
\mathrm{V}_{\mathrm{O}}=\mathrm{A} \cdot\left(\mathrm{~V}_{\mathrm{a}}-\mathrm{V}_{\mathrm{b}}\right)
$$

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{a}}=\sigma \cdot \mathrm{V}_{\mathrm{S}} \\
& \mathrm{~V}_{\mathrm{b}}=\alpha \cdot \mathrm{V}_{\mathrm{S}}+\beta \cdot \mathrm{V}_{\mathrm{O}} \\
& \mathrm{~V}_{\mathrm{O}}=\mathrm{A} \cdot \sigma \cdot \mathrm{~V}_{\mathrm{S}}-\mathrm{A} \cdot \alpha \cdot \mathrm{~V}_{\mathrm{S}}-\mathrm{A} \cdot \beta \cdot \mathrm{~V}_{\mathrm{O}} \\
& \mathrm{~V}_{\mathrm{O}}=\frac{\mathrm{A} \cdot(\sigma-\alpha) \cdot \mathrm{V}_{\mathrm{S}}}{1+\mathrm{A} \cdot \beta}
\end{aligned}
$$

solving for voltage gain,

$$
\mathrm{A}_{\mathrm{V}}=\frac{\mathrm{V}_{\mathrm{O}}}{\mathrm{~V}_{\mathrm{S}}}=\frac{\mathrm{A} \cdot(\sigma-\alpha)}{1+\mathrm{A} \cdot \beta}
$$

if $\mathrm{A} \rightarrow \infty$,

$$
\begin{aligned}
& \mathrm{A}_{\mathrm{V}}=\frac{\sigma-\alpha}{\beta} \\
& \mathrm{A}_{\mathrm{V}}=\frac{\sigma}{\beta}-\frac{\alpha}{\beta}
\end{aligned}
$$

using Eq:1.2-1 and Eq:1.2-2,

$$
\begin{aligned}
& \mathrm{A}_{\mathrm{V}}=\sigma \cdot\left(\frac{\mathrm{R}_{1} \cdot \mathrm{r}_{\mathrm{on}}^{\prime}+\mathrm{R}_{1} \cdot \mathrm{R}_{2}+\mathrm{R}_{2} \cdot \mathrm{r}_{\mathrm{on}}^{\prime}}{\mathrm{R}_{1} \cdot \mathrm{r}_{\mathrm{on}}^{\prime}}\right)-\frac{\mathrm{R}_{2}}{\mathrm{R} 1} \\
& \mathrm{~A}_{\mathrm{V}}=\sigma \cdot\left(1+\frac{\mathrm{R}_{2}}{\mathrm{r}_{\mathrm{on}}^{\prime}}+\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}}\right)-\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}}
\end{aligned}
$$

$$
\mathrm{A}_{\mathrm{v}}=\sigma \cdot\left(1+\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}}\right)+\sigma \cdot \frac{\mathrm{R}_{2}}{\mathrm{r}_{\mathrm{on}}^{\prime}}-\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}}
$$

from FET Junction Transistors, Carl David Todd, Wiley, p. 122
$\mathrm{r}_{\mathrm{on}}^{\prime}=\mathrm{r}_{\mathrm{on}} \cdot\left(1-\frac{\mathrm{V}_{\mathrm{GS}}}{\mathrm{V}_{\mathrm{P}}}\right)^{-1}$
Eq:1.2-6
$\mathrm{V}_{\mathrm{GS}}=$ Gate to source voltage
$\mathrm{V}_{\mathrm{P}}=$ Pinch-off voltage of JFET

$$
\mathrm{A}_{\mathrm{V}}=\sigma \cdot\left(1+\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}}\right)+\sigma \cdot \frac{\mathrm{R}_{2}}{\mathrm{r}_{\mathrm{on}}} \cdot\left(1-\frac{\mathrm{V}_{\mathrm{GS}}}{\mathrm{~V}_{\mathrm{P}}}\right)-\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}}
$$

from Eq:1.2-3,

$$
\begin{aligned}
& \sigma=\frac{\mathrm{R}_{2} \cdot \mathrm{r}_{\text {on }}}{\mathrm{R}_{1} \cdot \mathrm{r}_{\text {on }}+\mathrm{R}_{1} \cdot \mathrm{R}_{2}+\mathrm{R}_{2} \cdot \mathrm{r}_{\text {on }}} \\
& \mathrm{A}_{\mathrm{V}}=\frac{\mathrm{R}_{2} \cdot \mathrm{r}_{\text {on }}}{\mathrm{R}_{1} \cdot \mathrm{r}_{\text {on }}+\mathrm{R}_{1} \cdot \mathrm{R}_{2}+\mathrm{R}_{2} \cdot \mathrm{r}_{\text {on }}} \cdot\left(1+\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}}\right)+\frac{\mathrm{R}_{2}^{2}}{\mathrm{R}_{1} \cdot \mathrm{r}_{\text {on }}+\mathrm{R}_{1} \cdot \mathrm{R}_{2}+\mathrm{R}_{2} \cdot \mathrm{r}_{\mathrm{on}}} \cdot\left(1-\frac{\mathrm{V}_{\mathrm{GS}}}{\mathrm{~V}_{\mathrm{P}}}\right)-\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}}
\end{aligned}
$$

now if $r_{\text {on }}$ is very small,

$$
\begin{aligned}
& \frac{\mathrm{R}_{2} \cdot \mathrm{r}_{\text {on }}}{\mathrm{R}_{1} \cdot \mathrm{r}_{\text {on }}+\mathrm{R}_{1} \cdot \mathrm{R}_{2}+\mathrm{R}_{2} \cdot \mathrm{r}_{\text {on }}} \rightarrow 0 \\
& \frac{\mathrm{R}_{2}^{2}}{\mathrm{R}_{1} \cdot \mathrm{r}_{\text {on }}+\mathrm{R}_{1} \cdot \mathrm{R}_{2}+\mathrm{R}_{2} \cdot \mathrm{r}_{\text {on }}} \rightarrow \frac{\mathrm{R}_{2}}{\mathrm{R}_{1}}
\end{aligned}
$$

then,

$$
\begin{aligned}
& \mathrm{A}_{\mathrm{V}}=\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}} \cdot\left(1-\frac{\mathrm{V}_{\mathrm{GS}}}{\mathrm{~V}_{\mathrm{P}}}\right)-\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}} \\
& \mathrm{~A}_{\mathrm{V}}=-\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}} \cdot \frac{\mathrm{~V}_{\mathrm{GS}}}{\mathrm{~V}_{\mathrm{P}}}
\end{aligned}
$$

Eq:1.2-7
for $R_{2}=k \cdot R_{1}$,

$$
A_{V}=-k \cdot \frac{V_{G S}}{V_{P}}
$$

$\mathrm{V}_{\mathrm{GS}} \rightarrow \mathrm{V}_{\mathrm{C}}$

$$
A_{\mathrm{V}}=-\mathrm{k} \cdot \frac{\mathrm{~V}_{\mathrm{C}}}{\mathrm{~V}_{\mathrm{P}}}
$$

