VOLTAGE CONTROLLED AMPLIFIER ANALYSIS

1.0 Circuit



Figure 1.0-1

1.1 Analysis model



Figure 1.1-1

 $\begin{array}{ll} r_{on}' \mbox{ is the controlled FET on resistance with non-zero gate to source voltage,} & Ohms \\ r_{on} \mbox{ is the reference FET on resistance with zero gate to source voltage,} & Ohms \\ A \mbox{ is the operational amplifier open loop gain,} & V/V \end{array}$

1.2 Analysis

Define α , the attenuation factor from the signal input terminal to the amplifier inverting input, and β , the attenuation factor from the output terminal to the amplifier inverting input;

$$\alpha = \frac{\mathbf{R}_2 \cdot \mathbf{r}_{on}}{\mathbf{R}_1 \cdot \mathbf{r}_{on} + \mathbf{R}_1 \cdot \mathbf{R}_2 + \mathbf{R}_2 \cdot \mathbf{r}_{on}}$$
Eq:1.2-1

$$\beta = \frac{R_1 \cdot r_{on}}{R_1 \cdot r_{on} + R_1 \cdot R_2 + R_2 \cdot r_{on}}$$
 Eq:1.2-2

Define σ , the attenuation factor from the signal input terminal to the amplifier non-inverting input.

$$\sigma = \frac{\mathbf{R}_2 \cdot \mathbf{r}_{on}}{\mathbf{R}_1 \cdot \mathbf{r}_{on} + \mathbf{R}_1 \cdot \mathbf{R}_2 + \mathbf{R}_2 \cdot \mathbf{r}_{on}}$$
Eq:1.2-3

solving for V_o ,

$$V_{\rm O} = A \cdot \left(V_{\rm a} - V_{\rm b} \right)$$
 Eq:1.2-4

$$V_{a} = \sigma \cdot V_{S}$$

$$V_{b} = \alpha \cdot V_{S} + \beta \cdot V_{O}$$

$$V_{O} = A \cdot \sigma \cdot V_{S} - A \cdot \alpha \cdot V_{S} - A \cdot \beta \cdot V_{O}$$

$$V_{O} = \frac{A \cdot (\sigma - \alpha) \cdot V_{S}}{1 + A \cdot \beta}$$

solving for voltage gain,

$$A_{v} = \frac{V_{o}}{V_{s}} = \frac{A \cdot (\sigma - \alpha)}{1 + A \cdot \beta}$$
 Eq:1.2-5

if $A \rightarrow \infty$,

$$A_{v} = \frac{\sigma - \alpha}{\beta}$$
$$A_{v} = \frac{\sigma}{\beta} - \frac{\alpha}{\beta}$$

using Eq:1.2-1 and Eq:1.2-2,

$$A_{v} = \sigma \cdot \left(\frac{R_{1} \cdot r_{on} + R_{1} \cdot R_{2} + R_{2} \cdot r_{on}}{R_{1} \cdot r_{on}}\right) - \frac{R_{2}}{R_{1}}$$
$$A_{v} = \sigma \cdot \left(1 + \frac{R_{2}}{r_{on}} + \frac{R_{2}}{R_{1}}\right) - \frac{R_{2}}{R_{1}}$$

$$\mathbf{A}_{\mathrm{v}} = \boldsymbol{\sigma} \cdot \left(1 + \frac{\mathbf{R}_2}{\mathbf{R}_1} \right) + \boldsymbol{\sigma} \cdot \frac{\mathbf{R}_2}{\mathbf{r}_{\mathrm{on}}} - \frac{\mathbf{R}_2}{\mathbf{R}_1}$$

from FET Junction Transistors, Carl David Todd, Wiley, p.122

$$\mathbf{r}_{on} = \mathbf{r}_{on} \cdot \left(1 - \frac{\mathbf{V}_{GS}}{\mathbf{V}_{P}}\right)^{-1}$$
 Eq:1.2-6

$$\label{eq:VGS} \begin{split} V_{GS} &= Gate \ to \ source \ voltage \\ V_{P} &= Pinch-off \ voltage \ of \ JFET \end{split}$$

$$A_{v} = \sigma \cdot \left(1 + \frac{R_{2}}{R_{1}}\right) + \sigma \cdot \frac{R_{2}}{r_{on}} \cdot \left(1 - \frac{V_{GS}}{V_{P}}\right) - \frac{R_{2}}{R_{1}}$$

from Eq:1.2-3,

$$\sigma = \frac{R_2 \cdot r_{on}}{R_1 \cdot r_{on} + R_1 \cdot R_2 + R_2 \cdot r_{on}}$$

$$A_V = \frac{R_2 \cdot r_{on}}{R_1 \cdot r_{on} + R_1 \cdot R_2 + R_2 \cdot r_{on}} \cdot \left(1 + \frac{R_2}{R_1}\right) + \frac{R_2^2}{R_1 \cdot r_{on} + R_1 \cdot R_2 + R_2 \cdot r_{on}} \cdot \left(1 - \frac{V_{GS}}{V_P}\right) - \frac{R_2}{R_1}$$

now if ron is very small,

$$\frac{\mathbf{R}_{2} \cdot \mathbf{r}_{on}}{\mathbf{R}_{1} \cdot \mathbf{r}_{on} + \mathbf{R}_{1} \cdot \mathbf{R}_{2} + \mathbf{R}_{2} \cdot \mathbf{r}_{on}} \rightarrow 0$$

$$\frac{\mathbf{R}_{2}^{2}}{\mathbf{R}_{1} \cdot \mathbf{r}_{on} + \mathbf{R}_{1} \cdot \mathbf{R}_{2} + \mathbf{R}_{2} \cdot \mathbf{r}_{on}} \rightarrow \frac{\mathbf{R}_{2}}{\mathbf{R}_{1}}$$

then,

$$A_{v} = \frac{R_{2}}{R_{1}} \cdot \left(1 - \frac{V_{GS}}{V_{p}}\right) - \frac{R_{2}}{R_{1}}$$

$$A_{v} = -\frac{R_{2}}{R_{1}} \cdot \frac{V_{GS}}{V_{p}}$$
Eq:1.2-7

for $\mathbf{R}_2 = \mathbf{k} \cdot \mathbf{R}_1$,

$$A_{v} = -k \cdot \frac{V_{GS}}{V_{P}}$$
 Eq:1.2-8

 $V_{GS} \rightarrow V_C$

$$A_{v} = -k \cdot \frac{V_{c}}{V_{p}}$$
 Eq:1.2-9