Optimal Charging Scheduling by Pricing for EV Charging Station With Dual Charging Modes

Yongmin Zhang, Member, IEEE, Pengcheng You, Student Member, IEEE, and Lin Cai, Senior Member, IEEE

Abstract—With the increasing penetration of electric vehicles (EVs) and various user preferences, charging stations often provide several different charging modes to satisfy the various requirements of EVs. How to effectively utilize the charging capacity to minimize the service dropping rate is a pressing and open issue for charging stations. Given that EV owners are price-sensitive to the charging modes, we intend to design an optimal pricing scheme to minimize the service dropping rate of the charging station. First, we formulate the operation of a dual-mode charging station as a queuing network with multiple servers and heterogeneous service rates, and analyze the relationship between the service dropping rate of the charging station and the selections of EVs. Then, we formulate a customer attrition minimization problem to minimize the number of EVs that leave the charging station without being charged and propose an optimal pricing approach to guide and coordinate the charging processes of EVs in the charging station. The simulation has been conducted to evaluate the performance of the proposed charging scheduling scheme and show the efficiency of the proposed pricing scheme.

Index Terms—Charging stations, charging modes, queueing theory, charging scheduling, pricing.

I. INTRODUCTION

ELECTRIC vehicles (EVs) have been considered to be a key technology to cut down the massive greenhouse gas emissions from the transportation sector, and they are also expected to mitigate the fossil fuels scarcity problem [1]. Thanks to the policies and plans for promoting EVs from the regions and countries worldwide (e.g., the sales of EVs including PHEVs in US will reach 50% of total sales of mobile vehicles by 2030, and Europe has the similar targets [2]), the amount of EVs is expected to reach a sizable market share in the next decade.

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However, due to limited cruising range, EVs may require frequent recharging when they travel to a faraway destination, and hence charging convenience is one of the most important concerns for EV owners. In addition, due to the high cost of battery replacement, battery lifetime-related cost is another important concern for EV owners. Given the slow evolution of battery technologies, the key issue to is how to address these challenges without waiting for new battery technologies.

Charging stations play an important role in providing charging services to EVs. Typically, there exist two different charging modes for conventional public chargers: i) AC Level II (charging power typically is 10-22 kW); and ii) Direct Current Quick Charging (DCQC) (charging power typically is 50-120 kW). Given the EV charging requirement, AC Level II has a longer charging duration while DCQC has a shorter charging duration, which may further reduce battery lifetime. Because of deployment cost concerns, both the number of charging stations and the number of chargers in charging stations are limited. How to use limited charging station resources to satisfy the various EV charging requirements has attracted attention, such as developing intelligent charging station architectures [3]–[6], and optimizing the location and sizing of charging stations [7]–[11]. Nevertheless, the non-cooperative charging behaviors of EVs will lead to congestion at charging stations and reduce their operational efficiency.

To guide and coordinate the charging behaviors of EVs, there have been extensive researches on developing charging scheduling schemes, for EV charging stations [12]–[17], for battery swapping stations [18]–[20], and for charging stations with renewable energy [21]–[24]. However, these works assumed that all the chargers in the charging station are using the similar charging mode, such as AC Level II or DCQC. Since different EVs may have different charging behaviors and charging service requirements, i.e., short charging duration or long battery lifetime or both of them, the charging station with single charging mode has a low flexibility and adaptability to satisfy the requirements of multi-class customers, and thus limits the service quality.

To deal with the various EV charging requirements, the charging station can install two types of chargers, such as part of them with the AC Level II mode and part of them with the DCQC mode. Consequently, the charging station can provide different charging services to different classes of

customers based on their behaviors. Furthermore, guiding the EV owners to select adequate charging modes can reduce the congestion and improve the service quality of the charging station. This motivates us to design an optimal scheduling scheme for the charging station with dual charging modes to minimize the service dropping rate. To the best of our knowledge, this is the first paper addressing the charging scheduling problem for the charging station with dual charging modes by designing an optimal pricing scheme.

Generally, the selections of EV owners depend on several factors, i.e., service fee, charging duration, waiting time, etc [12]. Given that EV owners are price sensitive, we analyze the relationship between the service dropping rate and the service rate of the charging station and then design an optimal pricing scheme to guide and coordinate the charging processes of EVs, such that the number of EVs that leave the charging station without being charged can be minimized and the operation efficiency of the charging station can be improved. The contributions of this papers can be summarized as follows:

- We model the operation processes of the charging station with dual charging modes as a queuing network with multiple servers and heterogeneous service rates.
- Based on the queuing theory, we analyze the relationship between the selections of EVs and the service dropping rate of the charging station, and prove that the service dropping rate is a convex function of the service rates.
- We formulate a customer attrition minimization problem for the charging station and propose an optimal pricing scheme to guide and coordinate the charging processes of EVs to minimize the service dropping rate.
- Simulation results show that the proposed pricing scheme can reduce the service dropping rate of the charging station with dual charging modes significantly comparing to the charging station with a single charging mode.

The rest of the paper is organized as follows: Section II presents the operation models for the charging station, and formulates the charging scheduling problem as a customer attrition minimization problem based on queuing theory. The relationship between the selections of EVs and the service dropping rate of the charging station is analyzed, and an optimal pricing scheme is designed by utilizing the price sensitivity of EV owners in Section III. Section IV demonstrates the operational performance analysis based on simulation results. Finally, Section V concludes our work.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Considering a charging station, there are $N_1$ AC chargers and $N_2$ DC chargers to provide charging services for connected EVs.1 Given that the charging demand of one EV, both the service fee and the battery lifetime-related cost of the AC chargers are much lower than the DC chargers. But, the charging duration for the AC chargers is much longer. As shown in [12] and [25], the dominant factors that affect the selections of EVs are service fee, waiting time, battery lifetime-related cost, etc. In this paper, we assume that the service fee and the battery lifetime-related cost will affect the EVs’ selections while the waiting time will affect the service dropping rate of the charging station. When the corresponding queue length is too long, EV will leave the charging station without being charged. Otherwise, EV will be charged by one charger with the selected charging mode immediately or later.

For each EV, it needs to select one charging mode when it arrives at the charging station. Then, it will be charged when there is at least one available charger with the selected charging mode; otherwise, it needs to wait in the corresponding queue until one charger with the selected charging mode is available, or leaves the charging station directly without being charged if the corresponding queue length is too long. Here, the percentage of EVs that leave the charging station directly denotes the service dropping rate, which is one of the main factors related to the Quality of Service (QoS) of the charging station. Our objective is to design an optimal pricing scheme based on EV owners’ preferences to guide them to select adequate charging modes, such that the service dropping rate of the charging station can be minimized. The operation model and the queuing model for the charging station are shown in Fig. 1. A summary of notations is given in Table I.

Note that, given the ubiquitous communication networks, in the future, EVs and charging stations can exchange pricing and waiting time information remotely. Such that, an EV can decide which charging mode it prefers and whether go to the charging station for charging service or not based on its QoS. In this scenario, the queuing model and the proposed
we divide the whole day into time slot 1, and assume that the average EV arrival rate during peak vs. off-peak hours may be different, corresponding service fee. Let $p_A^t$ and $p_D^t$ respectively denote the probability for one EV selecting the AC mode and the DC mode during time slot $t$, respectively. The values of $p_A^t$ and $p_D^t$ satisfy

\begin{align}
\sum_{t=1}^{T} p_A^t + p_D^t &= 1, \\
0 &\leq p_A^t, \quad p_D^t \leq 1.
\end{align}

Let $\lambda_A^t$ and $\lambda_D^t$ denote the expected number of EVs that select the AC mode and the DC mode during time slot $t$, respectively. We have

$$\lambda_A^t = \lambda_t \bar{p}_A^t \quad \text{and} \quad \lambda_D^t = \lambda_t \bar{p}_D^t. $$

Generally, based on the existing data analysis in [27], the expected service time for one AC charger and one DC charger to fully charge a battery with the capacity of $B^C$, respectively. $T^A > T^D$ always holds. The mean service times for one EV selecting the AC mode and the DC mode are $ET^A/B^C$ and $ET^D/B^C$, respectively.

Based on the selections of EVs, they enter two separate queues: 1) one queue for EVs that select the AC mode, denoted by $Q_A^t$; 2) another queue for EVs that select the DC mode, denoted by $Q_D^t$. The queue lengths of $Q_A^t$ and $Q_D^t$ at time slot $t$ can be given by the following equations:

$$Q_A^t = \max(0, Q_{t-1}^A + \lambda_A^t - V_A^t - O_A^t - L_A^t), $$

$$Q_D^t = \max(0, Q_{t-1}^D + \lambda_D^t - V_D^t - O_D^t - L_D^t), $$

where $V_A^t$ and $V_D^t$ respectively denote the total number of EVs arriving at the charging station during time slot $t$, and $O_A^t$ and $O_D^t$ respectively denote the number of EVs that have been fully recharged by the AC chargers and the DC chargers and leave the charging station during time slot $t$, and $L_A^t$ and $L_D^t$ respectively denote the number of EVs that select the AC mode and the DC mode and leave the charging station without being charged due to the long corresponding queue length.

In this paper, the maximum queue length is used to denote the maximal tolerable queue length of the EV owners, denoted by $\tilde{Q}_A^t$ and $\tilde{Q}_D^t$ respectively. If $Q_A^t \leq \tilde{Q}_A^t$ and $Q_D^t \leq \tilde{Q}_D^t$, the coming EVs that select the AC (DC) mode will leave the charging station without being charged; otherwise, they will be charged immediately or wait in the corresponding queue. Let $L_t$ denote the service dropping rate of the charging station during time slot $t$. The expected value of $L_t$ can be given by

$$L_t = L_A^t + L_D^t, $$

where

$$L_A^t = \lambda_t \bar{p}_A^t \Pr(Q_A^t \geq \tilde{Q}_A^t), $$

$$L_D^t = \lambda_t \bar{p}_D^t \Pr(Q_D^t \geq \tilde{Q}_D^t). $$

Obviously, the service dropping rate $L_t$ depends on the selections of EVs $\lambda_A^t$ and $\lambda_D^t$, and the queue lengths $Q_A^t$ and $Q_D^t$, as well as the maximum queue lengths $\tilde{Q}_A^t$ and $\tilde{Q}_D^t$. Since the values of $\tilde{Q}_A^t$ and $\tilde{Q}_D^t$ are determined by EV owners, it is difficult for the charging station to change them. According to (4) and (5), $Q_A^t$ and $Q_D^t$ mainly depend on the selections of EVs $\lambda_A^t$ and $\lambda_D^t$, respectively, which can be tuned to reduce the service dropping rate $L_t$.

### B. Selection Model of EVs

Generally, due to the high construction cost of the DC mode, the service fee of the DC mode is much higher than that for

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$t$</td>
<td>The $t$-th time slot.</td>
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<tr>
<td>$\lambda_t$</td>
<td>Number of EVs that arrive at charging station during time slot $t$.</td>
</tr>
<tr>
<td>$\lambda_A^t$</td>
<td>Number of EVs that select the AC mode during time slot $t$.</td>
</tr>
<tr>
<td>$\lambda_D^t$</td>
<td>Number of EVs that select the DC mode during time slot $t$.</td>
</tr>
<tr>
<td>$\bar{p}_A^t$</td>
<td>The probability for EVs selecting the AC mode during time slot $t$.</td>
</tr>
<tr>
<td>$\bar{p}_D^t$</td>
<td>The probability for EVs selecting the DC mode during time slot $t$.</td>
</tr>
<tr>
<td>$B^C$</td>
<td>The expected battery capacity of each EV.</td>
</tr>
<tr>
<td>$C_A^t$</td>
<td>Service fee of the AC mode during time slot $t$.</td>
</tr>
<tr>
<td>$C_D^t$</td>
<td>Service fee of the DC mode during time slot $t$.</td>
</tr>
<tr>
<td>$Q_A^t$</td>
<td>The number of EVs that have been charged during time slot $t$.</td>
</tr>
<tr>
<td>$Q_D^t$</td>
<td>The lower bound of $C_D^t$ during time slot $t$.</td>
</tr>
<tr>
<td>$L_A^t$</td>
<td>The upper bound of $C_A^t$ during time slot $t$.</td>
</tr>
<tr>
<td>$Q_A^t$</td>
<td>The battery lifetime-related cost in the AC mode.</td>
</tr>
<tr>
<td>$Q_D^t$</td>
<td>The battery lifetime-related cost in the DC mode.</td>
</tr>
<tr>
<td>$E_A^t$</td>
<td>The expected charging requirement of each EV.</td>
</tr>
<tr>
<td>$E_D^t$</td>
<td>Total number of EVs that leave the charging station without being charged during time slot $t$.</td>
</tr>
<tr>
<td>$N_A^t$</td>
<td>Total number of EVs that leave the charging station during time slot $t$.</td>
</tr>
<tr>
<td>$N_D^t$</td>
<td>Total number of AC chargers.</td>
</tr>
<tr>
<td>$N_D^t$</td>
<td>Total number of the DC chargers.</td>
</tr>
<tr>
<td>$O_A^t$</td>
<td>Number of EVs fully charged by the AC mode during time slot $t$.</td>
</tr>
<tr>
<td>$O_D^t$</td>
<td>Number of EVs fully charged by the DC mode during time slot $t$.</td>
</tr>
<tr>
<td>$Q_A^t$</td>
<td>The queue length for the AC mode during time slot $t$.</td>
</tr>
<tr>
<td>$Q_D^t$</td>
<td>The queue length for the DC mode during time slot $t$.</td>
</tr>
<tr>
<td>$T_A^t$</td>
<td>The maximal queue length for the AC mode during time slot $t$.</td>
</tr>
<tr>
<td>$T_D^t$</td>
<td>The maximal queue length for the DC mode during time slot $t$.</td>
</tr>
<tr>
<td>$T_A^t$</td>
<td>Total time for recharging an empty battery to full in the AC mode.</td>
</tr>
<tr>
<td>$T_D^t$</td>
<td>Total time for recharging an empty battery to full in the DC mode.</td>
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the AC mode. Furthermore, the battery lifetime-related cost for
the AC mode is much lower than that for the DC mode [28].
Different EVs may have the different preferences, but are price
sensitive [16]. Since different charging stations always have
the similar service fee of the AC mode, we set the service fee
of the AC mode as a constant and adjust the service fee of
the DC mode.

Let $C_A^t$ and $C_D^t$ denote the service fee per kWh during time
slot $t$, and $C_A^B$ and $C_D^B$ denote the battery lifetime-related
cost per kWh, with the AC mode and the DC mode, respectively.
Here, $C_A^t < C_D^t$ and $C_A^B < C_D^B$ always hold. Generally,
the service fee of the DC mode can only be adjusted in a
given range. Let $C_D^L$ and $C_D^R$ denote the lower bound and upper
bound on the service fee of the DC charger. Thus, the service
fee $C_D^t$ should always satisfy

$$C_D^L \leq C_D^t \leq C_D^R. \quad (9)$$

As mentioned above, the selections of EVs mainly depend
on the service fees $C_A^t$ and $C_D^t$ and the battery lifetime-related
costs $C_A^B$ and $C_D^B$. Specifically, in this paper, we define $\bar{p}_A^t$ and
$\bar{p}_D^t$ as follows:

$$\bar{p}_A^t = -aC_A^t + b, \quad (10)$$

$$\bar{p}_D^t = 1 - \bar{p}_D^t, \quad (11)$$

where $a = \frac{1}{C_A^t + \beta - (C_D^B - C_A^B)}$, $b = \frac{1}{C_D^R}$, and $\beta$ denotes the
service time reduction cost of the DC mode comparing with
the AC mode. It can be seen that $a$ is a price-dependent term
and $b$ is a price-independent term.

To ensure $0 \leq \bar{p}_A^t, \bar{p}_D^t \leq 1$, we set $C_D^L = \bar{C}_D^L - (C_A^t + \beta -
(C_D^B - C_A^B))$, which usually is larger than 0. Such that, when
$C_D^L$ equals its lower bound $C_D^L$, all the coming EVs will select
the DC mode, i.e., $\bar{p}_D^t = 1$ and $\bar{p}_A^t = 0$; when $C_D^t = \bar{C}_D^L$,
all the coming EVs will select the AC mode, i.e., $\bar{p}_D^t = 0$
and $\bar{p}_A^t = 1$. Note that, our proposed algorithm also applies
to other probability models, such as linear or concave ones.

C. Service Dropping Rate of Charging Station

Let $\bar{p}_L^t$ denote the probability for EVs leaving the charging
station without being charged during time slot $t$. The expected
values of $\bar{p}_L^t$ can be given by

$$\bar{p}_L^t = Pr\{Q_A^t|Q_A^t \geq \bar{Q}_A^t\} \bar{p}_A^t + Pr\{Q_D^t|Q_D^t \geq \bar{Q}_D^t\} \bar{p}_D^t. \quad (12)$$

Thus, the expected value of the service dropping rate $L_t$ can be
rewritten as

$$L_t = \bar{\lambda}_t \bar{p}_L^t, \quad (13)$$
due to $\bar{\lambda}_A^t = \bar{\lambda}_t \bar{p}_A^t$ and $\bar{\lambda}_D^t = \bar{\lambda}_t \bar{p}_D^t$. It can be found that
the service dropping rate $L_t$ can be reduced by adjusting the
service fee $C_D^t$, which affects the values of $\bar{p}_A^t$ and $\bar{p}_D^t$.

D. Customer Attrition Minimization Problem

The service dropping rate is not only an important parameter
for the loss of benefit, but also one of the major factors for the
satisfaction of EV owners. To minimize the service dropping
rate of the charging station, we formulate a customer attrition
minimization problem. Since EV arrives at different time slot
follow a Poisson process, which has independent increments,
we just need to minimize the service dropping rate of the
charging station during each time slot, such that the total
service dropping rate can be minimized. By now, the customer
attrition minimization problem can be formulated as follows:

$$\text{P0: } \min_{C_D^t} \quad L_t = \lambda_t \bar{p}_L^t,$$

s.t. $C_D^L \leq C_D^t \leq C_D^R,$

$$\bar{p}_D^t = -aC_D^t + b, \quad (14)$$

$$\bar{p}_A^t + \bar{p}_D^t = 1, \quad (15)$$

$$1 \leq \bar{p}_A^t, \bar{p}_D^t \leq 0, \quad (16)$$

(4), (5), and (12).

The objective is to minimize the service dropping rate $L_t$ by
adjusting the service fee $C_D^t$. The first constraint defines the
available range of $C_D^t$. The constraints (14)-(16) describe the
relationship between the service fee $C_D^t$ and the selections
of EVs, $\bar{p}_A^t$ and $\bar{p}_D^t$. The other constraints show relationships
among $\bar{p}_L^t, Q_A^t, Q_D^t, \bar{p}_A^t$, and $\bar{p}_D^t$.

III. OPTIMAL EV CHARGING SCHEDULING SCHEME

To solve the customer attrition minimization problem,
we first analyze the performance measure of the queue system
to explore the possible range of optimal price and the relationship
between the service dropping rate $L_t$ and the service fee $C_D^t$. Then, we transform the primal problem into a convex
optimization problem. At last, we propose an optimal pricing
scheme to guide the EVs to select adequate charging modes,
such that the total service dropping rate of the charging station
can be minimized.

According to the definitions of $\bar{p}_A^t$ and $\bar{p}_D^t$, it can be found
that the relationships among $\bar{\lambda}_A^t, \bar{\lambda}_D^t$ and $C_D^t$ are linear.
For any given service fee $C_D^t$ and arrival rate $\bar{\lambda}_t, \bar{\lambda}_A^t$ and $\bar{\lambda}_D^t$
can be calculated according to (3), (10) and (11). Also, $C_D^L$ and
$\lambda_A^t$ can be obtained when the value of $\lambda_D^t$ is given. Hence,
in the following sections, we can replace the variable $C_D^t$ by
$\lambda_D^t$ for easy computation.

A. Performance Measures Based on Queue Theory

Let $\bar{\mu}_A^t$ and $\bar{\mu}_D^t$ denote the average service rate for one AC
charger and that for one DC charger, respectively. Based on the
system model, the values of $\mu_A^t$ and $\mu_D^t$, and their relationship
can be given by

$$\mu_A^t = \frac{B^C}{ET_A^C}, \quad (18)$$

$$\mu_D^t = \frac{B^C}{ET_D^C}, \quad (19)$$

$$\mu_A^t = \frac{T_A}{T_A T_D^C}. \quad (20)$$

Obviously, $\mu_A^t < \mu_D^t$ since $T_D^C < T_A^C$.

The charging processes of EVs at the charging station
can be formulated as two independent queuing networks,
i.e., $M/M/N_1$ with $\rho_A^t = \bar{\lambda}_A^t/(N_1 \bar{\mu}_A^t)$ and $M/M/N_2$ with
\( \rho_t^D = \lambda_t^D / (N_2 \mu_t^D) \), respectively. Here, \( \rho_t^A \) and \( \rho_t^D \) are the corresponding utilization factors for different charging modes. For the optimal service fee \( \tilde{C}_t^D \), we have the following Theorem:

**Theorem 1:** For any given arrival rate \( \lambda_t \), the available range for the optimal service fee \( \tilde{C}_t^D \) can be given by

\[
\begin{align*}
&\left\{ \begin{array}{ll}
\frac{N_1 \mu_t^A + b \lambda_t - \lambda_t}{a \lambda_t} \\
\frac{b \lambda_t - N_2 \mu_t^D}{a \lambda_t}
\end{array} \right. \\
&\left\{ \begin{array}{ll}
\frac{N_1 \mu_t^A + b \lambda_t - \lambda_t}{a \lambda_t} \\
\frac{b \lambda_t - N_2 \mu_t^D}{a \lambda_t}
\end{array} \right. \quad \text{if } \lambda_t \geq N_1 \mu_t^A + N_2 \mu_t^D; \\
&\left\{ \begin{array}{ll}
\frac{N_1 \mu_t^A + b \lambda_t - \lambda_t}{a \lambda_t} \\
\frac{b \lambda_t - N_2 \mu_t^D}{a \lambda_t}
\end{array} \right. \quad \text{otherwise}.
\end{align*}
\]

**Proof:** According to the queuing theory, if with infinite buffer, the necessary conditions for these two queue systems to be stable are \( \rho_t^A < 1 \) and \( \rho_t^D < 1 \). If the mean arrival rate is greater than the mean service rate, the necessary conditions for the stable queue systems cannot be satisfied, which makes the server keeping busy and the queue growing without bound\(^2\) [29]. Thus, based on the arrival rate \( \lambda_t \) and the service rate, we classify the available range of \( \tilde{C}_t^D \) into the following two cases:

**CASE I:** If \( \lambda_t \geq N_1 \mu_t^A + N_2 \mu_t^D \), there is no available service fee satisfying \( \frac{\lambda_t^D}{N_1 \mu_t^A} < 1 \) and \( \frac{\lambda_t^D}{N_2 \mu_t^D} < 1 \) simultaneously. If \( \rho_t^A \geq 1 \), the chargers with the AC mode will keep busy since their queueing system becomes overloaded and the service dropping rate will increase with respect to \( \lambda_t^A \), and \( L_t^A \approx \lambda_t^A - N_1 \mu_t^A \); otherwise, the service dropping rate \( L_t^A \approx \lambda_t^A \Pr(Q_t^A | Q_t^A \geq \tilde{Q}_t^A) = \lambda_t^A (1 - \Pr(Q_t^A | Q_t^A < \tilde{Q}_t^A)) \). The parameters for \( L_t^D \) are similar to those for \( L_t^A \). Hence, the service dropping rate \( L_t \) can be given by the following cases:

- If \( \lambda_t^A > N_1 \mu_t^A \) and \( \lambda_t^D < N_2 \mu_t^D \), we have
  \[
  L_t \approx \lambda_t - N_1 \mu_t^A - N_2 \mu_t^D; \quad (21)
  \]
- If \( \lambda_t^A \geq N_1 \mu_t^A \) and \( \lambda_t^D \geq N_2 \mu_t^D \), we have
  \[
  L_t \approx \lambda_t - N_1 \mu_t^A - N_2 \mu_t^D; \quad (22)
  \]
- If \( \lambda_t^A < N_1 \mu_t^A \) and \( \lambda_t^D > N_2 \mu_t^D \), we have
  \[
  L_t \approx \lambda_t - N_2 \mu_t^D - \lambda_t^D \Pr(Q_t^A | Q_t^A < \tilde{Q}_t^A). \quad (23)
  \]

Since \( \Pr(Q_t^A | Q_t^A < \tilde{Q}_t^A) < 1 \) and \( \Pr(Q_t^D | Q_t^D < \tilde{Q}_t^D) < 1 \), (22)<(21) and (22)<(23) always hold. To minimize the service dropping rate \( L_t \), \( \rho_t^A \geq 1 \) and \( \rho_t^D \geq 1 \) should be satisfied simultaneously. Thus, the available range for the optimal service fee \( \tilde{C}_t^D \) is

\[
\begin{align*}
&\lambda_t \tilde{p}_t^A \geq N_1 \mu_t^A \quad \Rightarrow \quad \tilde{C}_t^D \geq \frac{N_1 \mu_t^A + b \lambda_t - \lambda_t}{a \lambda_t}, \\
&\lambda_t \tilde{p}_t^D \geq N_2 \mu_t^D \quad \Rightarrow \quad \tilde{C}_t^D \leq \frac{b \lambda_t - N_2 \mu_t^D}{a \lambda_t}.
\end{align*}
\]

**CASE II:** If \( \lambda_t < N_1 \mu_t^A + N_2 \mu_t^D \), it means that there exists an optimal service fee \( \tilde{C}_t^D \) that can satisfy \( \rho_t^A < 1 \) and \( \rho_t^D < 1 \) simultaneously. If \( \rho_t^A < 1 \) and \( \rho_t^D < 1 \), it means that \( \lambda_t^A < N_1 \mu_t^A \) and \( \lambda_t^D < N_2 \mu_t^D \) always hold, and the service dropping rate \( L_t \) can be calculated by

\[
\begin{align*}
L_t &= \lambda_t \Pr(Q_t^A | Q_t^A \geq \tilde{Q}_t^A) + \lambda_t^D \Pr(Q_t^D | Q_t^D \geq \tilde{Q}_t^D) \\
&= \lambda_t - \lambda_t^A \Pr(Q_t^A | Q_t^A < \tilde{Q}_t^A) - \lambda_t^D \Pr(Q_t^D | Q_t^D < \tilde{Q}_t^D). \quad (24)
\end{align*}
\]

It can be found that (24)<(21) and (24)<(23) always hold. Hence, the optimal service fee \( \tilde{C}_t^D \) should satisfy

\[
\begin{align*}
&\lambda_t \tilde{p}_t^A < N_1 \mu_t^A \Rightarrow \tilde{C}_t^D < \frac{N_1 \mu_t^A + b \lambda_t - \lambda_t}{a \lambda_t}, \\
&\lambda_t \tilde{p}_t^D < N_2 \mu_t^D \Rightarrow \tilde{C}_t^D > \frac{b \lambda_t - N_2 \mu_t^D}{a \lambda_t}.
\end{align*}
\]

The available range for the optimal service fee \( \tilde{C}_t^D \) is obtained.

According to the available range for the optimal service fee \( \tilde{C}_t^D \) given by Theorem 1, if the arrival rate \( \lambda_t \) satisfies \( \lambda_t \geq N_1 \mu_t^A + N_2 \mu_t^D \), we have the following Lemma for the optimal service fee \( \tilde{C}_t^D \):

**Lemma 1:** The optimal service fee \( \tilde{C}_t^D \) can be any value in the available range \([\frac{N_1 \mu_t^A + b \lambda_t - \lambda_t}{a \lambda_t}, \frac{b \lambda_t - N_2 \mu_t^D}{a \lambda_t}]\) when \( \lambda_t \geq N_1 \mu_t^A + N_2 \mu_t^D \).

**Proof:** For the service dropping rate \( L_t \) given by (22), it can be found that the minimal value of the service dropping rate \( L_t \) is a constant when \( \lambda_t \geq N_1 \mu_t^A + N_2 \mu_t^D \), and the only condition for the optimal service fee \( \tilde{C}_t^D \) is to make sure that \( \rho_t^A \geq 1 \) and \( \rho_t^D \geq 1 \) are satisfied simultaneously. Thus, the optimal service fee \( \tilde{C}_t^D \) can be any value in the available range \([\frac{N_1 \mu_t^A + b \lambda_t - \lambda_t}{a \lambda_t}, \frac{b \lambda_t - N_2 \mu_t^D}{a \lambda_t}]\).

Therefore, we only need to design the optimal pricing scheme for the problem when \( \lambda_t < N_1 \mu_t^A + N_2 \mu_t^D \).

Let \( n_1 \) and \( n_2 \) denote the steady state of the charging processes of the AC mode and the DC mode, in which there are \( n_1 \) and \( n_2 \) customers in their corresponding systems, include the customers in service, respectively. Let \( p_t^A(n_1) \) denote the probability that in steady state the number of customers present in the AC mode is \( n_1 \) during time slot \( t \) and \( p_t^D(n_2) \) denote the probability that in steady state the number of customers present in the DC mode is \( n_2 \) during time slot \( t \). Then, we have

\[
\begin{align*}
p_t^A(n) &= \begin{cases} 
\text{if } n \leq N_1; \\
\text{if } n > N_1; 
\end{cases} \\
p_t^D(n) &= \begin{cases} 
\text{if } n \leq N_2; \\
\text{if } n > N_2; 
\end{cases}
\end{align*}
\]

where

\[
p_t^A(0) = \sum_{n=0}^{N_1-1} \frac{(N_1 p_t^A)^n}{n!} = \frac{N_1!}{(1 - p_t^A)^N_1}, \quad (27)
\]
\[ p_t^D(0) = \sum_{n=0}^{N-1} \left( N_2 p_t^D(n) \right)_n + \left( N_2 p_t^D(n_n) \right)_n \frac{1}{1 - \rho^D} \].

In this paper, we use the steady-state distribution of EVs to model the service processes of the charging station and solve the customer attrition minimization problem.

**B. Service Dropping Rate of Charging Station**

Since the maximal queue lengths \( \hat{Q}_t^A \) and \( \hat{Q}_t^D \) for EVs during time slot \( t \) are constant, the service dropping rate \( L_t \) can be given by

\[
L_t = \sum_{n_1=N_1+\hat{Q}_t^A}^{\infty} \lambda_t^A p_t^A(n_1) + \sum_{n_2=N_2+\hat{Q}_t^D}^{\infty} \lambda_t^D p_t^D(n_2)
\]

where \( \sum_{n_1=N_1+\hat{Q}_t^A}^{\infty} \lambda_t^A p_t^A(n_1) = \frac{\lambda_t^A (p_t^A(n_1+\hat{Q}_t^A))}{1-\rho^A} \) denotes the blocking traffic for the queue of the AC mode and \( \sum_{n_2=N_2+\hat{Q}_t^D}^{\infty} \lambda_t^D p_t^D(n_2) = \frac{\lambda_t^D (p_t^D(n_2+\hat{Q}_t^D))}{1-\rho^D} \) denotes the blocking traffic for the queue of the DC mode. According to the definitions of \( p_t^A(n) \) and \( p_t^D(n) \) given by (25) and (26), it can be found that \( p_t^A(n) \) and \( p_t^D(n) \) also depend on the values of \( \lambda_t^A \) and \( \lambda_t^D \). Hence, the service dropping rate \( L_t \) mainly depends on the values of \( \lambda_t^A \) and \( \lambda_t^D \).

**C. Problem Transformation**

According to Theorem 1 and the relationship among \( \lambda_t^A, \lambda_t^D, \bar{\rho}_t^A, \bar{\rho}_t^D \) and \( C_t^D \) given by (3), (10) and (11), the optimal \( \lambda_t^D \) should satisfy

\[
\lambda_t - N_1 \mu^A < \lambda_t^D < N_2 \mu^D.
\]

By now, the primal Problem P0 can be transformed to the following problem P1, in which the variable is \( \lambda_t^D \), i.e.,

**P1:** \( \min_{\lambda_t^D} L_t, \)

s.t. \( \lambda_t - N_1 \mu^A < \lambda_t^D < N_2 \mu^D. \)

In this problem, the goal is to find the optimal \( \lambda_t^D \) to minimize the service dropping rate \( L_t \), and the available range for \( \lambda_t^D \) is given by constraint (32). If the objective function \( L_t \) is a convex function of \( \lambda_t^D \), Problem P1 can be transformed into a typical convex optimization problem. Thus, we first establish that, if the problem is a convex optimization problem, the solution of this problem is indeed toward the global optimum [30]. To solve this problem, we first prove that convexity of the service dropping rate \( L_t \) with respect to \( \lambda_t^D \), and then propose an optimal pricing scheme to obtain the optimal price \( \hat{C}_t^D \).

**D. Proof of Convexity**

According to (29), the service dropping rate \( L_t \) is

\[
L_t = \lambda_t^A p_t^A(n_1') \frac{1}{1 - \rho_t^A} + \lambda_t^D p_t^D(n_2') \frac{1}{1 - \rho_t^D},
\]

where \( n_1' = N_1 + \hat{Q}_t^A \) and \( n_2' = N_2 + \hat{Q}_t^D \). Due to \( \lambda_t^A = N_1 \mu^A \rho_t^A \) and \( \lambda_t^D = N_2 \mu^D \rho_t^D \), we have

\[
L_t = L_t^A + L_t^D,
\]

where

\[
L_t^A = \frac{N_1 \mu^A p_t^A(n_1')}{1 - \rho_t^A} = \frac{N_1 \mu^A p_t^A(0) N_1^N (\rho_t^A)^{n_1'+1}}{N_1!} \frac{1}{1 - \rho_t^A}.
\]

\[
L_t^D = \frac{N_2 \mu^D p_t^D(n_2')}{1 - \rho_t^D} = \frac{N_2 \mu^D p_t^D(0) N_2^N (\rho_t^D)^{n_2'+1}}{N_2!} \frac{1}{1 - \rho_t^D}.
\]

It can be found that \( L_t^A \) and \( L_t^D \) have the similar structure. Thus, we first prove that the second part \( L_t^D \) is an increasing and convex function of \( \lambda_t^D \) and then can derive that the first part \( L_t^A \) is a decreasing and convex function of \( \lambda_t^A \) due to \( \lambda_t - \lambda_t^A = \lambda_t^D \). Because \( \rho_t^A \) is a linear function of \( \lambda_t^A \) and such a variable substitution will not change the convexity of the objective function, we take \( L_t^D \) as the objective function and \( \rho_t^D \) as the variable in the following part. For the second part \( L_t^D \), we have the following theorem:

**Theorem 2:** The service dropping rate \( L_t^D \) is an increasing and convex function of the service rate \( \rho_t^D \).

**Proof:** The proof can be found in Appendix A. ■

Since \( L_t^A \) and \( L_t^D \) have the similar structure, we have the following Lemma for the relationship between \( L_t^A \) and \( \rho_t^A \):

**Lemma 2:** The service dropping rate \( L_t^A \) is a decreasing and convex function of the service rate \( \rho_t^A \).

**Proof:** The proof can be found in Appendix B. ■

**Theorem 3:** The service dropping rate \( L_t \) is a convex function of \( \lambda_t^D \).

**Proof:** Since \( L_t = L_t^A + L_t^D \), \( \frac{\partial^2 L_t^A}{\partial (\rho_t^A)^2} > 0 \) and \( \frac{\partial^2 L_t^D}{\partial (\rho_t^D)^2} > 0 \), we have \( \frac{\partial^2 L_t}{\partial (\rho_t^D)^2} > 0 \). Since \( \lambda_t^D = \rho_t^D N_2 \mu^D \), \( \frac{\partial^2 L_t}{\partial (\rho_t^D)^2} = N_2 \mu^D > 0 \) always holds. Hence, \( \frac{\partial^2 L_t}{\partial (\rho_t^D)^2} = \frac{(N_2 \mu^D)^2 \frac{\partial L_t}{\partial \lambda_t^D}}{\partial (\rho_t^D)^2} > 0 \). Thus, the service dropping rate \( L_t \) is a convex function of \( \lambda_t^D \). ■

Because the relationship between \( \lambda_t^D \) and \( C_t^D \) is linear, the service dropping rate \( L_t \) also is a convex function of \( C_t^D \).

**E. Optimal Pricing Scheme**

Since the objective function \( L_t \) is a convex function of \( \lambda_t^D \) and the constraint (32) is a linear constraint for \( \lambda_t^D \), the transformed Problem P1 is a convex optimization problem. Since this optimization problem will be solved by the charging station, it can be solved using existing centralized tools, such as fmincon function [31] or CVX toolbox [32] in Matlab. We omit the details of how to solve this problem.

By solving Problem P1, the optimal \( \lambda_t^D \) can be obtained, then the optimal pricing \( \hat{C}_t^D \) can be calculated by

\[
\hat{C}_t^D = \frac{b}{a} - \frac{\lambda_t^D}{a \lambda_t}.
\]

The process for obtaining the optimal price \( \hat{C}_t^D \) can be sketched as Algorithm 1.

**Theorem 4:** The minimal \( L_t \) can be achieved by the optimal pricing scheme shown in Algorithm 1.
Algorithm 1 Optimal Pricing Scheme for the Charging Station

Initialization $\lambda_1, N_1, \mu^A, N_2, \mu^D, Q^A, Q^D$

- If $\lambda_1 \geq N_1 \mu^A + N_2 \mu^D$
  
  Chooses $\lambda^D_1$ randomly in $[N_2 \mu^D, \lambda_1 - N_1 \mu^A]$;
  
- Else
  
  Obtains the optimal $\lambda^D_1$ by solving Problem P1;
  
- end

Calculates the optimal price $\hat{C}^D_1$ by (35);

return $\hat{C}^D_1$

---

$\text{Fig. 2. The EV arrivals of the charging station.}$

Proof: According to Theorem 1 and Lemma 1, if $\lambda_1 \geq N_1 \mu^A + N_2 \mu^D$, the minimal service dropping rate $L_t \approx \lambda_1 - N_1 \mu^A - N_2 \mu^D$ can be obtained by any value in the available range, i.e., $\hat{C}^D_t \in \left[ \frac{N_1 \mu^A + b \lambda_1 - \lambda_2}{a \lambda_2}, \frac{b \lambda_2 - N_2 \mu^D}{a \lambda_2} \right]$. If $\lambda_1 < N_1 \mu^A + N_2 \mu^D$, since the customer attrition minimization problem is a convex optimization problem according to Theorem 3, there exists an unique optimal solution, which can be obtained by solving Problem P1 [30]. Thus, the proposed optimal pricing scheme in Algorithm 1 can minimize the service dropping rate $L_t$.

IV. PERFORMANCE EVALUATION

In order to demonstrate the performance of the proposed algorithm, we take one charging station with dual charging modes and time-varying arrivals of EVs as a case study, and then analyze the effects of the number of chargers, the maximal waiting queue length, and the arrival rate on the optimal pricing of the charging station.

A. Case Study

Consider a charging station with $N_1 = 15$ AC chargers and $N_2 = 8$ DC chargers. The average service rates for each AC charger and DC charger are $\mu^A = \frac{2}{3}$ per hour and $\mu^D = \frac{12}{5}$ per hour, respectively. According to [33], the cost for replacing a 24kWh battery is about $5500$, and the lifetime with the AC mode and the DC mode are about 650 and 500 cycles, respectively. Thus, we can derived that $C^A_t \approx 0.35/kWh$ and $C^D_t \approx 0.46/kWh$. The maximal queue lengths are $\hat{Q}^A_t = 10$ and $\hat{Q}^D_t = 8$. We set $C_t \equiv 0.15/kWh$, $\hat{C}^A_t = 0.08/kWh$, $\beta = 0.02/kWh$, and $E = 16kWh$. The arrivals of EVs during different time slot can be found in Fig. 2. Assume that there exists another two construction plan: i) charging station with the AC mode, in which all the DC chargers (including the waiting space) are replaced by the AC chargers, and all the

$\text{Fig. 3. Optimal price } \hat{C}^D_t \text{ where } C_t^A = \$0.15/kWh \text{ and selections of EVs at different time. (a) Optimal price } \hat{C}^D_t. \text{ (b) Selection of EVs.}$

$\text{Fig. 4. The numbers of EVs that leave the charging station without being charged and the corresponding expected queue lengths at different times. (a) } L_t^A \text{ and } L_t^D. \text{ (b) } Q_t^A \text{ and } Q_t^D.$

EVs can only select the AC mode, i.e., $\hat{N}_t = 31$ and $\hat{\hat{Q}}^A_1 = 10$; ii) charging station with the DC mode, in which all the AC chargers are replaced by the DC chargers, i.e., $\hat{N}_2 = 23$ and $\hat{\hat{Q}}^D_t = 18$, while the service fee of the DC mode is $0.63/kWh$.

The optimal price $\hat{C}^D_t$ of the charging station with dual charging modes and the expected selections of EVs can be found in Fig. 3. It can be found that the available range for the optimal price changes and the optimal price of the DC mode is time-varying due to time-varying EV arrival rate. With the increase of the arrival rate $\lambda_t$, the available range for optimal price will be narrowed, which means that the selections for the optimal price become less, and the optimal price $\hat{C}^D_t$ for the DC mode will be decreased, such that more EVs will select the DC mode. However, since the service fees in the charging stations with single mode are constants, all the EVs in i) charging station with the AC mode can only select the AC mode while only part of EVs in ii) charging station with the DC mode will select the DC mode due to its high battery lifetime-related cost.

The total service dropping rates for different charging stations and the expected queue lengths can be found in Fig. 4. It can be found that the optimal pricing scheme can minimize the service dropping rate since it can guide the EVs selecting the suitable charging mode to improve the utilization of chargers in the charging station. For i) the charging station with the AC mode, part of EVs leaves due to its limited service ability and the long waiting queue, while for ii) the charging station with the DC mode, part of EVs will not select this charging station due to its high battery lifetime-related cost and high charging service fee. For the charging station with dual charging modes under optimal pricing scheme, by adjusting
the service fee of the DC mode, both the number of EVs that leave the charging station without being charged and the expected queue lengths can be minimized, especially when the arrival rate is high.

With same arrival rates and construction space, i) charging station with the AC mode has the smallest service ability and longest waiting queue, ii) charging station with the DC mode has the largest service ability and the smallest waiting queue, and charging station with dual charging modes under optimal pricing scheme has a middle service ability and waiting queue. In addition, i) charging station with the AC mode has the largest service dropping rate due to limited service ability, and ii) charging station with the DC mode has a higher service dropping rate due to its high battery lifetime-related cost, and the charging station with dual charging modes under optimal pricing scheme can minimize the service dropping rate, which has a higher flexibility and adaptability to deal with the time-varying arrival rate of EVs.

The expected service dropping rate for different charging stations can be found in Table II. It can be found that the charging station with dual charging modes under the proposed pricing scheme can minimize the total service dropping rate of the charging station. The charging station with the AC mode has the highest service dropping rate due to its limited charging service rate even when it has 31 chargers. The charging station with the DC mode has a middle service dropping rate due to its high battery fee and high battery lifetime-related cost. As we know, since the charging station with the DC mode has enough charging service ability to service more EVs, reduce its service fee can cut down the service dropping rate. However, we found that only when the charging station with the DC mode reduce its the service fee to $0.59/kWh, which is much lower than the service fee in charging station with dual charging modes, it has the similar service dropping rate of the charging station with dual charging modes.

### Table II

<table>
<thead>
<tr>
<th>Chargers</th>
<th>5-6am</th>
<th>7-8pm</th>
<th>9-10am</th>
<th>11-12pm</th>
<th>1-2pm</th>
</tr>
</thead>
<tbody>
<tr>
<td>All AC mode</td>
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<td>11</td>
<td>4.04</td>
<td>28.60</td>
<td>18.60</td>
</tr>
<tr>
<td>All DC mode</td>
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<td>11.50</td>
<td>7.80</td>
<td>15.60</td>
<td>13.60</td>
</tr>
<tr>
<td>Dual Modes</td>
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<td>0.26</td>
<td>0.02</td>
<td>7.40</td>
<td>1.89</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chargers</th>
<th>3-4pm</th>
<th>5-6pm</th>
<th>7-8pm</th>
<th>9-11pm</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>All AC mode</td>
<td>35.00</td>
<td>30.00</td>
<td>3.04</td>
<td>0</td>
<td>130.10</td>
</tr>
<tr>
<td>All DC mode</td>
<td>18.00</td>
<td>16.20</td>
<td>7.50</td>
<td>1.50</td>
<td>93.60</td>
</tr>
<tr>
<td>Dual Modes</td>
<td>13.6</td>
<td>7.51</td>
<td>0.01</td>
<td>0</td>
<td>30.69</td>
</tr>
</tbody>
</table>

### B. Relationship Between Arrival Rate and System Performance

The maximal service ability of the charging station with dual charging modes is 25.2 EVs/hour. In order to explore the performance of our proposed pricing scheme, we set the arrival rate from [1, 35] and respectively show the optimal price $\hat{C}_A^D$ and its available range, the minimal service dropping rates $L_{A^D}$ and $L_{D^D}$, the selections of EVs $\lambda_A^D$ and $\lambda_D^D$, and the expected queue lengths for the AC mode and the DC mode in Fig. 5.

From the simulation results in Fig. 5(a), it can be found that, when the arrival rate of EVs is smaller than the maximal service ability of the charging station, with the increase of the arrival rate $\lambda$, the available range for the optimal price becomes narrow to ensure that both of the utilization factors $\rho_A^D$ and $\rho_D^D$ are smaller than 1, and the optimal price $\hat{C}_A^D$ decreases; otherwise, the available range for the optimal price becomes wider to ensure that both of the utilization factors $\rho_A^D$ and $\rho_D^D$ are larger than 1, and the optimal price $\hat{C}_A^D$ can be any value in the available range. Specially, when the arrival rate of EVs is smaller than the maximal service ability of the charging station, the charging station needs to select an optimal pricing by solving Problem P1 to minimize the total service dropping rate for both the AC mode and the DC mode; and when the arrival rate of EVs is larger than the maximal service ability of the charging station, the charging station just needs to select one available value in the available range since any value in the available range will obtain the same service dropping rate.

Figs. 5(b)-5(d) show the selections of EVs, the service dropping rates and the expected queue lengths, respectively. With the increase of EV arrival rate, the selections of EVs for both the AC mode and the DC mode, the service dropping rate and the expected queue lengths will be increased. In order to minimize the service dropping rate, it can be found that the selections of EVs for both the AC mode and the DC mode increase smoothly. Also, the service dropping rate grows when the arrival rate of EVs is larger than a certain threshold and the expected queue lengths reach their upper bounds of the charging station. Since the service rate of the AC mode is much lower than the service rate of the DC mode, more EVs are assigned to select the DC mode to minimize the total service dropping rate.

### C. The Effects of $N_1$ and $N_2$ on Service Dropping Rate $L_1$

To explore the effect of the charging facilities in the charging station on the service dropping rate $L_1$, we fix the
EV arrival by \( \lambda \), and the minimal service dropping rate \( L_t \) is shown in Fig. 6(a); 2) fixing \( N_2 = 8 \) while adjusting \( N_1 \) from 1 to 20, the minimal service dropping rate \( L_t \) is shown in Fig. 6(b). It can be found that the minimal service dropping rate \( L_t \) in the first simulation is decreasing much faster than that in the second one. That is because the service rate of each DC charger is much higher than that of each AC charger. For a charging station with limited space, more DC chargers can reduce the service dropping rate, but may decrease the number of EVs due to its high battery lifetime-related cost.

**D. The Effects of \( \bar{Q}^D \) and \( \bar{Q}^A \) on Service Dropping Rate \( L_t \)**

Since the maximal queue length affects the minimal service dropping rate \( L_t \), we conduct several simulations to demonstrate the effects of \( \bar{Q}^A \) and \( \bar{Q}^D \) on the minimal service dropping rate \( L_t \). First, we fix \( \bar{Q}^A = 10 \) and adjust \( \bar{Q}^D \) from 1 to 20, whose minimal service dropping rate \( L_t \) is shown in Fig. 7(a). Then, we fix \( \bar{Q}^D = 8 \) and adjust \( \bar{Q}^A \) from 1 to 20, whose minimal service dropping rate \( L_t \) is illustrated in Fig. 7(b). Obviously, the service dropping rate \( L_t \) in the first simulation will decrease more quickly than that in the second simulation. That is because the service rate for the DC chargers is much higher than that of the AC charger. However, increasing the waiting space of the charging station may not improve the service dropping rate significantly, since the service dropping rate mainly depends on the service ability of the charging station.

**V. CONCLUSION**

In this paper, we modeled the operation of the charging station with dual charging modes as a queuing network with the multiple servers and heterogeneous service rates, and analyzed the relationship between the service dropping rate and the selections of EVs. Then, by making use of price sensitiveness of EV owners, we designed an optimal pricing scheme to guide and coordinate the charging processes of EVs to minimize the service dropping rate of the charging station. Simulation results are provided to demonstrate the efficiency of the proposed charging scheduling scheme.

In our future work, we will consider the optimal pricing scheme for the charging station, where the arrival rate of the charging station depends on the service fee and the EVs can change their selections when the selected queue length is too long. Also, we intend to design an algorithm to determine the optimal number of chargers with dual charging modes based on the distribution of EVs, which can maximize the total profit of the charging stations and improve the service quality of the charging station.

**APPENDIX A**

**Proof:** For the special case when \( \bar{Q}^D = 0 \), according to the Erlang’s C formula [34], the loss probability for \( M/M/N/N \) can be given by

\[
B(N_2, N_2 \rho_i^D) = \rho_i^D(0) \frac{N_2^{N_2} (\rho_i^D)^{N_2}}{N_1 !(1 - \rho_i^D)^{N_2}}
\]

\[
= \sum_{n=0}^{N_2} \frac{(N_2 \rho_i^D)^n}{n!} \left( \frac{(1 - \rho_i^D)^{N_2+n}}{N_2!(1 - \rho_i^D)^{N_2}} \right)
\]

\[
= \left( \sum_{n=0}^{N_2} \frac{(1 - \rho_i^D)}{n!(N_2 \rho_i^D)^{N_2-n}} + 1 \right)^{-1}. \tag{36}
\]

The existing works [35], [36] have proved that \( B(N_2, N_2 \rho_i^D) \) is an increasing and convex function of \( \rho_i^D \). When \( N_2 \) is given. Thus, both of the first and the second derivatives \( B(N_2, N_2 \rho_i^D) \) with respect to \( \rho_i^D \) are larger than zero, i.e., \( \partial B(N_2, N_2 \rho_i^D) / \partial \rho_i^D > 0 \) and \( \partial^2 B(N_2, N_2 \rho_i^D) / (\partial \rho_i^D)^2 > 0 \).

According to (34), \( L_t^D \) can be rewritten as

\[
L_t^D = N_2 \mu^D (\rho_i^D) \hat{Q}^D + 1 B(N_2, N_2 \rho_i^D). \tag{37}
\]

The first derivative \( L_t^D \) with respect to \( \rho_i^D \) is

\[
\partial L_t^D \over \partial \rho_i^D = N_2 \mu^D \left( \hat{Q}^D \right) \hat{Q}^D + 1 B(N_2, N_2 \rho_i^D)
\]

\[
+ \left( \rho_i^D \right) \hat{Q}^D + 1 \frac{\partial B(N_2, N_2 \rho_i^D)}{\partial \rho_i^D}.
\]

Since \( N_2 \mu^D > 0 \), \( \partial B(N_2, N_2 \rho_i^D) / \partial \rho_i^D > 0 \), \( B(N_2, N_2 \rho_i^D) > 0 \), \( (\rho_i^D) \hat{Q}^D + 1 > 0 \), and \( \partial B(N_2, N_2 \rho_i^D) / \partial \rho_i^D > 0 \), we have

\[
\partial L_t^D \over \partial \rho_i^D > 0.
\]
The second derivative $L_D^2$ with respect to $\rho^D$ is
\[
\frac{\partial^2 L_D}{\partial (\rho^D)^2} = N_2 \mu D \left( \frac{\partial^2 (\rho^D)^2}{\partial (\rho^D)^2} + \partial B(N_2, N_2 \rho^D) \right) + 2 \frac{\partial (\rho^D)^{-1}}{\partial (\rho^D)^2} B(N_2, N_2 \rho^D) + (\rho^D)^{-1} \frac{\partial B(N_2, N_2 \rho^D)}{\partial (\rho^D)^2}. \tag{38}
\]
Since \( \frac{\partial^2 (\rho^D)^{2+1}}{\partial (\rho^D)^2} = \frac{\partial (\rho^D)^{-1}}{\partial (\rho^D)^2} \) and \( \frac{\partial B(N_2, N_2 \rho^D)}{\partial (\rho^D)^2} > 0 \), and \( \frac{\partial^3 B(N_2, N_2 \rho^D)}{\partial (\rho^D)^3} > 0 \), \( \frac{\partial^2 (\rho^D)^{2+1}}{\partial (\rho^D)^2} > 0 \) always holds.

Since \( \frac{\partial L_D}{\partial \rho^D} > 0 \) and \( \frac{\partial^2 L_D}{\partial (\rho^D)^2} > 0 \) always hold, the service dropping rate $L_D^2$ is an increasing and convex function of the service rate $\rho^D$.

**APPENDIX B**

**Proof:** Since \( \lambda^A = L_1 - \lambda^D \), \( \rho^A = \lambda^A / N_1 \mu^A \) and \( \rho^D = \lambda^D / N_2 \mu^D \), we have
\[
\rho^A = \frac{L_1 - N_2 \mu^D \rho^D}{N_1 \mu^A}. \tag{39}
\]
and the first derivation of $\rho^A$ with respect to $\rho^D$ is
\[
\frac{\partial \rho^A}{\partial \rho^D} = -\frac{N_2 \mu^D \rho^D}{N_1 \mu^A}. \tag{40}
\]
Since $L_D^1$ has the same structure with $L_A^1$, it can be easily proved that \( \frac{\partial L_D^1}{\partial \rho^D} > 0 \) and \( \frac{\partial^2 L_D^1}{\partial (\rho^D)^2} > 0 \). Due to \( \frac{\partial^2 \rho^D}{\partial (\rho^D)^2} < 0 \), we have \( \frac{\partial L_D^1}{\partial \rho^D} < 0 \) and \( \frac{\partial^2 L_D^1}{\partial (\rho^D)^2} > 0 \). Thus, the service dropping rate $L_D^1$ is a decreasing and convex function of $\rho^D$.

**REFERENCES**


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