

# Topology Control for Wireless Sensor Networks

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## ABSTRACT

We consider a two-tiered Wireless Sensor Network (WSN) consisting of sensor clusters deployed around strategic locations and base-stations (BSs) whose locations are relatively flexible. Within a sensor cluster, there are many small sensor nodes (SNs) that capture, encode and transmit relevant information from the designated area, and there is at least one application node (AN) that receives raw data from these SNs, creates a comprehensive local-view, and forwards the composite bit-stream toward a BS. In practice, both SN and AN are battery-powered and energy-constrained, and their node lifetimes directly affect the network lifetime of WSNs. In this paper, we focus on the *topology control* process for ANs and BSs, which constitute the upper tier of a two-tiered WSN. We propose approaches to maximize the *topological* network lifetime of the WSN, by arranging BS location and inter-AN relaying optimally. Based on an algorithm in Computational Geometry, we derive the optimal BS locations under three topological lifetime definitions according to mission criticality. In addition, by studying the intrinsic properties of WSNs, we establish the upper and lower bounds of their maximal topological lifetime. When inter-AN relaying becomes feasible and favorable, we continue to develop an optimal parallel relay allocation to further prolong the topological lifetime of the WSN. An equivalent serialized relay schedule is also obtained, so that each AN only needs to have one relay destination at any time throughout the mission. The experimental performance evaluation demonstrates the efficacy of topology control as a vital process to maximize the network lifetime of WSNs.

## Categories and Subject Descriptors

C.2.1 [COMMUNICATION NETWORKS]: Network Architecture and Design—*Network topology, Wireless communication*

## General Terms

Algorithms, Design, Performance

## Keywords

Wireless sensor networks, topology control, network lifetime

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## 1. INTRODUCTION

Recent advances in MEMS technologies and low-power short-range radios have enabled a rapid development of Wireless Sensor Networks (WSNs) in the last few years, which predicts a ubiquitous deployment in the near future [1, 2]. In a two-tiered WSN considered in this paper, small (even tiny) sensor nodes (SNs) are deployed in clusters around strategic locations to capture essential information in terms of video or audio stream, temperature reading, motion measure, and so on. In addition, there is at least one application node (AN) in the same cluster. This AN is responsible for receiving raw data generated by SNs, creating a local-view by exploring the application-specific correlation among raw data, and forwarding the composite bit-stream toward a base-station (BS). The BS, which may be relatively distant from these ANs, is equipped with sophisticated processing and storage capabilities to further increase the value of individual local-views by creating a comprehensive global-view for the whole WSN. The BS can also serve as a gateway for WSNs to exchange data and control information with other networks. Besides many interesting applications such as real-time monitoring, search and rescue, emergence response, and field surveys enabled by their wired counterparts [3, 4], WSNs have some attractive features, such as quick on-demand (re)deployment and virtually free of coverage constraints.

In recent years, WSNs and wireless ad hoc networks [5, 17], have sparked numerous research interests in almost every layer of the network protocol stack. There are extensive efforts in energy-conscious media access control (MAC) [35, 36, 37, 38], variable topology multi-hop routing [11, 14, 34], localized flow and error control [16, 26], domain-specific application design [33], *etc.*. Traditional computer networks may more-or-less have similar concerns, but the distinct characteristics of WSNs and the like warrant a rethinking of the conventional design in network architectures, services, and protocols. As expected, these efforts will help pave the road for a more efficient WSN deployment.

There are some unique application requirements that further distinguish WSNs from others. First, SNs and ANs are usually battery-powered. It is unlikely, if not impossible, to recharge them economically once they are deployed in the field. In many instances, low-cost SNs are even disposable after the mission is over. If a single SN runs out of energy, its AN may still have the capability to reconstruct a comprehensive local-view by data generated by other correlated SNs. However, if an AN runs out of energy, the whole cluster coverage is totally lost, which can jeopardize the entire mission in some cases. Therefore, we are more concerned about the energy constraint of ANs. Although ANs can have better initial energy provisioning than SNs, ANs also consume energy at a considerably higher rate due to the transmission of bit-streams over much longer distances. On the other hand, BSs can have extra recharge-

able power supplies to facilitate additional computational and communicational needs. Second, once being sparsely deployed around strategic locations, SN/AN clusters are stationary or have very low mobility. However, we may have the flexibility to locate the recyclable BSs and arrange communication activities among ANs and BSs. The location and communication arrangement can be determined before the WSN initialization, or can be adjusted throughout the mission. Third, the practical value, or *utility*, of a WSN heavily depends on the time duration from the network initialization to a point when the WSN fails to maintain *enough* alive ANs that construct and feed live local-views. Here, we are particularly interested in the lifetime of the whole WSN, which has to be collectively determined by the lifetimes of *certain* ANs. There are many factors involved when determining the node lifetime: an energy-conscious MAC can avoid the energy wasted by consistent channel sensing and frequent transmission collisions; an energy-aware routing can balance the power consumption among ANs and route around dead or dying ANs; an energy-favored flow and error scheme can have an asymmetric design with most of its control overhead at BSs that are not energy constrained. Other non-communication-related activities such as node operating and local-view composition can also affect the lifetime of a particular AN.

In this paper, we focus on the distance-dominated communication-related power consumption in WSNs. In particular, we are interested in *topology control*, an underlying process even below the traditional layer structure. Given a geographical coverage, topology control determines where to place SNs, ANs and BSs, and how to arrange communication among ANs and BSs, in order to accomplish a mission effectively and efficiently. The network lifetime under this process is referred to as *topological lifetime*. Our goal is to maximize the topological lifetime of a WSN with regard to a given mission and a certain amount of initial energy.

The main contributions of this paper are two-fold. First, according to mission criticality, we analytically obtain the optimal BS locations for a WSN without relying on inter-AN relaying. We also study the intrinsic properties of WSNs and develop the upper and lower bounds of maximal topological lifetime to enable a quick assessment of energy provisioning feasibility and topology control necessity. Second, if the BS has been located and inter-AN relaying is applicable, we theoretically obtain the optimal relay allocation such that the topological lifetime is further prolonged. We then transform the parallel relay allocation into a serialized relay schedule with the same optimality, where each AN only needs to have one relay destination at any time.

The remainder of this paper is organized as follows. In Section 2, we give the system architecture of a two-tiered WSN, its AN power consumption and energy dissipation models, and three definitions of topological network lifetime. In Section 3, by extending an algorithm in Computational Geometry, we present the approaches to locate BS optimally without relying on inter-AN relaying. More importantly, we discover some intrinsic properties of WSNs and derive the upper and lower bounds of maximal topological lifetime. Locating BS without inter-AN relaying provides a *hard* bottom-line even when relaying is application-undesirable, energy-unfavorable, or infeasible in operation due to the limitations in transceiver designs or concerns in inter-AN trustworthiness. In Section 4, we introduce an approach to arrange inter-AN relaying optimally if relaying does become favorable and feasible. First, we define relaying feasibility and select relay candidates. We then obtain an optimal relay allocation by assuming that ANs have the capability to communicate in parallel with multiple AN/BSs. Finally, we transform the parallel allocation into a serialized relay schedule without loss of optimality. In Section 5, we discuss the applica-

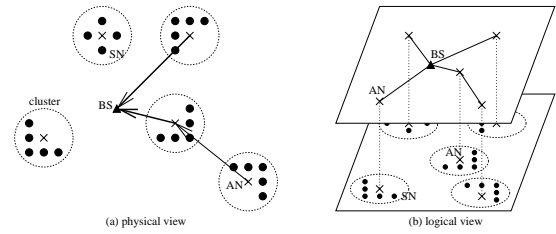


Figure 1: A two-tiered architecture of WSNs

bility and extensibility of the proposed approaches to handle heterogeneous ANs and constrained BSs and to incorporate the cluster placement and partition techniques. We present related work in Section 6, with a focus on topology and lifetime-centric efforts in wireless ad hoc and sensor networks. Section 7 concludes this paper and points out the directions for our future work.

## 2. SYSTEM MODEL

### 2.1 Two-tiered Wireless Sensor Networks

A two-tiered WSN, as shown in Fig. 1(a), consists of a number of SN/AN clusters and at least one BS. In each cluster, there are many SNs and at least one AN. SNs are responsible for all sensing-related activities: once triggered by an internal timer or an external event, an SN starts to capture live information that will be encoded by the SN and directly transmitted to an AN in the same cluster. SNs are small, low cost and disposable, and can be densely deployed within a cluster. SNs do not communicate with other SNs in the same or other clusters, and usually are independently operated. ANs, on the other hand, have much more responsibilities than SNs. First, an AN receives raw data from all active SNs in the cluster. It may also instruct SNs to be in sleep, idle, or active state if some SNs are found to always generate uninterested or duplicated data, thereby allowing these SNs to be reactivated later when some existing active SNs run out of energy. Second, the AN creates an application-specific local-view for the cluster by exploring the correlation among data generated by SNs. Excessive redundancy in raw data can be alleviated by ANs; the fidelity of captured information will be enhanced. Third, the AN forwards the composite bit-stream toward a BS that generates a comprehensive global-view for the entire WSN coverage. Optionally, ANs can be involved in inter-AN relaying if such activity is applicable and favorable.

The two-tiered architecture of WSNs is motivated by the latest advances in distributed signal processing and source coding [7], which offer a better balance among metrics such as reliability, redundancy and scalability in WSNs. Under this architecture, the goal of lower-tier SNs and their ANs is primarily to *gather data* as effectively as possible; upper-tier ANs and BSs are designed to *move information* as efficiently as possible. ANs, which extract useful information and compose local-views, are the logical bridge for these two tiers, as shown in Fig. 1(b). With this functionality partition, we can optimize the performance of each tier separately, since they are designed for different purposes and have different concerns. Practically, both SNs and ANs are battery-powered. Although ANs can carry more initial energy, they also consume energy at a much higher rate due to the transmission of bit-streams to BSs that are comparatively far away. When an SN runs out of energy, its AN may still have the capability to reconstruct a comprehensive local-view with other related SNs; but if the AN runs out of energy, the whole coverage of the cluster will be totally lost

**Table 1: Notations**

symbol	description
$V_N$	a set of $N$ ANs of a WSN
$v_i$	an AN at $(x_i, y_i)$ on a plane
$b$	base station (for notation convenience, $v_0 = b$ )
$d_i$	the Euclid distance from $v_i$ to $b$
$d_{j,k}$	the Euclid distance from $v_j$ to $v_k$
$D$	diameter of $V_N$ , <i>i.e.</i> , $D = \max\{d_{i,k}\}$
$r_i(t)$	data rate generated locally by $v_i$ at time $t$ , or $r_i$ if $r_i(t)$ is time-invariant
$r_{i,j}(t)$	data rate relayed from $v_i$ to $v_j$ at time $t$
$p_i(t)$	power consumption of $v_i$ at time $t$ , or $p_i$ if $p_i(t)$ is time-invariant (Sec. 2.2)
$e_i(t)$	remaining energy for $v_i$ at time $t$ (Sec. 2.2)
$e_i(0)$	initial energy allocation for $v_i$
$l_i$	node lifetime of $v_i$
$L$	network lifetime without relaying (Sec. 2.3)
$V_C$	critical node set (Sec. 2.3 and Sec. 3)
$V_S$	supporting node set (Sec. 2.3 and Sec. 3)
$R$	network lifetime with node relaying (Sec. 4)
$RC_i$	relay candidates set for $v_i$ (Sec. 4.1)
$RR_i$	parallel relay routes allocation for $v_i$ in terms of $\{(v_i, v_k, r_{i,k})\}$ (Sec. 4.2)
$\phi_{i,k}$	energy quota to relay data from $v_i$ to $v_k$ (Sec. 4.3)
$RS_i$	serialized relay routes schedule for $v_i$ in terms of $\{(time, v_i, v_k)\}$ (Sec. 4.3)

from the viewpoint of BSs, even when some SNs in that cluster still have energy left. Therefore, in the rest of this paper, we mainly focus on energy constraints of ANs.

Once being deployed, an AN can obtain and report its own location by using its on-board GPS receiver, through triangulation with a few reference points [43], or as instructed by network operator during manual deployment. ANs are in sleep state initially, unless they are activated by the on-board wake-up circuit. Then they are instructed with mission schedules, aggregation schemes, and relay routes to cooperatively accomplish the mission with other SN/AN clusters. An SN/AN cluster may undergo the sleep-idle-active cycle repeatedly during its lifetime until the AN exhausts its on-board energy. Once being activated, the AN should feed live local-views or view changes to other ANs or, eventually, to BSs. According to a specific mission, all ANs can be activated at the same time, or they can be activated independently. The first style is referred to as synchronized activation; the second one is unsynchronized. An AN can be left in the active state once it is activated, or it can be in the active and inactive (including sleep and idle) states alternatively. The first mode is referred to as continuous activation; the second one is discrete. Although different missions can choose different activation styles and activation modes, from the viewpoint of topological lifetime, an unsynchronized discrete mission can always be converted into an *equivalent* synchronized continuous mission, as will be soon discussed in Section 2.2.

During the topology control process, once ANs have been placed, an immediate challenge is to locate BSs so that network lifetime can be maximized. We assume that ANs can communicate with BSs independently, and that BSs are always reachable for ANs as long as ANs can draw enough transmission power from their remaining energy supply. This property, and the characteristics of steady live local-views created by ANs, suggest a deterministic MAC scheme such as TDMA employed by ANs. Although an

SN, depending on the amount of sensible information available at a certain moment, can send raw data in burst to its AN, the aggregated live local-views should be relatively smooth and in low volume, whereas the TDMA scheme can save extra control overhead and power consumption encountered by a contention-based MAC scheme. However, our study does not rely on any specific MAC schemes, since topology control is even under the regular MAC layer. After BSs are located, and if inter-AN relaying is applicable, BSs can calculate relay schedules, and instruct ANs to communicate cooperatively to achieve a longer network lifetime. In Table 1, we list some frequently-used symbols in this paper.

## 2.2 Power and Energy Models

Communication is a dominant factor in power consumption for WSNs where live local-views are transmitted over the air. Thus, we focus on the communication-related activities for the battery-powered ANs, since BSs are not energy constrained. For an AN to transmit a composite bit-stream at rate  $r$  over the Euclid distance  $d$ , the minimal transmitter power consumption  $p_t$  is

$$p_t(r, d) = r(\alpha_1 + \alpha_2 d^n), \quad (1)$$

where  $\alpha_1$  is the distance-independent term (*i.e.*, the power consumed in the transmitter circuit), and  $\alpha_2$  captures the distance-dependent one. (1) mainly considers the path loss of exponent  $n$ , and usually  $2 \leq n \leq 4$  for the free-space and short-to-medium-range radio communication. Even with a more complicated model (*e.g.*, including multi-path fading and geographical shadowing effects), the proposed approaches still apply, as long as the distance-related power consumption can be isolated empirically.

For an AN to receive a bit-stream at rate  $r$  from other ANs, the power consumed in the receiver circuit is

$$p_r(r) = r\beta. \quad (2)$$

Therefore, for an AN to relay a bypassing bit-stream at  $r$  and to forward it further over distance  $d$ , the power consumption is

$$p_f(r, d) = p_r(r) + p_t(r, d) = r[(\beta + \alpha_1) + \alpha_2 d^n]. \quad (3)$$

If an AN has the capability to aggregate 2 incoming streams at  $r_1$  and  $r_2$  into 1 outgoing stream at  $r$  where  $r_1 + r_2 > r$ , the aggregation power consumption is

$$p_a(r_1 + r_2; r) = (r_1 + r_2 - r)\gamma. \quad (4)$$

In summary, if an AN generates a bit-stream at  $r_0(t)$  itself, aggregates  $j$  incoming streams at  $r_{i,0}(t)$  where  $1 \leq i \leq j$ , relays  $m$  bypassing streams at  $r_{k,0}(t)$  where  $j+1 \leq k \leq j+m$ , and transmits an outgoing stream at  $r(t)$  to another AN or a BS which is  $d$  away, the total communication-related power consumption is

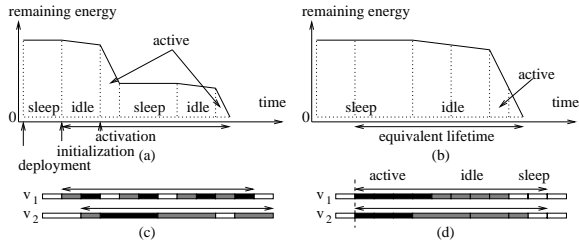
$$\begin{aligned} p(t) &= p_r(\sum_{i=1}^{j+m} r_{i,0}(t)) \\ &+ p_a(\sum_{i=1}^j r_{i,0}(t); r(t) - r_0(t) - \sum_{i=j+1}^{j+m} r_{i,0}(t)) \\ &+ p_t(r(t), d). \end{aligned}$$

The initial energy allocated for an AN is denoted as  $e(0)$ . With a linear energy dissipation model, node lifetime  $l$  is defined by

$$\int_{t=t_0}^{t_0+l} p(t) dt = e(0), \quad (5)$$

where  $t_0$  is the time when the AN is initialized. Even with a non-linearity model for conventional batteries (*e.g.*, battery lifetime is determined by both battery capacity and discharge current raised to the Peuker constant), as long as we can derive  $l$  from  $e(0)$  and  $p(t)$  empirically, the proposed approaches can still apply.

From the viewpoint of remaining energy, as shown in Fig. 2, an unsynchronized discrete mission can always be transformed into



**Figure 2: Activation styles and modes: (a) discrete, (b) continuous, (c) unsynchronized, and (d) synchronized**

an equivalent synchronized continuous mission [8]. For example, Fig. 2(a) represents a discrete mission. If we group all of the sleep, idle, and active periods together, we have Fig. 2(b), which is a continuous mission equivalent in remaining energy. In Fig. 2(c), two ANs,  $v_1$  and  $v_2$ , have unsynchronized activation cycles. However, we can always rearrange the transformed continuous missions to make sure that they are synchronized at least once. The convertibility is due to the additive property of consumed energy, which is the integral of power consumption over time according to (5). In the rest of this paper, we mainly focus on a synchronized continuous mission, whose results can be extended to a general mission with arbitrary activation styles and modes.

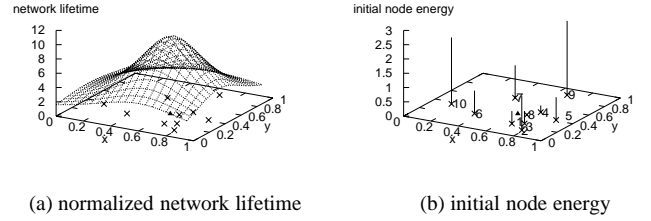
### 2.3 Topological Lifetime Definition

For a WSN of  $N$  ANs placed on a plane ( $V_N = \{v_i = (x_i, y_i)\}$ ), given the initial energy allocation  $e_i(0)$  at  $v_i$ , which generates a bit-stream at rate  $r_i > 0$ , the node lifetime is  $l_i$ . For topology control, we are interested in network lifetime ( $L$  or  $R$ ) from the network initialization to a point when the WSN cannot maintain enough ANs alive to continue the given mission. The goal of topology control is to maximize the topological lifetime with regard to a certain amount of initial energy provisioning.

According to the *criticality* of a specific mission, we have the following three definitions of topological lifetime:

- $N$ -of- $N$  lifetime  $L_N$ : Mission fails when any AN runs out of energy, *i.e.*,  $L_N = \min\{l_i\}$  for  $1 \leq i \leq N$ . The first ANs that run out of energy are denoted as *critical nodes* in  $V_C$ .
- $K$ -of- $N$  lifetime  $L_N^K$ : Mission survives as long as there are at least  $K$  ANs alive ( $K \leq N$ ), or mission fails when the  $(N-K+1)$ -th AN runs out of energy, *i.e.*,  $L_N^K = \min_{N-K+1}\{l_i\}$ .
- $m$ -in- $K$ -of- $N$  lifetime  ${}^m L_N^K$ : Mission survives as long as all  $m$  supporting ANs in  $V_S$  are alive, and overall at least  $K$  ANs are alive ( $m \leq K \leq N$ ). Conversely, mission fails if any AN in  $V_S$  or the  $(N-K+1)$ -th AN in  $V_N \setminus V_S$  runs out of energy, *i.e.*,  ${}^m L_N^K = \min\{L_m | V_S, L_{N-m}^K | V_N \setminus V_S\}$ .

For  $L_N$ , it is implied that every AN is vital and cannot be substituted by others; while for  $L_N^K$ , there is a certain amount of redundancy among ANs. Even if some ANs fail, their responsibilities can be taken by nearby ANs, so that the WSN still has the capability to carry on its mission. Normally,  $K$  is close to  $N$ , otherwise, the deployment has too much redundancy. Obviously,  $L_N \leq L_N^K$  when  $K \leq N$ , and when  $K = N$ , it degenerates to  $L_N$ . Less obviously,  $L_N^K$  is *not* the time when the  $(N-K+1)$ -th AN runs out of energy under the  $L_N$  definition, which will be clarified in Section 3.2. The pre-specified  $m$  supporting ANs are vital to the mission, so they play an important role in determining the network lifetime. Since  $m \leq K \leq N$ ,  $L_N \leq {}^m L_N^K \leq L_N^K$ . When  $m = 0$ , it degenerates to  $L_N^K$ . Also  ${}^m L_N^K \leq {}^m L_N^m$  for the same  $V_S$  since  $m \leq K$ .



**Figure 3: Flexible initial energy allocation**

Maximizing the topological lifetime  $L_N$  is equivalent to maximizing  $\min\{l_i\}$  for  $1 \leq i \leq N$ , where  $\min\{l_i\}$  is the lifetime of the critical ANs. In the next two sections, we adopt a 2-step strategy to approach this problem. First, we locate the BS without relying on inter-AN relaying; second, we further prolong the network lifetime by arranging inter-AN communication properly if relaying becomes applicable. We will also discuss the strategy and feasibility of combining these two steps into one.

### 3. LOCATING BASE-STATION

We begin with locating one BS  $b$  (or  $v_0 = (x_0, y_0)$  for notation convenience) for  $V_N$ . At this stage, we do not rely on inter-AN relaying for the following reasons: relaying may be undesirable for applications, energy inefficient (*e.g.*, the power saving due to a smaller  $d$  in (1) may not compensate the extra overhead in (2) from system viewpoint), or is practically infeasible due to the limitations in MAC and AN designs (*e.g.*, a simplex or half-duplex MAC with a very limited on-board buffer) or trust concerns (*i.e.*, BSs cannot accept any data from an AN that does not generate them). More importantly, the BS location under this context gives a *hard* bottom-line even if relaying fails during the operation.

Before obtaining the optimal BS location, we first notice that the initial energy allocation plays an important role in topology control for WSNs. Fig. 3(a) gives the normalized lifetime  $L_N$  with a random BS location for a sample WSN of  $N = 10$  ANs (identified by numbered crosses) scattered in a unit square. A filled triangle on the plane denotes the BS location when the network lifetime is *maximized* through numerical search. Here, we assume that the initial energy *can* be allocated proportionally for an AN according to its actual power consumption, and that the total energy provisioning for the whole network is  $N$  units. Let  $n = 2$  for an easy geometrical illustration. Assume that only the distance-related power consumption is considered, *i.e.*,  $\alpha_1 = 0$ . Therefore,  $l_i = \frac{e_i(0)}{r_i \alpha_2 d_i^2}$ , and the network lifetime shown in Fig. 3(a) is normalized with regard to  $1/(r \alpha_2)$ . Fig. 3(b) plots the initial AN energy allocation *w.r.t.* the *best* BS location found in Fig. 3(a). The farther away an AN is from the BS, the more initial energy it should be granted, so that all ANs will run out of energy at the exactly same time.

Unfortunately, in reality, we rarely have such flexibility and granularity to allocate the initial energy. Homogeneous *off-shelf* ANs may have different models, but their initial on-board energy (*e.g.*, number of batteries) is proportional to the bit-rate at which they generate, *i.e.*,  $l_i = \frac{E(0)}{d_i^2}$  and  $E(0) = \frac{e_i(0)}{r_i \alpha_2}$  for all ANs. In Fig. 4(a), node lifetime is normalized with regard to  $E(0)$ , and is compatible that in Fig. 3(a). This figure plots network lifetime under the fixed initial energy allocation scheme for the same sample network, *i.e.*, one unit per node. The *maximal* normalized topological lifetime through numerical search drops dramatically from 10.794 to 5.083 unit time, as listed in Table 2. Even with the *best* BS location,

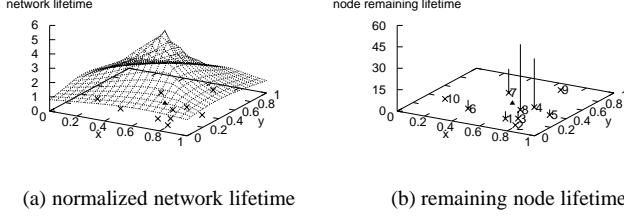


Figure 4: Fixed initial energy allocation

**Table 2: Statistics on flexible and fixed allocation schemes**

$L_N$	min	mean	median	max	25%-75%
flexible	1.337	4.520	3.942	10.794	2.722-5.800
fixed	0.722	1.789	1.546	5.083	1.250-2.123

when the whole network cannot continue its mission, many ANs still have considerable energy left (but wasted) to keep themselves alive for a while (e.g., 45.831 unit remaining lifetime for  $v_8$ ), as Fig. 4(b) shows. Table 2 also indicates that topology control is vital for WSNs, because regardless of energy allocation schemes, a randomly located BS is unlikely to achieve maximal network lifetime. Since the fixed initial energy allocation scheme is employed in practice, we adopt this scheme for the rest of this paper.

### 3.1 N-of-N Lifetime

Although exhaustive numerical searches like the ones used in Fig. 3 and Fig. 4 can find the *best* BS location approximating the optimal one, this approach becomes prohibitively expensive when we have a large coverage and need fine precision. In the following section, we will develop an approach that locates BS optimally and only imposes the least complexity. Fig. 4(b) suggests that the optimal BS location actually is determined by few *critical* ANs that run out of energy first. To maximize  $\min\{l_i = \frac{E(0)}{d_i^n}\}$ , it is equivalent to minimize  $d = \max\{d_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2}\}$  where  $(x_0, y_0)$  is the location of  $b$ , and  $L_N = \frac{E(0)}{d^n}$ . Given the definition of  $d$ , the optimal  $b$  should locate at the center of a circle  $C$  with minimal radius  $d$  to enclose  $V_N$  and cross  $V_C$ .

#### 3.1.1 Properties and bounds

First with a *naive* approach, we can show that  $C$  always exists. As Fig. 5 sketches, we can find a circle  $C_0$  large enough to enclose

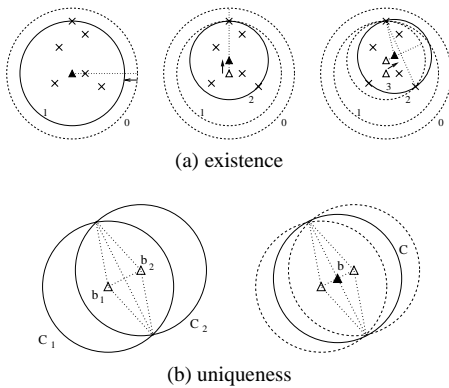


Figure 5: Properties on  $C$  under  $L_N$  definition

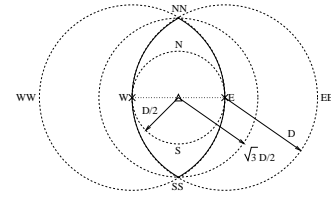


Figure 6: Upper and lower bounds of  $d$

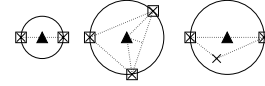


Figure 7:  $C$  for small  $N$

all ANs in  $V_N$ . Then we keep the center intact and reduce the radius until there is a smaller circle  $C_1$  across an AN in  $V_N$ . Next we move the center toward that AN, and keep it crossed, which effectively reduces the radius, until an even smaller circle  $C_2$  crosses another AN in  $V_N$ . If these two ANs are co-diameter, we can no longer shrink the circle, and  $C = C_2$ . Otherwise, we move the center toward the line determined by these two ANs, and keep these two ANs crossed, which also reduces the radius, until another even smaller circle  $C_3$  crosses the third AN in  $V_N$ . Now  $C = C_3$ , as at most 3 nodes determine a circle on the plane (there might be more than 3 node crossed by  $C_3$  when  $V_N$  degenerates).

Next, we show that  $C$  exists uniquely. Assume that  $C_1$  and  $C_2$  with the minimal radius  $d$  centered at  $b_1$  and  $b_2$ , respectively, as shown in Fig. 5(b). Since all ANs are enclosed by both  $C_1$  and  $C_2$ , they should be within the area intersected by  $C_1$  and  $C_2$ . However, this area can be enclosed by a smaller circle with the radius  $\sqrt{d^2 - (\frac{d_{b_1, b_2}}{2})^2} \leq d$ . This contradicts the assumption that  $C_1$  and  $C_2$  have the minimal radius unless  $d_{b_1, b_2} = 0$ , or  $b_1 = b_2$ . The uniqueness of  $b$  means that the optimal BS location under the  $L_N$  definition is deterministically unique, so is the  $\max L_N$ .

Finally, we derive the upper and lower bounds of  $d$  as shown in Fig. 6. Let node  $E$  and  $W$  be the diameter ANs of  $V_N$ , i.e., they are  $D$  away. Obviously  $d \geq \frac{D}{2}$ . Otherwise,  $C$  can not enclose both  $E$  and  $W$  at the same time. All ANs should be within the area  $E$ - $NN$ - $W$ - $SS$ - $E$  where  $NN$  and  $SS$  are  $D$  away from both  $E$  and  $W$ . We find that the area can be enclosed by a circle with radius  $\frac{\sqrt{3}D}{2}$  and centered at the middle point of  $E$  and  $W$ . Therefore,  $\frac{D}{2} \leq d \leq \frac{\sqrt{3}D}{2}$ . With the bounds of  $d$ , we can obtain the bounds of  $\max L_N$ , and easily assess whether the energy allocation is feasible to maintain a WSN for a certain period of time. If a mission requires a lifetime longer than the  $\max L_N$  upper bound, we need either to consider inter-AN relaying, or introduce multiple BSs. In Section 3.1.2, we will show how to tighten the upper bound of  $d$ .

#### 3.1.2 Algorithmic approach

The existence property suggests a *brute force* approach to find  $C$  and  $b$ . Since  $C$  is determined by 2 or 3 ANs in  $V_N$ , there are at most  $\binom{N}{2} + \binom{N}{3}$  such circles. By pairwise comparing those enclosing all ANs with regard to their radius, we should find  $C$  in finite steps. However, this approach still imposes a computational challenge when  $N$  is relatively large.

Therefore, we need develop an algorithmic approach with the least complexity to obtain  $C$ . Fig. 7 gives some examples if  $N \in \{2, 3\}$ . When  $N = 2$ ,  $V_C = V_2$ , and  $b$  is at the middle point of

**Table 3: Recursive algorithm to determine  $C$**

```

1  proc c {NC VC}
2    if |VC|==3 || |NC|==0
3      return [c_3 VC]
4    v = [lindex NC 0]
5    NC' = [lrange NC 1 end]
6    C' = [c NC' VC]
7    if v ∉ C'
8      VC' = [lappend VC v]
9      return [c NC' VC']
10   else return C'
11 endproc
12 c VN NULL

```

these two ANs. When  $N = 3$ , there are two subcases.

1. If all 3 angles determined by these 3 ANs are no greater than  $\pi/2$ ,  $V_C = V_3$ . The largest angle should be no less than  $\pi/3$  and face the longest edge no greater than  $D$ . Therefore, the view angle from  $b$  to the longest edge is no less than  $2\pi/3$ . With the cosine rule, we have  $D \geq \sqrt{3}d$ , *i.e.*,
$$\frac{D}{2} \leq d \leq \frac{\sqrt{3}D}{3}. \quad (6)$$
2. If there is 1 (and at most 1) angle at AN  $v$  greater than  $\pi/2$ ,  $v$  is non-critical and  $V_C = V_3 \setminus \{v\}$ .  $|V_C| = 2$ , *i.e.*, it degenerates to the case with 2 critical nodes.

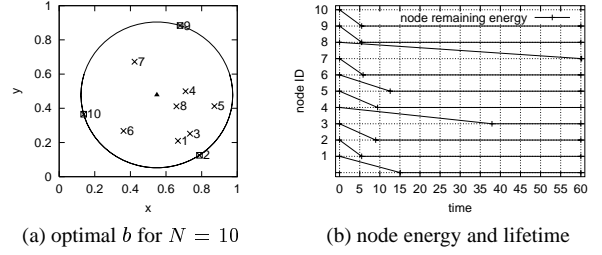
Fig. 7 suggests a recursive approach to obtain  $C$ . Assume that we have  $C_{N-1}$  for  $V_{N-1}$ , and that its critical set is  $V_{C_{N-1}}$ . For the  $N$ -th AN  $v$ , if  $v$  is enclosed by  $C_{N-1}$ , then  $v \in V_N \setminus V_{C_N}$  and  $C_N = C_{N-1}$ . Otherwise,  $v \in V_{C_N}$ . As long as we have 2 or 3 critical ANs for any  $V_N$ , we can get  $C_N$  geometrically. This recursive approach is based on Welzl's algorithm [42] in Computational Geometry, and is outlined by Tc1-like pseudo code in Table 3.

$c\{NC\ VC\}$  is the procedure to find  $C$  for non-critical node set  $NC$  and critical node set  $VC$ . If there are 3 nodes in  $VC$  or  $NC$  is empty,  $C$  can be derived as Fig. 7 shows (line 3 in Table 3). Otherwise, randomly pick a  $v$  from  $NC$  and let  $NC' = NC \setminus \{v\}$ , *i.e.*,  $|NC'| = |NC| - 1$ . We call  $c\{NC' VC\}$  recursively (line 6). After obtaining the  $C'$  for  $NC'$  and  $VC$ , we check whether  $v$  is enclosed by  $C'$ . If yes,  $C'$  is also applicable for  $NC$  and  $VC$  (line 10). If not,  $v$  is a critical node for  $NC$  and  $VC$ . Let  $VC' = VC \cup \{v\}$ , and we call  $c\{NC' VC'\}$  recursively again (line 9). Line 12 gives the initial call on  $c$  when we do not have any knowledge on  $VC$  (*i.e.*,  $VC = \text{NULL}$ ) and treat all nodes as non-critical in  $NC$  ( $NC = VN$ ).

Since at most 3 critical nodes in  $V_N$  determine a  $C_N$ , if we have a perfectly randomized  $NC$  at each step, the probability of line 9 being executed is  $p \leq \frac{3}{|NC|}$  (if  $V_N$  degenerates, it actually speeds up the algorithm in Table 3). Assume that the complexity of  $c$  with  $i$  known critical nodes and  $j$  assumed non-critical nodes is  $O_{i,j}$ , then

$$O_{i,j} \leq O_{i,j-1} + O(1) + \frac{3-i}{j} O_{i+1,j-1}, \quad (7)$$

where  $O(1)$  is the constant cost (*i.e.*, in lines 2, 4, 5, 7 and *etc.*) for each recursion of  $c$ . If we treat  $c_3\{VC\}$  as an atomic operation, *i.e.*,  $O_{3,\cdot} = O(1)$ , the time complexity of  $c\{VN\ \text{NULL}\}$  is



**Figure 8:  $N$ -of- $N$  lifetime**

$O_{0,N} \sim O(N)$ . Actually, this is the *best* complexity that we can achieve algorithmically [42], since we need to examine every AN in  $V_N$  individually to determine whether it belongs  $V_C$  or not, which mandates the *least* achievable complexity  $O(N)$ .

For the sample WSN, the  $C$  and the optimal BS location (filled triangle) obtained through the algorithm described in Table 3 are illustrated in Fig. 8. The critical ANs are additionally denoted with rectangles, and  $V_C = \{v_2, v_9, v_{10}\}$ .  $\max L_N = 5.504$  unit time, which is better than the *best* one (5.083 unit time) obtained through an exhaustive search in Fig. 4(a). According to  $D$ , we can obtain the upper and lower bounds of  $\max L_N$ ,  $L_N^+ = 6.812$  and  $L_N^- = 5.110$  unit time, respectively. If the desired network lifetime is less than 5.110 unit time, just placing BS at the middle point of  $V_N$ 's diameter can satisfy the mission requirement. If the desired lifetime is higher than 6.812 unit time, we even do not need to run the algorithm, since the result won't satisfy the requirement at all. This quick assessment can speed up topology control by avoiding some unnecessary or *doomed* computations.

### 3.2 $K$ -of- $N$ Lifetime

The optimal BS location under the  $L_N^K$  lifetime definition should also exist, but the uniqueness property of such optimal locations cannot be guaranteed for  $L_N^K$  with some easy-to-find counterexamples (*e.g.*, a *dumbbell*-like coverage).

The existence property also suggests a *brutal force* approach to obtain  $C_N^K$ , *i.e.*, choosing any  $K$  ANs from  $V_N$  and executing the  $L_K$  approach repeatedly. Since there are  $\binom{N}{K}$  combinations, by comparing them pairwise,  $\max L_N^K = \max\{L_K | \forall V_K \subset V_N\}$ . However, the definition of  $V_C$  and the property of  $|V_C| \leq 3$  for a general  $V_N$  can help us develop a better approach.

When removing an AN  $v$  from  $V_N$ , if  $v \notin V_C$ , network lifetime, as well as the optimal BS location, will *not* change. Only if a  $v \in V_C$  is removed, the optimal BS location may change accordingly to prolong network lifetime. Since  $2 \leq |V_C| \leq 3$  for a general  $V_N$ , we can build a 2-3 search tree that removes 1 critical AN at each step (if  $V_N$  degenerates, removing a redundant critical node neither prolongs network lifetime nor changes the optimal BS location). The span and depth of the search tree are limited, and we have at most  $3^{N-K}$  leaf node-sets to compare. When  $N$  is large and  $K$  is close to  $N$ , this approach is much more efficient than the *brutal force* approach. We can further enhance this approach if we have logged the history when building  $C_N$ . Since many node-sets in this search tree have already been examined, we can obtain the BS location and its critical AN set for these node-sets directly.

Fig. 9(a) shows the optimal BS location under  $L_N^7$  for the sample WSN. After ANs  $\{v_7, v_9, v_{10}\}$  run out of energy, ANs  $\{v_2, v_5, v_6\}$  become critical for the remaining 7 ANs. Fig. 9(b) further plots  $L_N^K$  for  $K \in \{3, 4, \dots, 10\}$ . The vertical axis in log scale is the time when the  $i$ -th AN, indexed by the horizontal axis, runs out of energy. For  $K = 7$ , we find that the 4-th AN runs out

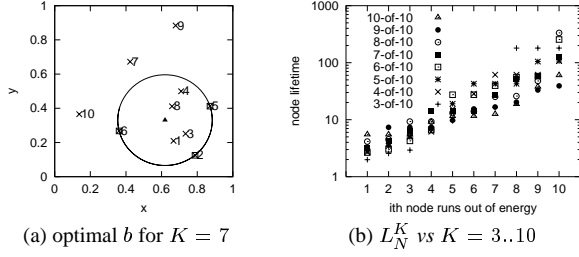


Figure 9:  $K$ -of- $N$  lifetime

of energy at 14.191 unit time after the network initialization, so  $\max L_{10}^7 = 14.191$  unit time. Clearly, the smaller  $K$ , the larger  $L_{10}^K$  as assessed in Section 2.3. Fig. 9 also validates the proposition that  $L_N^K$  is *not* the time when the  $(N-K+1)$ -th AN runs out of energy under the  $L_N$  lifetime. In this example, the 4-th AN under  $L_{10}^0$  runs out of energy at 9.186 unit time.

### 3.3 $m$ -in- $K$ -of- $N$ Lifetime

The optimal BS location for  ${}^m L_N^K$  should also exist, but not necessarily uniquely, since it is a special case of  $L_N^K$  by definition. For  ${}^m L_N^K$ , once a supporting AN runs out of energy, the whole mission fails immediately. As with the *brutal force* approach for  $L_N^K$ , we can find the optimal BS location by choosing  $K-m$  ANs from  $N-m$  non-supporting ANs and using  $L_N^K$  together with  $m$  supporting ANs. In total there are  $\binom{N-m}{K-m}$  combinations, and  $\max {}^m L_N^K = \max \{L_N^K\}$  by comparing them pairwise.

A better approach for  ${}^m L_N^K$  can be obtained by slightly modifying the  $L_N^K$  algorithm. When removing an AN from  $V_C$ , if it is also a *supporting* AN in  $V_S$ , the spanning of the current branch in search tree stops. Therefore, the resultant search tree is no longer always complete. To obtain  $\max {}^m L_N^K$ , we only need to compare the existing leaf node-sets. Under the  ${}^m L_N^K$  definition, the approach to find the optimal BS location may stop even when the remaining ANs are more than  $K$ , *i.e.*,  ${}^m L_N^K$  is no longer *tight* on  $K$  (as both  $V_C$  and  $V_S$  determine network lifetime). Since we only have a partial tree to search, it is clear that  $\max {}^m L_N^K \leq \max L_N^K$ .

## 4. ARRANGING INTER-AN RELAYING

After determining the optimal BS location, we can further prolong network lifetime if inter-AN communication (or relaying) is energy-favorable and feasible. In this section, we first define the relay candidates for a given AN. Then we obtain an optimal parallel relay allocation through Linear Programming. Finally, we introduce an approach to convert the parallel relay allocation, which requires an AN to communicate with all of its relays simultaneously, into a serialized relay schedule, with which an AN only needs to communicate with one of its relays at any time.

### 4.1 Relay Candidate Selection

As discussed in Section 3, the critical ANs run out of energy first. To further prolong network lifetime, it is necessary to find the relay candidates for critical ANs first.

#### 4.1.1 One-dimension relaying

Assume for a critical AN  $v_1 \in V_C$ , there is a non-critical AN  $v_2 \in V_N \setminus V_C$  between  $v_1$  and  $b$ .  $v_2$  can be a relay candidate for  $v_1$ , if  $v_2$  has energy left when  $v_1$  runs out of energy, *i.e.*,  $e_2 - \frac{e_1 p_2}{p_1} > 0$ . As shown in Fig. 10(a),  $v_2$  relays  $x$  portion of the data generated by  $v_1$ . In this stage, we assume that relay is always favorable, *i.e.*,

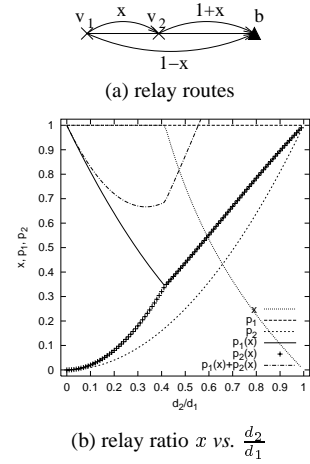


Figure 10: One-dimension relaying

$\beta = 0$  in (3). The communication-related power consumption is

$$p_1(x) = r_1[x(d_1 - d_2)^2 + (1-x)d_1^2]$$

at  $v_1$  and

$$p_2(x) = (r_1 x + r_2)d_2^2$$

at  $v_2$ . For  $v_1$ , its node lifetime with relaying is  $l_1(x) = \frac{e_1}{p_1(x)}$ , and for  $v_2$ ,  $l_2(x) = \frac{e_2}{p_2(x)}$ . By increasing  $x$  from 0 to 1, *i.e.*,  $v_2$  relays more data for  $v_1$ ,  $l_2(x)$  is reduced. This process stops either  $x = 1$  or  $l_2(x) = l_1(x)$ . In the former case,  $v_2$  still has energy left when  $v_1$  runs out of energy. In the latter case,  $v_2$  cannot take more data from  $v_1$ , otherwise  $v_2$  runs out of energy first.

Fig. 10(b) plots the optimal  $x$  as a function of  $\rho = \frac{d_2}{d_1}$  when  $E_1(0) = E_2(0) = 1$  unit. If  $p_1(x) = p_2(x)$ , *i.e.*,

$$x(d_1 - d_2)^2 + (1-x)d_1^2 = (1+x)d_2^2,$$

or

$$d_1^2 - 2x d_1 d_2 - d_2^2 = 0.$$

When  $x = 1$ ,

$$d_1^2 - 2d_1 d_2 - d_2^2 = 0,$$

or

$$\rho^2 + 2\rho - 1 = 0.$$

Therefore  $\rho = \sqrt{2} - 1$  when the optimal  $x$  becomes 1.

As shown in Fig. 10(b), when  $d_2 \leq (\sqrt{2} - 1)d_1$ ,  $v_1$  should use  $v_2$  as its full relay, *i.e.*,  $x = 1$ . When  $(\sqrt{2} - 1)d_1 \leq d_2 \leq d_1$ , the optimal  $x$  decreases gradually. When  $d_2 = d_1$ ,  $x = 0$ , *i.e.*,  $v_2$  is no longer a relay candidate for  $v_1$ , since they are the same distance away from  $b$ . In Fig. 10(b),  $p_1$  and  $p_2$  are power consumption without relaying. To minimize the power consumption at  $v_1$ ,

$$\begin{cases} x = 1 \\ \rho = \sqrt{2} - 1 \end{cases}, \quad (8)$$

and  $\min p_1(1) = [(2 - \sqrt{2})d_1]^2$ . When  $x = 1$ , to minimize the total power consumption  $p_1(x) + p_2(x)$  at  $v_1$  and  $v_2$ ,

$$\begin{aligned} p_1(1) + p_2(1) &= (d_1 - d_2)^2 + 2d_2^2 \\ &= 3(d_2 - \frac{d_1}{3})^2 + \frac{2d_1^2}{3}, \end{aligned}$$

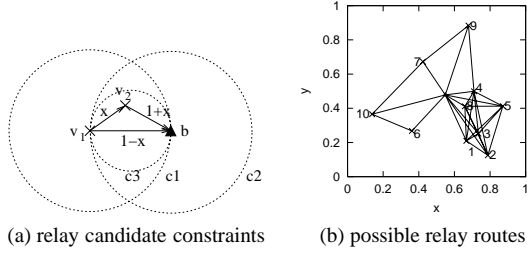


Figure 11: Two-dimensional relaying

i.e.,  $\min\{p_1(1) + p_2(1)\} = \frac{2d_1^2}{3}$  when

$$\begin{cases} x = 1 \\ \rho = \frac{1}{3} \end{cases} \quad (9)$$

These two equations can be used to locate the *best* relay for an AN to minimize its own (8) or the total (9) power consumption for the AN and its relay, respectively. These equations can also assist the SN/AN cluster placement process when dedicated relay nodes are introduced to increase network lifetime.

#### 4.1.2 Two-dimension relaying

Unfortunately, determining relaying routes and relay rates on a plane becomes much more complicated. We will use Linear Programming (LP) to solve the 2-dimension relay allocation. However, the computational complexity may become an obstacle when  $N$  is large, since there are in total  $N^2$  possible relay routes. Therefore, we have to develop some constraints to preselect the relay candidates for an AN so that LP complexity is affordable.

Consider a homogeneous WSN of ANs with unit  $r$  and  $e$ , as shown in Fig. 11(a), there are several possible constraints for an AN  $v_1$  to choose its relay candidate  $v_2$ .

1. *closer to  $v_1$* :  $v_1$  does not choose an AN which is indeed farther away from  $v_1$  than  $b$ , i.e.,  $d_1 > d_{1,2}$ .
2. *relay toward  $b$* :  $v_1$  does not choose an AN which is farther away from  $b$  than  $v_1$ , i.e.,  $d_1 > d_2$ .
3. *energy conservativeness*: optionally,  $v_1$  does not choose  $v_2$  as its relay if the energy saving at  $v_1$  cannot compensate the extra overhead ( $\hat{p}_2$ ) at  $v_2$ , i.e.,  $p_1 - p_{1,2} > \hat{p}_2$ .

The first constraint (c1 in Fig. 11(a)) excludes any ANs that are more *expensive* to reach for  $v_1$ . The second constraint (c2) excludes any ANs that are farther away from  $b$ : since under the  $e$  and  $r$  assumptions, they are more *critical* than  $v_1$ . The final constraint (c3) is optional and only applicable when ANs also need to conserve total energy consumption. c1 and c2 do not change the optimality of network lifetime for homogeneous WSNs, but c2 has such a potential when the initial energy and data rate among ANs are significantly different. When preselecting relay candidates, which constraints are used to filter out *bad* relays depends on specific applications. Here, we adopt constraint c1 and c2.

Table 4 outlines an algorithm to preselect relay candidates and form relay routes in a WSN. Initially, the relay candidate set RC is empty (line 1), and a non-relayed set NR is built (line 3) and then sorted (line 4) by the node distance to the BS. For the first AN  $v$  (line 6) in NR, we examine whether there is a relay candidate  $r$  for this AN in RC (line 8) according to the chosen constraints  $\mathcal{R}$ . If so, the relay route  $\{v \ r \ 0\}$  is added to the relay route set RR. After

Table 4: Algorithm to preselect relay candidates

```

1  set RC NULL
2  foreach v in VN
3    lappend NR {v dv}
4  set NR [lsort -index 1 NR]
5  while NR
6    set v [lindex NR 0]
7    foreach r in RC
8      if r ∈ R(v)
9        lappend RR_v {v r 0}
10   set NR [lrange NR 1 end]
11   lappend RC v

```

all ANs in RC have been examined,  $v$  is removed from NR (line 10) and added to RC (line 11). When NR becomes empty, RR contains all possible relay routes under the chosen constraints. Since there are  $N$  ANs, and each AN can be a relay for other ANs, the time complexity for this algorithm is  $O(N^2)$ . However, it is better to have a preselecting process, instead of leaving this complexity for LP. Fig. 11(b) gives all possible relay routes for the sample WSN with the chosen constraints c1 and c2.

After obtaining the relay routes, we need to determine the amount of data relayed through each route, as we did with the relay ratio  $x$  in Section 4.1.1. The relay routes and their data rate are referred to as relay *rate* allocation. The allocation is optimal if network lifetime can be maximized with relaying.

## 4.2 Parallel Relay Routes

To obtain the optimal relay allocation, we first assume that an AN has the capability to transmit data to multiple relay candidates simultaneously (or in *parallel* relaying).

Consider an AN  $v$  that is a relay for ANs  $\{v_1^r, v_2^r, \dots, v_m^r\}$ , and  $v$  has its own relay candidates  $\{v_1^t, v_2^t, \dots, v_n^t\}$ .  $v$  generates a bit-stream at rate  $r$  itself, and relays for  $v_i^r$  at  $r_i^r$ . It then transmits an outgoing stream at  $r_j^t$  to its relay candidate  $v_j^t$ . Therefore,

$$\sum_{i=1}^m r_i^r + r = \sum_{j=1}^n r_j^t, \quad (10)$$

i.e., the rate of incoming streams plus the rate of the self-generated stream should equal to the rate of outgoing streams, since all live local-views should be forwarded to the BS (we do not consider flow aggregation now, since it is application specific). This property is referred to as *flow conservation*.

Let  $e$  be the initial energy that  $v$  has, and  $\epsilon$  be the remaining energy that  $v$  has when the WSN fails to carry on its mission.  $e - \epsilon$  is the energy to receive flows at  $r_i^r$  from  $v_i^r$  and transmit flows at  $r_j^t$  to  $v_j^t$  throughout network lifetime. Let  $p_i^r$  and  $p_j^t$  be the power consumption to receive and transmit these flows. We have

$$R(\sum_{j=1}^m p_j^r + \sum_{k=1}^n p_k^t) + \epsilon = e,$$

where  $R$  is the network lifetime with inter-AN relaying. This property is referred to as *energy conservation*. When the network fails to carry on its mission, the remaining energy  $\epsilon \geq 0$ . This equation can be further rewritten as

$$\frac{\sum_{j=1}^m p_j^r + \sum_{k=1}^n p_k^t}{e} + s = \frac{1}{R}, \quad (11)$$

where  $s = \frac{\epsilon}{eR} \geq 0$  is treated as a *slack* variable.

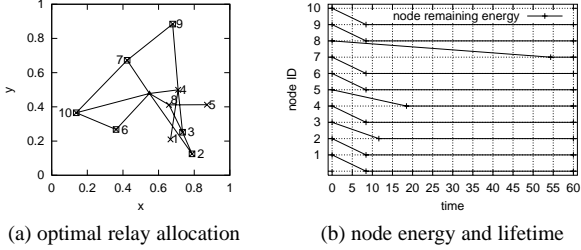
Now we can formulate a Linear Programming problem with the objective of maximizing the network lifetime  $R$ , i.e.,

$$\min \frac{1}{R} = \frac{\sum_{j=1}^m p_j^r + \sum_{k=1}^n p_k^t}{e_q} + s_1 \quad (12)$$



**Table 5: Relay routes and rate allocation  $r_{i,j}$**

$i$	$r_i$	$r_{i,1}$	$r_{i,2}$	$r_{i,3}$	$r_{i,4}$	$r_{i,5}$	$r_{i,6}$	$r_{i,7}$	$r_{i,8}$	$r_{i,9}$	$r_{i,10}$	$r_{i,b}$	$e_i$	$\epsilon_i$	$p_i$	$l_i$
1	1	-			1.386				0			0	1	$\approx 0$	$\approx 0.119$	$\approx 8.420$
2	1	0	-	0.386	0	0			0		0.614	0.614	1	0	0.119	8.420
3	1	0		-	1.386				0		0	0	1	0.033	0.086	11.609
4	1				-				0		4.501	4.501	1	0	0.119	8.420
5	1	0.386		0	0.506	-			1.107		0	0	1	0.065	0.054	18.475
6	1						-				1.495	1.495	1	0	0.119	8.420
7	1							-			2.230	2.230	1	0	0.119	8.420
8	1								-		1.107	1.107	1	0.100	0.018	54.359
9	1				0.222				0.778		0	0	1	0	0.119	8.420
10	1						0.495	0.452			-	0.053	1	0	0.119	8.420



**Figure 12: Parallel relay allocation**

with the following constraints at each AN  $v_i$

$$\text{S.T.} \begin{cases} \sum_{j=1}^m r_{j,i}^r + r_i - \sum_{k=1}^n r_{i,k}^t = 0 \\ \frac{\sum_{j=1}^m p_{j,i}^r + \sum_{k=1}^n p_{i,k}^t}{e_i} + s_i - \frac{1}{R} = 0 \end{cases}, \quad (13)$$

where  $r_i$  and  $e_i$  are the data rate and initial energy that  $v_i$  generates and carries, respectively. In this formulation, we have  $N$  flow conservation constraints and  $N$  energy conservation constraints, *i.e.*,  $2N$  constraints in total (term  $\frac{1}{R}$  can be removed from (13) by linking energy constraints at any two nodes, which results an equivalent LP with  $2N - 1$  resultant constraints in total).

If we do not preselect relay candidates in Section 4.1, we have  $N^2$  relay routes (including the final routes to the BS).  $N^2$  variables will add considerable computational overhead when solving this problem. Therefore, the problem formulation has a high complexity, despite the fact that LP itself is expensive to solve in time complexity. Since we cannot reduce the number of constraints, we will try to reduce the number of total variables (routes).

If  $v_i$  chooses  $v_j$  as its relay,  $v_j$  should not choose  $v_i$  as its relay, since it is energy-inefficient to *bounce* traffic between AN/BSs, *i.e.*, there are at most  $\frac{N(N+1)}{2}$  preselected relay routes. With the constraints in Section 4.1.2, we can further reduce the number of routes to be considered in LP formulation.

Table 5 gives the optimal relay *rate* allocation with the preselected relay candidates. Blank entry in  $r_{i,j}$  denotes the routes not in the RR set, and 0 denotes the routes in the RR set but not in the optimal relay allocation set. Since self-relay is not energy conscious, it is denoted as - in Table 5. Positive  $r_{i,j}$  is the actual relay allocation when the network lifetime is maximized. A quick statistics can tell among all 100 relay routes, there are 29 preselected relay routes, and LP finally chooses 16 optimal relay routes, which are shown in Fig. 12(a), for the maximized network lifetime of 8.420 unit time. ANs with the shortest lifetime in Fig. 8(b), *i.e.*,  $\{v_2, v_9, v_{10}\}$ , now have a longer lifetime by transmitting a portion of their data to nearby relays, at the cost of other ANs such as  $\{v_1, v_4, v_8\}$  that now have a shorter lifetime, as shown in Fig. 12(b).

**Table 6: Comparison on Linear Programming complexity**

	# constr.	# variables	# iterations	max $R_N$
full LP	20	100	55	8.420
enhanced LP	20	29	21	8.420

**Table 7: Comparison on random or optimal base station location with optimal relay allocation**

	random $b$ (median)	random $b$ (maximum)	optimal $b$	optimal $b$ w/ optimal relay
$L_N$	1.546	5.083	5.504	$R_N=8.420$

Table 6 compares the complexity of the regular LP formulation and the enhanced LP formulation with preselected routes. They both have 20 constraints, *i.e.*, 1 flow and 1 energy conservation for each AN. For the full LP formulation, there are 100 relay routes considered, while for the enhanced LP, only 29 routes are chosen after preselecting the relay candidates. The number of relay routes considered is translated into the number of total variables in the LP formulation. The more variables, the higher complexity to solve the problem. With the full LP, it takes 51 iterations to find the optimal allocation, while with the enhanced LP, it only takes 21 iterations. They both eventually obtain the optimal relay allocation with the same network lifetime. With the candidate preselecting approach developed in Section 4.1, we can considerably speed up the LP problem-solving process as indicated in Table 6.

Table 7 lists the topological network lifetime achieved through random BS location, optimal BS location without relaying, and optimal BS location with optimal relay allocation. We can see the substantial efficacy of the proposed topology control approaches. To be distinct from Section 3, the  $N$ -of- $N$  topological lifetime with relaying is denoted as  $R_N$ . For the sample WSN, the optimal BS location with optimal relay allocation can improve network lifetime by 445% over the random BS location without relaying.

ANs  $\{v_2, v_4, v_6, v_7, v_9, v_{10}\}$  are critical and run out of energy first in the optimal relay allocation for the sample WSN. It is worth pointing out that for non-critical ANs  $\{v_1, v_3, v_5, v_8\}$ , their relay allocation can vary from the one shown in Fig. 12(a) and Table 5, unless they run out of energy first. In addition, the optimal relay allocation may not be unique due to the different initial LP solution, but they all give the same network lifetime.

For the  $K$ -of- $N$  and  $m$ -in- $K$ -of- $N$  lifetime definitions, the ANs outside  $C_N^K$  or  ${}^m C_N^K$  run out of energy first, since they directly

communicate with  $b$ . When relaying is favorable and feasible, we give preference to the ANs inside  $C_N^K$  or  ${}^m C_N^K$ , and preselect relay candidates and obtain the optimal relay allocation in a manner similar to those described in Section 4.1 and Section 4.2. We do not allow the ANs outside  $C_N^K$  or  ${}^m C_N^K$  to use ANs inside  $C_N^K$  or  ${}^m C_N^K$  as their relay. Since we are dealing with a smaller number of vital ANs, we can expect that  $R_N \leq {}^m R_N^K \leq R_N^K$ .

### 4.3 Serialized Relay Schedule

Although we have obtained the optimal *parallel* relay allocation in Section 4.2, doing so mandated the assumption that an AN always has the capability to transmit data to multiple relay nodes simultaneously. This requirement can impose a technical challenge in the radio transceiver design when a transmitter only tunes to either a specific time slot, or a frequency band, or a code sequence. Therefore, it is necessary to develop a *serialized* relay schedule, so that an AN transmits its data to at most one node (including BS) at any time. At a predetermined time, the AN switches its time slot, frequency band, or code sequence, and communicates with the next relay node. Since turnaround operations are *expensive*, we expect one *switch* for each relay during the whole network lifetime.

The proposed serialization approach is based on relay *energy* allocation, not relay *rate*, *power*, or *time* allocation. Although energy allocation is an integral of power and time allocations, only energy (data) allocation is an *invariant* during the serialization process, as shown in Fig. 2. In a parallel relay allocation, an AN  $v$  transmits a bit-stream at rate  $r_1$  to its relay node  $v_1$ ,  $r_2$  to  $v_2$ , and so on.  $e$  and  $\epsilon$  have the same definitions as those in Section 4.2. During the network lifetime  $R$ , the energy allocation (or *quota*) for  $v_k$  at  $v$  is

$$\phi_k = \frac{(e-\epsilon)p_k}{p_1+p_2+\dots+p_m}, \quad (14)$$

where  $p_k$  is the power to transmit a bit-stream at  $r_k$  to  $v_k$ .

During the whole network lifetime, the AN  $v$  has the flexibility to choose which relay to use at a certain time and how long it uses that node as its relay, as long as the flow conservation and the energy quota are both satisfied. For example, once the WSN is initialized,  $v$  can randomly pick an AN  $v_1$  in its relay set, and transmit all data it has, including the data it generates and the data relayed for others, until it exhausts the energy quota  $\phi_1$  for  $v_1$ .  $v_1$  will then be removed from the relay set. Then it picks another unchosen node  $v_2$  in its remaining relay set and exhausts the energy quota  $\phi_2$  for  $v_2$ . This process repeats until the relay set becomes empty. No matter in which order the relay nodes are chosen,  $v$  always achieves the same network lifetime, as shown in [8].

Table 8 outlines the approach to obtain the serialized relay schedule. Procedure `addr{v dr}` is used when an AN  $v_j$ , which  $v$  relays data for, changes its data relayed from  $v_j$  to  $v$ , by  $\Delta r$ . If  $v$  is the BS, such change has no impact, since  $b$  is not energy constrained (line 2). Otherwise,  $v$  cancels its next `switch` event (line 4) and sets up a new one (line 9) according to the remaining energy quota of its current relay  $v_2$  and the updated outgoing data rate  $r_t$ . This procedure is called recursively for  $v_2$  and its relay.

Procedure `switch{v}` determines the actual relay schedule. It is called when the current relay  $v_2$  has exhausted its energy quota. Therefore, relay  $v_2$  is updated by `addr{v2 -rt}` (line 14) since the data rate from  $v$  to  $v_2$  drops from  $r_t$  to 0. Then, the next relay node for  $v$  is retrieved from the relay list, and a new `switch` event is set up according to the energy quota for the new relay and the current outgoing data rate of  $v$ . For the new relay  $v'_2$  and its relays, `addr` is called recursively again (line 20) since the data rate relayed from  $v$  to  $v'_2$  jumps from 0 to  $r_t$ .

Code from line 22 to 28 calculates the energy quota (EQ) for each relay of a given node  $v$ , according to the output of the can-

**Table 8: Algorithm to calculate relay schedule**

```

1  proc addr {v dr}
2    if v == b
3      return
4    cancel switch v
5    set v2 [lindex EQ_v 0 1]
6    set e2 [lindex EQ_v 0 2]
7    update e2 in EQ_v
8    set rt [expr rt+dr]
9    at now+ $\frac{e_2}{p(r_t,d_v,v_2)}$  switch v
10   addr v2 dr
11 endproc

12 proc switch {v}
13   set v2 [lindex EQ_v 0 1]
14   addr v2 -rt
15   set EQ_v [lrange EQ_v 1 end]
16   set v2 [lindex EQ_v 0 1]
17   set e2 [lindex EQ_v 0 2]
18   at now+ $\frac{e_2}{p(r_t,d_v,v_2)}$  switch v
19   lappend RS {now v v2}
20   addr v2 rt
21 endproc

22 foreach v VN
23   set ps 0
24   foreach {v t rr} RR_v
25     set rs [expr ps+pt(rr,dt)]
26     set eq  $\frac{e-\epsilon}{ps}$ 
27     foreach {v t rr} RR_v
28       lappend EQ_v {v t eq:pt}
29     set EQ_v [lsort -ran 1 EQ_v]
30     set EQ_v [concat { } EQ_v]
31 foreach v VN
32   switch v

```

didate preselection in Section 4.1 and the rate allocation in Section 4.2. When applicable, line 29 randomizes relays in the list, so that it is less likely that multiple ANs choose the same AN as their relay. Line 30 intentionally prefixes a *dummy* relay at the beginning of the EQ list, so that we can execute a pseudo `switch{v}` at the network initialization and switch from the *dummy* relay to a *real* relay in  $V_N$ . Assume that `add{v r}` have the complexity  $O(1)$ , then each `switch{v}` has the complexity  $O(N)$  since a relay path at most has  $N-1$  intermediate relays to  $b$ . Therefore, the total time complexity to obtain the relay schedule is  $O(N|RR|)$ . This schedule actually can be calculated in a distributed manner at each AN, if  $b$  dispatches the energy quota to ANs, unless the schedule needs to be coordinated with the mission schedule at  $b$ .

Fig. 13(a) plots the resultant serialized relay schedule. The numbered cross denotes when an AN chooses another node as its new relay, and the unnumbered cross denotes when the AN serves as a relay for other ANs and the incoming data rate changes. For example, at the network initialization,  $v_{10}$  chooses  $b$  as its relay for 0.446 unit time. When its remaining energy drops to 0.918 unit,  $v_{10}$  has used up the energy quota for  $b$  and switches to the next relay  $v_6$ . At 4.616 unit time,  $v_{10}$  has used up the quota for  $v_6$ , and switches to  $v_7$  until at 8.420 unit time the network fails to carry on its mission due to multiple ANs (including  $v_{10}$ ) running out of energy.

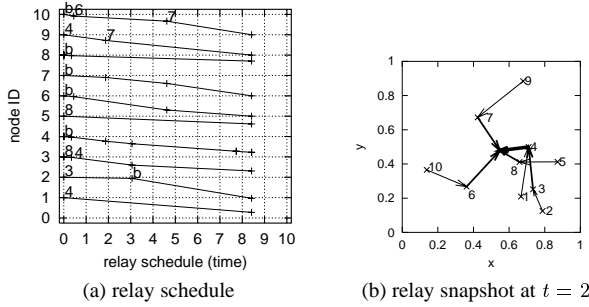


Figure 13: Serialized relay schedule

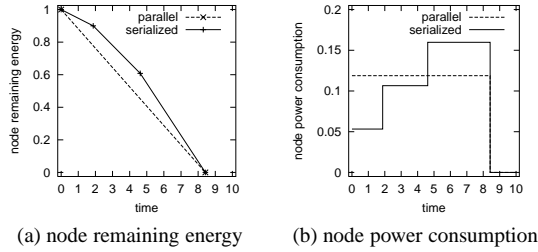


Figure 14: Equivalency between parallel and serial relay ( $v_7$ )

Although AN  $v_7$  has not changed its relay (the BS) throughout the network lifetime, its power consumption also changes due to different ANs using it as their relay. During  $[0, 1.873]$  unit time, no other ANs use  $v_7$  as a relay;  $v_7$  has the least power consumption for an outgoing bit-stream at 1 unit rate. Then at 1.873 unit time,  $v_9$  starts to use  $v_7$  as its relay. Therefore,  $v_7$  begins to have a higher power consumption (or a larger drop slope of its remaining energy) with a 2-unit outgoing flow. After 4.616 unit time, both  $v_9$  and  $v_{10}$  use  $v_7$  as their relay.  $v_7$  now has the highest power consumption in its lifetime for a 3-unit outgoing flow. At 8.420 unit time,  $v_7$  exhausts its energy, and at the same point the network fails to carry on its mission. Fig. 13(b) gives a snapshot of the relay schedule at  $t = 2.0$  unit time. The arrow shows the relay direction; the line width implies the data rate. For the serialized relaying, at any time,  $V_N$  always forms a tree rooted at  $b$ .

A certain amount of energy allocated for  $v_k$  at  $v$  represents the amount of data transferred from  $v$  to  $v_k$ . Since the total energy and energy allocation for each relay are identical in either parallel or serialized relaying, the amount of data transferred should also be the same. A formal proof of this equivalency appears in [8]. Therefore, an AN has the same lifetime with parallel or serialized relaying, as shown in Fig. 14(a) for AN  $v_7$ . With parallel relaying, the remaining energy at  $v_7$  decreases at a constant rate throughout its node lifetime. With serialized relaying, the remaining energy at  $v_7$  decreases at different rates according to its current power consumption shown in Fig. 14(b). Although the remaining energy curves for parallel and serialized relaying are different most of the time, they meet again at the network lifetime  $R_N$ .

## 5. FURTHER DISCUSSIONS

We have presented the approaches to obtain the optimal BS location, and the optimal relay arrangement if inter-AN communication is feasible and favorable, to maximize the topological lifetime of a WSN with the given initial energy provisioning. In this section, we

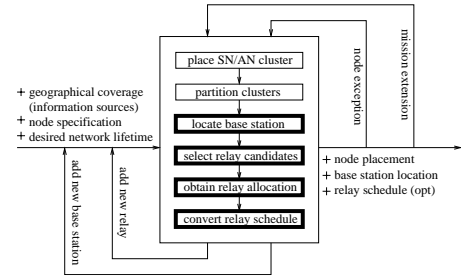


Figure 15: Topology control iterations for WSN

further discuss the applicability and extensibility of these proposed approaches in a more practical context.

### 5.1 Topology Control Process

Fig. 15 illustrates the relationship among these approaches and their positions in the whole topology control process. Given a geographical coverage  $\mathcal{C}$  and the information sources  $\mathcal{S}$  to be monitored, the first step is to collocate SN/AN clusters  $\mathcal{V}$  with  $\mathcal{S}$ , which gives a proper coverage [31]. With the incremental cluster grouping techniques, some SN/AN clusters are then grouped into a WSN partition  $V_N$  which is served by one BS  $b$ .

After that, the approaches proposed in this paper become applicable, as shown in the highlighted blocks in Fig. 15. First, we can quickly assess whether the current energy provisioning is feasible to achieve the desired network lifetime  $\mathcal{T}$ . If  $\mathcal{T}$  is below the lower bound of  $L_N$ , *i.e.*,  $\mathcal{T} < \mathcal{L}_N^-$ , the BS can be placed at the middle point of  $V_N$ 's diameter directly. If  $\mathcal{T}$  is above the upper bound of  $L_N$ , *i.e.*,  $\mathcal{T} > \mathcal{L}_N^+$ , topology control either needs to introduce more BSs (*i.e.*, with a smaller  $D$ ), or has to negotiate with the application for  $L_N^K$  or  $m L_N^K$ . With the given lifetime definition, the optimal  $b$  is obtained through the algorithms proposed in Sections 3.1, 3.2, and 3.3, respectively. If  $L \geq \mathcal{T}$  for every BS  $b$ , the topology control process finishes with the optimal BS locations.

If  $L < \mathcal{T}$ , topology control can either adjust the SN/AN cluster partition (*i.e.*, changing  $D$ ), or request more BSs. If inter-AN relaying is application desirable, energy favorable, and most importantly, operationally feasible, topology control can invoke the approaches in Section 4.1 to preselect relay candidates. With the LP approach in Section 4.2, network lifetime can be prolonged to  $R$ . If  $R \geq \mathcal{T}$ , this relay allocation is acceptable and will be converted into a serialized relay schedule, according to the approach designed in Section 4.3. Then topology control exits with both the optimal BS location and optimal relay schedule.

However, if  $R < \mathcal{T}$ , the topology control has to rely on its last two resorts: more BSs or dedicated relay nodes. Although we did not address the node placement and partition problem in this paper, the relay candidate selection criteria in Section 4.1 can assist in the process of deciding where to place the additional dedicated relay nodes. It turns out that topology control actually is an interactive process with multiple iterations. During the course of network operation, nodes may fail and be substituted by other nodes, and the mission may get extended. These changes require a revisit of some blocks in the topology control process depicted in Fig. 15.

### 5.2 Practical Considerations

In the previous discussions, we focused on the distance-related portion of power consumption and its role in topology control. In a practical WSN, other non-distance-related power consumptions may become non-negligible, *e.g.*, the energy consumed within the transmitter or receiver circuit, as well as in the data processing

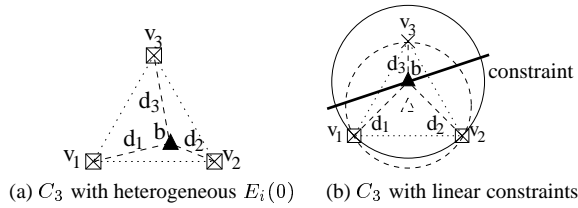


Figure 16: Extensibility on approaches for optimal  $b$

and view composition components. Node homogeneity may not always be guaranteed, especially when we consider the WSN re-deployment scenarios (*i.e.*, new nodes join the network long after old nodes are initialized and activated). Also, transmission power consumption may take a path loss exponent greater than 2 and include other portions to combat multi-path, shadowing, interference and other effects. A third geometry dimension might be introduced when node elevation varies considerably.

The approaches proposed in this paper are extensible to accommodate these challenges. For example, we can still find the optimal BS location determined by a few *critical* nodes that run out of energy first. When we have  $b_{N-1}$  for  $N-1$  nodes, for the  $N$ -th node, if its lifetime with regard to  $b_{N-1}$  is no less than the shortest one in those  $N-1$  nodes, the  $N$ -th node is non-critical and  $b_{N-1}$  is also optimal for the  $N$ -th node. Otherwise, this node is critical, and the recursive algorithm outlined in Table 3 still applies. Instead of  $d$  for a minimal enclosing circle, the actual node lifetime is considered to determine whether a node can be *logically* enclosed. With node heterogeneity, we no longer have a circle to illustrate the process, as shown in Fig. 16(a) where  $E_1(0) : E_2(0) : E_3(0) = 2 : 1 : 3$ , and  $\frac{E_i(0)}{p_i}$  is a constant for all critical nodes in  $V_C = V_3$ . Fig. 16(b) illustrates the case of optimal  $b$  with a linear constraint, *i.e.*,  $b$  should be located along a given line (*e.g.*, an avenue across buildings). Here,  $v_3$  is no longer critical. The dashed triangle denotes the optimal BS location without the linear location constraint.

For the relay selection, instead of the Euclid distance used in criteria  $\{c_1, c_2, c_3\}$  in Section 4.1, we replace it with the node lifetime criterion: how *expensive* it is for a node to use a relay. For example, a node should not choose the node that is more expensive than  $b$  as its relay. Within this schema, the LP formulation is similar, and we still can obtain the optimal relay allocation. The serialization process is based on actual energy allocation, so it will not be challenged by heterogeneity in practice. ANs count the energy quota for the current relay, and switch to another relay when the energy quota for the current one has been exhausted, while non-transmission-related energy consumption can be set aside earlier.

The proposed approaches on BS location and inter-AN relaying arrangement, along with other blocks in topology control such as node placement and partition, give us the capability to maximize network lifetime *topologically*. Although we assumed that topology control is done before the network initialization, further improvement can be introduced by adaptively updating topology control throughout the whole mission. For example, the BS may have certain mobility, and may change its location when some ANs are dead or about to run out of energy. In this paper, we adopt a 2-stage approach: first, determine the BS location; then, find the inter-AN relaying arrangement. Another attempt can allow the BS to change its location while rearranging inter-AN communication, to achieve an even longer network lifetime. At each step, the approaches to obtain the optimal relay arrangement still apply.

## 6. RELATED WORK

Mobile ad hoc networks (MANETs) and WSNs have attracted extensive research interests in recent years. A comprehensive survey on WSNs can be found in [20] and the references therein. The research challenges and directions for MANETs can be found in [5]. Although sharing many similarities with MANET/WSNs, a two-tiered WSN has its unique underlying structure and application scenarios such as mission-driven AN/BS placement. Compared to MANET/WSNs, and despite their potentially wide applications, two-tiered WSNs have not yet been heavily explored in the literature. In the following section, we take both MANETs and regular WSNs, as the reference to review related work.

In the infrastructure-less MANETs, user mobility, network connectivity and node reachability are major concerns. From the perspective of network protocols, source or intermediate nodes need to know how to route packets toward their destinations. Many unicast and multicast routing protocols have been developed and evaluated [10, 11, 12, 14]. Multi-hop wireless communication in MANETs requests reconsiderations of designs in radio transceiver, media access [35, 38, 36], and error and flow controls in link [44] or higher layers [19]. Many existing wireless MAC protocols, *e.g.*, IEEE 802.11a/b/g and 802.15.1, were originally designed for wireless LANs/PANs; their applicability and achievable performance in MANETs and WSNs have been questioned and investigated [36]. Since user mobility plays an important role in MANETs, mobility management [9], location service [15], and their impact on routing and MAC protocols [13] have been studied as well.

WSNs can have a hybrid wireless multi-hop and BS-centric infrastructure; they are more challenged by the limited on-board energy for disposable sensor and application nodes. With regard to multi-hop, they share many similarities with MANETs in routing [21] and media access protocols [37], along with additional energy constraints [18]. Unlike the user autonomy and flow independence properties of MANETs, information collected in WSNs has to be aggregated somewhere to magnify its value. Therefore, the information flows from the data *sources*, such as sensor or surveillance nodes, are more structured (*i.e.*, in many-to-one) toward *sinks*, such as application nodes, cluster heads, or BSs, although the underlying network paths may or may not be fixed. In addition, information collected by redundant and substitutable sources in dense WSNs have considerable redundancy and inconsistency, so data fusion [32, 34] along the path or at the sink is very attractive. Application-specific fusion enables more sophisticated data [26] and node management functionalities inside WSNs, to reduce the unnecessary communication overhead. Due to the flow concentration, scalability can become an obstacle [6] for a large number of sensors, and techniques to introduce heterogeneity [46], hierarchy [45], clustering [16, 26], localization [41], and location-awareness [39] are developed. These application characteristics have added new dimensions to explore in MAC and multi-hop routing, and there are many results in the recent literature.

Although a two-tiered WSN has many aspects in common with MANETs and regular WSNs, its tiered structure still brings in some unique characteristics. For example, most research activities in WSNs assume a dense and microsensor deployment. Microsensors have very limited energy provisioning to capture scalar-only data such as temperature and motion triggered by external events. But for a two-tiered WSN, ANs are much more capable than the ordinary microsensors (SNs), as they are required to construct and feed live local-views to BSs when they are activated. With the considerable coverage of a single SN/AN cluster, there is no need to have a very dense deployment of SN/AN clusters (generally, SN/AN clusters are placed with the proximity of designated scenes). Due to

this sparse deployment, the inter-AN distance is comparable with the dimension of coverage, and scalability is manageable even with a few BSs and a certain number of homogeneous ANs. Based on these facts, the lifetime of an AN is dominated by its distance-related communication power consumption. Therefore, topology control that determines the distance from ANs to BSs and chooses relays according to the inter-AN distance, plays a vital role in maximizing the network lifetime of WSNs.

There are a few lifetime and topology-focused research activities in the literature. Bhardwaj *et al.* [22] derived the lifetime upper bound of information harvest sensor networks that convey probabilistic data from a point, a line, or an area source. They also gave simulation-based evaluations to validate the tightness of the derived bound. In [23], they further explicitly formulated the optimal role assignment as the maximal network flow problem, again in data harvest networks for which the BS is presumably being located already. In this paper, our first goal is to determine the optimal BS location to maximize topological lifetime even when relaying is infeasible. Three definitions of topological lifetime are proposed according to the criticality of a mission. Instead of harvesting from probabilistic information sources, when being activated, WSNs should consistently offer an in-situ, real-time and steady global-view of the whole network.

In [21], Chang *et al.* proposed a family of flow augmentation algorithms, which redirect data flows among nearby nodes to balance their energy consumption in a distributed but empirical manner. Our approach is a centralized one due to the application nature of WSNs. After the BS is located, if inter-AN relaying is feasible, we first select relay candidates and then obtain the optimal parallel relay allocation. In contrast to previous work in this area, we further convert the relay allocation into a serialized relay schedule with the equivalent optimality, and allow ANs to choose their relays locally according to *energy quota*. Therefore, an AN only needs to have one relay destination at any time. In WSNs, BSs can have certain mobility (*e.g.*, mounted on vehicles), and have sophisticated processing and storage capabilities to accommodate centralized topology control functionalities.

Other topology-related research mainly focused on multi-hop routings in WSNs. For example, [30] considered fixed topologies of {4, 6, 8}-neighbor in a 2-dimension plane and 6-neighbor in a 3-dimension space, and proposed a power-aware routing scheme to reduce the total, and even the per-node, power consumption. In this paper, we consider an arbitrary node placement on a plane, without any geometrical constraints on the node neighborhood. In practice, the location of SN/AN clusters is determined by specific missions, not by topology control. [29] considered adjusting the transmitter output power to create a desired topology for connectivity and bi-connectivity. [29] also observed that a poor topology can only offer a small portion of the achievable lifetime, but they focused on multi-hop networks without any common sinks like those in WSNs. [28] proposed a sparse topology and energy management (STEM) technique that aggressively puts nodes in sleep mode and only wakes them up when they are needed to forward data. It also explored the equivalency of nearby nodes for data forwarding. However, in WSNs, due to their application characteristics, once being activated, the already-sparsely-deployed ANs usually cannot be forced into sleep. Otherwise, the designated local-views are lost. [27] proposed a distributed cone-based topological control to maintain the global connectivity with minimum power paths in multi-hop ad hoc networks. [25] considered a distributed algorithm to determine whether a node should be awake or asleep, depending on how many of its neighbors will get benefit and how much remaining energy it has. The focus in these work, *i.e.*, the

purpose of topology control, is different from the one that we have in this paper. Instead of minimizing the power consumption for individual nodes or along a forwarding path, we want to minimize the power consumption of those ANs that dominate the lifetime, or utility, of the whole WSN. Overall, the WSNs under our consideration are BS-centric with the optional multi-hop relaying, where SN/AN clusters with a certain amount of initial energy are sparsely deployed in designated areas without significant redundancy.

## 7. CONCLUSIONS

In this paper, we have analytically obtained the optimal BS location for a two-tiered WSN to maximize its topological lifetime under several definitions according to mission criticality. When inter-node communication is application desirable, energy favorable, and indeed feasible, we have further obtained the optimal relay allocation to prolong topological lifetime. In addition, we theoretically derived the upper and lower bounds of maximal topological lifetime by using some intrinsic properties of WSNs. We also converted the parallel relay allocation into a serialized relay schedule so that any node only needs to have one relay destination at any time. Experimental evaluation has demonstrated the efficacy of topology control as a vital process for WSNs, and it also validates the optimality of proposed approaches.

For future work, the main focus will be on the other two blocks in the topology control diagram shown in Fig. 15: node placement and partition techniques, and their impact on BS location and inter-node communication arrangements. We will also consider others scenarios, such as dynamic deployment and redeployment, as well as hierarchical and heterogeneous WSNs.

## Acknowledgment

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