

Indirect Load Shaping for CHP Systems Through Real-Time Price Signals

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Abstract—Direct or centralized loading shaping in smart grid has been heavily investigated. However, it is usually not clear how the users are compensated by providing load shaping services. In this paper, we will discuss indirect load shaping in a distributed manner. On one hand, we aim to reduce the users' energy cost by investigating how to fully utilize the battery pack and the water tank for the combined heat and power (CHP) systems. We first formulate the queueing models for the CHP systems and then propose an algorithm based on the Lyapunov optimization technique, which does not need any statistical information about the system dynamics. The optimal control actions can be obtained by solving a nonconvex optimization problem. We then discuss when it can be converted into a convex optimization problem. Since the CHP battery pack queue and water tank queue are correlated, the capacity relationship between them is further explored considering different queue weights. On the other hand, based on the users' reaction model, we propose an algorithm with a time complexity of $O(\log n)$ to determine the real-time price for the power company to effectively coordinate all the CHP systems and provide distributed load shaping services.

Index Terms—Combined heat and power (CHP), Lyapunov optimization, smart grid.

I. INTRODUCTION

EXTENSIVE research has been done aiming to reduce the users' electricity bill by taking the advantage of the real-time price (RTP) and the elasticity of certain appliances. However, it has been argued that without an appropriate RTP to coordinate all the elastic loads, these algorithms may lead to new peaks which are undesirable [1]. In order to solve the problem, one approach is to control the elastic load directly by a central controller. For example, in [2], [3], and [21], the heating, ventilating, and air conditioning (HVAC)s can provide load shaping services if the ON/OFF states of each HVAC can be controlled by a control center directly. Others discussed how to determine the RTP to provide indirect load shaping mainly from a game theory perspective. In these papers, the authors usually assumed that the users make decisions according to a certain utility function. However, how to design appropriate utility functions is still an open problem.

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In this paper, we are motivated to design an indirect load shaping service framework through RTP, which can help both the users and the power companies save cost. Our framework can be divided into two parts. On one hand, we design a combined heat and power (CHP) system scheduling algorithm which reacts to the current RTP and help users save their total energy bill. The reason why we choose CHP systems is that CHP systems can generate both electricity and thermal energy simultaneously from a single fuel source, and can achieve a much higher energy efficiency than generating electricity and heat separately [4], [5]. The use of CHP systems can also reduce greenhouse gas emissions. As a result, CHP systems are becoming increasingly popular.

On the other hand, the value of the RTP will affect the charging and discharging of the battery packs of the CHP systems, and thus influence the total load. In other words, the power company can make the CHP systems provide load shaping services by adjusting the RTP. The key issue is to find an appropriate RTP based on the derived CHP system scheduling algorithm such that all the CHP systems can be effectively coordinated.

The contributions of this paper are threefold. First, we propose a comprehensive model from the perspective of a commercial customer, which incorporates both the electricity and thermal energy queues. We investigate the relationship of these two queues to minimize the average cost. Second, we propose an algorithm to approximately achieve the optimal average cost, considering the limited capacities of the battery pack and the water tank. The algorithm does not require any statistical information of the system dynamics such as electricity, hot water demands, etc. To obtain the optimal scheduling decision, we discuss when we can use the specific features of the problem to turn a nonconvex optimization problem into a convex one which can be solved in real time. Third, we discuss how to set the appropriate RTP to coordinate all the CHP systems indirectly to provide load shaping services. The time complexity of the proposed searching algorithm is $O(\log n)$.

The rest of this paper is organized as follows. Section II discusses the existing CHP economic dispatch (CHPED) problems, the application of Lyapunov optimization in smart grid, and the state-of-the-art approaches to determine the RTP. A general description of the CHP system architecture is given in Section III. Then, we discuss the design details of the proposed algorithm in Section IV. In Section V, we discuss how to determine the optimal RTP to coordinate all the CHP systems.

TABLE I
NOTATIONS

Symbol	Description
$B(t)$	the SOC level of the battery
C_{char}	the maximum charge rate of the battery
$C_e(t)$	electricity price at time t
$D(t)$	the amount of electricity discharged from the battery
$G_t(t)$	the amount of electricity bought to supply the user demand
$G_s(t)$	the amount of electricity bought to charge the battery
$L_e(t)$	the electricity demands from users
$L_t(t)$	the target load of load shaping service
$L_w(t)$	the hot water demands from users
$P_a(t)$	the amount of the natural gas consumed by the boiler
$P_c(t)$	the amount of natural gas consumed by the CHP
$r(t)$	a tradeoff between the amount of electricity used to charge the battery and that sold to the grid
Δt	one-slot Lyapunov drift
T	the number of time slots
$W(t)$	the water level in the water tank in slot t
η_{ce}	the conversion efficiency from natural gas to electricity charged to the battery
η_{cg}	the conversion efficiency from natural gas to hot water
η_{co}	the electricity conversion efficiency sold back to the grid
η_{ag}	the conversion efficiency from natural gas to hot water
η_s	the battery charging efficiency
ω	the weight between the battery queue and the water tank queue

Performance evaluation is given in Section VI, followed by the conclusion in Section VII.

For easy reference, the symbols used in this paper are summarized in Table I.

II. RELATED WORK

To provide both electricity and heat economically, the design and operation strategies of CHP systems have been well investigated. Hawkes and Leach [6] discussed operating strategies, such as heat and electricity load following, for three micro-CHP technologies. Nanaeda *et al.* [7] evaluated four typical operation modes in a hotel based on measured electric and heating loads. Dentice *et al.* [8] analyzed the utilization of micro-CHP systems in conjunction with domestic household appliances. Lokurlu *et al.* [9] analyzed the cost for different fuel-cell systems. These works tried to find the most cost-effective strategies from a system view, and do not consider the detailed control policies.

The CHPED problem, first raised in [10], aimed to find the optimal operation point of CHP with minimum energy cost such that both electricity and heat demands were met. However, in the CHPED problems, optimization was performed to minimize the cost in each time slot. No energy buffer was used to minimize the long-term cost. In addition, it did not consider the stochastic nature of energy demand. Recently, Tasdighi *et al.* [4] formulated an mixed-integer linear programming problem to optimize the operation of micro-CHP-based microgrids. Different from this paper, they used a time-of-use electricity price model and the energy requirement of future elastic load was assumed to be available through prediction.

There are also several works which use the Lyapunov optimization technique to construct low complexity energy storage management policies. Neely *et al.* [11] minimized

the time average cost from the perspective of one user, and guaranteed the worst-case delay for each elastic load. Urgaonkar *et al.* [12] used uninterruptible power systems in the data center to reduce the electricity bill in a RTP environment. Their model did not consider renewable energy sources. Guo *et al.* [13] investigated how to use a household battery to minimize the average electricity cost, considering both inelastic and elastic load. Instead of guaranteeing the worst-case delay, Huang *et al.* [14] guaranteed that the percentage of the delayed elastic load was less than a threshold. These works discussed above only considered one energy buffer, however, the system model discussed in this paper includes two energy buffers, the battery pack and the water tank, which are correlated by the CHP system. With two dependent queues, the system model is more complicated and we need to solve a nonconvex optimization problem to obtain the optimal control policy. In addition, we illustrate the relationship between the capacity of these two energy buffers and the minimum required capacity to achieve the optimal performance. This paper focuses on the problems closely related to the unique features of the CHP systems. Some well-studied applications of Lyapunov optimization in smart grid, such as elastic load queue, worst-case delay, etc., are not discussed here due to the space limit.

How to determine an appropriate RTP to coordinate all the “selfish” users is also a challenging task. Wu *et al.* [15] proposed a pricing scheme to stimulate a large group of electrical vehicle users to provide frequency regulation based on game theory. Gao *et al.* [16] extended their work by considering different users’ preferences under the presence of information asymmetry using contract theory. One problem in the above work is that the users’ preferences may keep changing over time, and are highly related to the state of charge (SoC) of the electrical vehicles’ battery. Therefore, the performance gain of the above user preference learning algorithm may be limited. In order to obtain a better user reaction model to price signals, Chen *et al.* [17] proposed an iterative method based on a leader and follower level game theory that needs frequent information exchange, which may lead to a high communication overhead. In this paper, different from the game theory approach, we propose a fast algorithm to determine the optimal RTP which can effectively coordinate all the CHP system for load shaping services. Specifically, the proposed algorithm is based on the CHP operation model obtained in the first part of this paper, which aims to minimize users’ long-term average cost, meanwhile meeting the users’ variable demands in each time slot. In our system model, although these CHP systems may belong to different owners, they make their operation decisions independently according to the current real-time electricity price. In other words, they do not compete or collaborate with each other.

III. CHP SYSTEM MODEL

In this section, we summarize the mathematical models of the CHP system in [18]. The major difference between the two CHP system models is that here we consider the battery pack queue and the water tank queue are of different weight.

of the heat needed to reheat the water tank as the additional heat demand in the form of hot water from the users.

The amount of hot water stored in the water tank, which is the queue length of the water tank, evolves according to the following equation:

$$W(t+1) = W(t) - L_w(t) + \eta_{cg}P_c(t) + \eta_{ag}P_a(t) \quad (7)$$

where $W(t)$ is the water level in the water tank in slot t .

Since the amount of the water stored in the water tank should always be bounded by the size of the water tank, we have $0 \leq W(t) \leq W_{\max}$, where W_{\max} is the capacity of the water tank. In addition, since we assume that the hot water demand in each time slot will not exceed $L_{w,\max}$, to ensure that users' demand can always be met even in the worst-case situation, i.e., the hot water demand is always $L_{w,\max}$, we assume the following constraint:

$$L_{w,\max} \leq \eta_{ag}P_{a,\max} \quad (8)$$

where $P_{a,\max}$ is the maximum amount of the natural gas used by the boiler in each time slot, and $L_{w,\max}$ is the upper bound of the hot water demand in each time slot.

D. Control Objective

In each time slot, the total energy cost for the CHP system is the sum of the electricity and natural gas cost minus the amount of the electricity sold to the grid

$$f(t) = C_e(t)\{G_l(t) + G_s(t) - (1 - r(t))\eta_{co}P_c\} + C_g\{P_c + P_a\} \quad (9)$$

where C_g is the natural gas price.

The control objective is to find a control policy determining the amount of the electricity and natural gas dispatched in each time slot, so as to minimize the long-term average energy cost

$$f_{\text{avg}} = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{i=0}^{T-1} \mathbb{E}\{f(i)\}. \quad (10)$$

IV. CHP SYSTEM SCHEDULING ALGORITHM

In this section, we assume that the electricity and hot water demands in each time slot $L_e(t)$ and $L_w(t)$ are independent. The proposed algorithm in this section will solve the following problems. First, given the current states of the CHP system, including the electricity and hot water demand, battery and water tank storage level, electricity price in the current time slot, etc., how to obtain the optimal control decisions with a low computational complexity and can adapt to the stochastic system dynamics? Second, what is the minimum capacity of the battery pack and water tank we should have to achieve a given performance requirement? Third, since the CHP can generate both electricity and heat, the battery pack and water tank queues specified in (1) and (7) are dependent. What is the relationship between the capacity of the battery pack and that of the water tank?

According to the system architecture and control objective described in Section III, the problem can be formulated as the following stochastic network optimization problem.

Problem 1:

$$\min_{D(t), r(t), G_l(t), G_s(t), P_c(t), P_a(t)} P_1 = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{f(t)\} \quad (11)$$

subject to

$$B(t+1) = B(t) - D(t) + \eta_s G_s(t) + r(t)\eta_{ce}P_c(t) \quad (12)$$

$$W(t+1) = W(t) - L_w(t) + \eta_{cg}P_c(t) + \eta_{ag}P_a(t) \quad (13)$$

$$0 \leq B(t) \leq B_{\max} \quad (14)$$

$$0 \leq W(t) \leq W_{\max} \quad (15)$$

$$L_e(t) = G_l(t) + D(t) \quad (16)$$

$$0 \leq \eta_s G_s(t) + r(t)\eta_{ce}P_c(t) \leq C_{\text{char}} \quad (17)$$

$$0 \leq r(t) \leq 1, P_c(t), G_s(t) \geq 0 \quad (18)$$

$$0 \leq D(t) \leq D_{\max}. \quad (19)$$

The above problem cannot fit into the traditional stochastic network optimization framework directly mainly because of the battery and water tank capacity constraints (14) and (15). Specifically, stochastic network optimization can only guarantee that the average energy generation equals the average consumption in the long term, but cannot provide a hard bound on the difference between the generation and consumption in any time slot. To solve this problem, we take the expectation on the two sides of (12) and (13), which leads to the following relaxed problem.

Problem 2:

$$\min_{D(t), r(t), G_l(t), G_s(t), P_c(t), P_a(t)} P_1 = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{f(t)\} \quad (20)$$

$$\text{subject to } \overline{D(t)} = \eta_s \overline{G_s(t)} + \eta_{ce} \overline{r(t)P_c(t)} \quad (21)$$

$$\overline{L_w(t)} = \eta_{cg} \overline{P_c(t)} + \eta_{ag} \overline{P_a(t)} \quad (22)$$

and (16)–(19).

Problem 2 fits the stochastic network optimization framework, so we can solve it using existing algorithms [19]. Obviously, only if the solutions to Problem 2 can meet the constraints (14) and (15) for $\forall t \in T$, they are also feasible to Problem 1. To reach this objective, we define two constants θ and ε . The intuition is that by adjusting these two constants appropriately, we can make the solutions to Problem 2 also be feasible to Problem 1.

To start, we define two queues $E(t)$ and $X(t)$

$$E(t) = B(t) - \theta \quad (23)$$

$$X(t) = W(t) - \varepsilon. \quad (24)$$

The constants θ and ε are two queue offsets, which are used to guarantee that the two queues $B(t)$ and $W(t)$ are bounded.

From (12) and (13), we can obtain the queueing dynamics

$$E(t+1) = E(t) - D(t) + \eta_s G_s(t) + r(t)\eta_{ce}P_c(t) \quad (25)$$

$$X(t+1) = X(t) - L_w(t) + \eta_{cg}P_c(t) + \eta_{ag}P_a(t). \quad (26)$$

We then define the Lyapunov function $Q(t) = (1/2)E(t)^2 + (1/2)w^2X(t)^2$, where w represents the weight between these two queues. The conditional one-slot Lyapunov drift is

$$\Delta(t) = \mathbb{E}\{Q(t+1) - Q(t)|E(t), X(t)\}. \quad (27)$$

According to (25) and (26), by squaring both sides, we have

$$\begin{aligned} \Delta(t) &\leq 0.5 \max \left[\left(\eta_s G_{s,\max} + \eta_{ce} P_{c,\max} \right)^2, D_{\max}^2 \right] \\ &\quad - E(t) [D(t) - \eta_s G_s(t) - r(t) \eta_{ce} P_c(t)] \\ &\quad + 0.5 w^2 \max \left[\left(\eta_{cg} P_{c,\max} + \eta_{ag} P_{a,\max} \right)^2, L_{w,\max}^2 \right] \\ &\quad - w^2 X(t) [L_w(t) - \eta_{cg} P_c(t) - \eta_{ag} P_a(t)] \\ &= B - E(t) [D(t) - \eta_s G_s(t) - r(t) \eta_{ce} P_c(t)] \\ &\quad - w^2 X(t) [L_w(t) - \eta_{cg} P_c(t) - \eta_{ag} P_a(t)] \end{aligned}$$

where $P_{c,\max}$ is the maximum amount of the natural gas that can be used by the CHP in each time slot, and B is a constant and defined as

$$\begin{aligned} B &= 0.5 \max \left[\left(\eta_s G_{s,\max} + \eta_{ce} P_{c,\max} \right)^2, D_{\max}^2 \right] \\ &\quad + 0.5 w^2 \max \left[\left(\eta_{cg} P_{c,\max} + \eta_{ag} P_{a,\max} \right)^2, L_{w,\max}^2 \right]. \end{aligned} \quad (28)$$

According to the stochastic network optimization framework, in order to make the two queues $E(t)$ and $X(t)$ mean rate stable, we must minimize the drift $\Delta(t)$. In addition, our control objective is to minimize the average cost. So, we use a constant V to represent the tradeoff between these two objectives. Then the drift plus penalty function can be written as follows:

$$\begin{aligned} \Delta(t) + V \mathbb{E}\{f(t)\} &\leq B - E(t) \mathbb{E}\{D(t) - \eta_s G_s(t) - r(t) \eta_{ce} P_c(t) | E(t)\} \\ &\quad - w^2 X(t) \mathbb{E}\{L_w(t) - \eta_{cg} P_c(t) - \eta_{ag} P_a(t) | X(t)\} \\ &\quad + V \mathbb{E}\{C_e(t) [G_l(t) + G_s(t) - (1 - r(t)) \eta_{co} P_c] \\ &\quad \quad + C_g [P_c + P_a]\}. \end{aligned} \quad (29)$$

We then substitute $G_l(t)$ in (29) according to (16), and after some manipulation we can obtain

$$\begin{aligned} \Delta(t) + V \mathbb{E}\{f(t)\} &\leq B + V \mathbb{E}\{C_e(t) L_e(t) | E(t)\} - \mathbb{E}\{L_w(t) w^2 X(t) | X(t)\} \\ &\quad - \mathbb{E}\{D(t) [E(t) + VC_e(t)] | E(t)\} \\ &\quad + \mathbb{E}\{G_s(t) [\eta_s E(t) + VC_e(t)] | E(t)\} \\ &\quad + \mathbb{E}\{P_c(t) [r(t) \eta_{ce} E(t) + \eta_{cg} w^2 X(t) \\ &\quad \quad - (1 - r(t)) \eta_{co} VC_e(t) + C_g V] | E(t), X(t)\} \\ &\quad + \mathbb{E}\{P_a(t) [\eta_{ag} w^2 X(t) + VC_g] | X(t)\}. \end{aligned} \quad (30)$$

Based on the ‘‘min-drift’’ principle of the Lyapunov optimization approach, the main idea of the proposed algorithm is to minimize the right-hand side of (30) over all the feasible control policies in each time slot. In other words, at the beginning of each time slot, we observe the system states $B(t)$, $W(t)$, $L_e(t)$, $L_w(t)$, and $C_e(t)$, determine the value of $B + V \mathbb{E}\{C_e(t) L_e(t) | E(t)\} - \mathbb{E}\{L_w(t) w^2 X(t) | X(t)\}$, and then solve the following problem.

Problem 3:

$$\min G_s(t) H_s(t) + P_c(t) H_c(r(t)) + P_a(t) H_a(t) - D(t) H_d(t) \quad (31)$$

$$\text{subject to } 0 \leq \eta_s G_s(t) + r(t) \eta_{ce} P_c(t) \leq C_{\text{char}} \quad (32)$$

$$0 \leq r(t) \leq 1, P_c(t), G_s(t) \geq 0 \quad (33)$$

$$0 \leq D(t) \leq D_{\max} \quad (34)$$

where

$$\begin{aligned} H_s(t) &= \eta_s E(t) + VC_e(t) & H_a(t) &= \eta_{ag} w^2 X(t) + VC_g \\ H_d(t) &= E(t) + VC_e(t) & H_c(r(t)) &= H_r(t) r(t) + H_b(t) \\ H_r(t) &= \eta_{ce} E(t) + \eta_{co} VC_e(t) \\ H_b(t) &= \eta_{cg} w^2 X(t) - \eta_{co} VC_e(t) + C_g V. \end{aligned} \quad (35)$$

Note that (31) contains the product of $P_c(t)$ and functions of $r(t)$, so Problem 3 is a nonconvex optimization problem because its Hessian matrix is not always positive definite. When looking into the structure of Problem 3, we can find that $D(t)$ and $P_a(t)$ can be easily obtained according to the value of $H_d(t)$ and $H_a(t)$. If $H_d(t) \geq 0$, then $D(t) = D_{\max}$; otherwise $D(t) = 0$. If $H_a(t) \leq 0$, then $P_a(t) = P_{a,\max}$; otherwise $P_a(t) = 0$. Therefore, we only have to solve the following subproblem:

$$\begin{aligned} \min G_s(t) H_s(t) + P_c(t) [H_r(t) r(t) + H_b(t)] \\ \text{subject to (32) and (33)}. \end{aligned} \quad (36)$$

Suppose that (32) is not active and $0 < r(t) < 1$. Since $H_c(r(t))$ is a linear function of $r(t)$, then we can always increase or decrease $r(t)$ to make (36) smaller. Therefore, either (32) is active or $r(t)$ equals 0 or 1.

Suppose that (32) is active. We can replace $G_s(t)$ using (32) in (36) and obtain an equivalent objective function

$$\begin{aligned} \min \left(\eta_{co} - \frac{\eta_{ce}}{\eta_s} \right) VC_e(t) P_c(t) r(t) \\ + \frac{C_{\text{char}}}{\eta_s} [\eta_s E(t) + VC_e(t)] + P_c(t) H_b(t). \end{aligned} \quad (37)$$

Obviously, since $C_e(t), P_c(t) \geq 0$, if $\eta_{co} \geq \eta_{ce}/\eta_s$, then $r(t) = 0$; otherwise $r(t)$ should be as large as possible. If $\eta_{ce} P_{c,\max} \leq C_{\text{char}}$, then $r(t)$ can be 1 and we can use this fact to convert Problem 3 to a linear optimization problem by substituting $r(t) = 0$ and $r(t) = 1$ into Problem 3, respectively, and choose the minimum value. Otherwise, $r(t)$ should be in the range of $[C_{\text{char}}/\eta_{ce} P_{c,\max}, 1]$. Since this range is not large, we can use a search algorithm to obtain the optimal solution.

Next we need to prove that the solutions to Problem 3 are also feasible to Problem 1. In other words, the solutions to Problem 3 can meet constraints (14) and (15) for $\forall t \in T$.

Theorem 1: Suppose that θ and ε are defined in (38) and (39), respectively

$$\theta = \frac{VC_{e,\max}}{\eta_s} + \min\{D_{\max}, L_{e,\max}\} \quad (38)$$

$$\varepsilon = \frac{VC_g}{w^2 \eta_{ag}} + L_{w,\max}. \quad (39)$$

Then through minimizing Problem 3, we can have the following results:

$$0 \leq B(t) \leq \theta + C_{\text{char}}, \quad \forall t \in T \quad (40)$$

$$0 \leq W(t) \leq \max \left\{ \begin{aligned} &\varepsilon + \frac{\eta_{ce}\theta - VC_g}{w^2\eta_{cg}} + w^2\eta_{cg}P_{c,\max}, \varepsilon \\ &+ \frac{\eta_{co}VC_{e,\max} - VC_g}{w^2\eta_{cg}} + w^2\eta_{cg}P_{c,\max}, \varepsilon \\ &+ w^2\eta_{cg}P_{c,\max} + w^2\eta_{ag}P_{a,\max} \end{aligned} \right\}, \forall t \in T \quad (41)$$

given that the above relationships are satisfied at $t = 0$.

The proof of this theorem can be found in [18], and is omitted due to the space limit.

From Theorem 1, we can see the needed capacity of the water tank is related to the queue weight w , so we can minimize the requirement on the water tank capacity by adjusting w^2 . To simplify the notation, we define

$$\frac{R}{w^2} + Sw^2 = \max \left\{ \begin{aligned} &\frac{\eta_{ce}\theta - VC_g}{w^2\eta_{cg}} + w^2\eta_{cg}P_{c,\max} \\ &\times \frac{\eta_{co}VC_{e,\max} - VC_g}{w^2\eta_{cg}} + w^2\eta_{cg}P_{c,\max} \end{aligned} \right\} \quad (42)$$

$$Tw^2 = w^2\eta_{cg}P_{c,\max} + w^2\eta_{ag}P_{a,\max}. \quad (43)$$

Our goal is to find the right parameter w^2 to minimize $\max\{R/w^2 + Sw^2, Tw^2\}$. Since the electricity price is typically much higher than the gas price for the same amount of energy, we have $\eta_{co}VC_{e,\max} - VC_g > 0$. Therefore, R , S , and T are all positive. Define $w = \hat{w}$ when $R/w^2 + Sw^2$ reaches its minimum value $M(\hat{w})$. We may reach the minimum of $\max\{R/w^2 + Sw^2, Tw^2\}$ in two cases.

- 1) $T\hat{w}^2 \leq M(\hat{w})$: We reach the minimum when $w = \hat{w}$.
- 2) $T\hat{w}^2 > M(\hat{w})$: We reach the minimum when $R/w^2 + Sw^2 = Tw^2$, and choose a smaller w^2 .

V. DETERMINE THE REAL-TIME PRICE

From Section IV, we notice that the CHP systems make charging or discharging decisions based on the current battery SoC and water tank level, the power and hot water demand in the current time slot, and the current real-time electricity price. Therefore, by setting appropriate RTP, the power company can control the charging and discharging process of all the CHP systems and make them provide load shaping service. In this section, we aim to design an algorithm to determine this RTP to coordinate all the CHP systems. The following assumptions are made. First, the control center in the power company can obtain the status of all the CHP systems, including the current battery SoC and water tank level, and the electricity and hot water demand in the next time slot through two-way communications. Second, there is a sufficient number of CHP systems to provide the load shaping service.

To get some insights into the CHP system reaction model with respect to different electricity price and system status, we first consider a simplified system as shown in Fig. 2(a).

By using a similar approach as that in Section IV, the charging or discharging process of the simplified system is

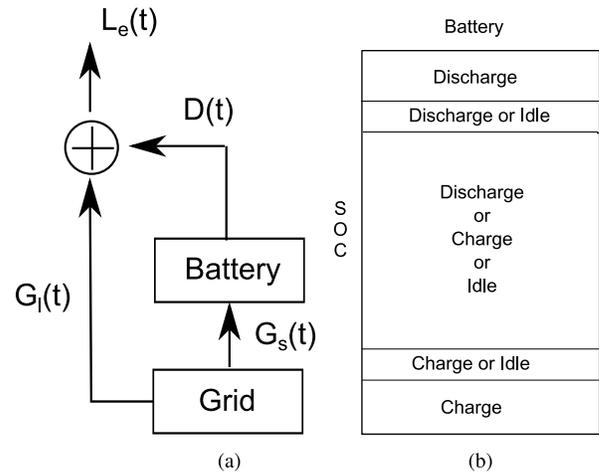


Fig. 2. Simplified battery system. (a) System model. (b) Battery model.

determined by solving the following optimization problem:

$$\min_{D(t), G_s(t)} G_s(t)[\eta_s E(t) + VC_e(t)] - D(t)[E(t) + VC_e(t)] \quad (44)$$

$$\text{subject to } 0 \leq B(t) \leq \theta + C_{\text{char}} \quad (45)$$

where

$$\theta = \frac{VC_{e,\max}}{\eta_s} + \min\{D_{\max}, L_{e,\max}\} \quad (46)$$

$$E(t) = B(t) - \theta, \quad 0 < \eta_s \leq 1. \quad (47)$$

To obtain the behavior of this simplified system, we consider the following cases.

- 1) $B(t) \geq \theta$: To minimize (44), we have $G_s(t) = 0$, and $D(t) = D_{\max}$. In other words, the battery will always discharge.
- 2) $B(t) \leq \min\{D_{\max}, L_{e,\max}\}$: To minimize (44), we have $G_s(t) = C_{\text{char}}$, and $D(t) = 0$. In other words, the battery will always charge.
- 3) $\theta - VC_e(t)/\eta_s \leq B(t) \leq \theta - VC_e(t)$: To minimize (44), we have $G_s(t) = 0$, and $D(t) = 0$. In other words, the battery will remain idle, i.e., neither charge nor discharge.
- 4) $\theta - VC_e(t) \leq B(t) < \theta$: To minimize (44), we have $G_s(t) = 0$. In other words, the battery will discharge or remain idle.
- 5) $\min\{D_{\max}, L_{e,\max}\} < B(t) \leq \theta - VC_{e,\min}(t)/\eta_s$: To minimize (44), we have $D(t) = 0$. In other words, the battery will charge or remain idle.

Notice that $G_s(t)$ and $D(t)$ cannot be positive at the same time. We can prove it by contradiction. If $G_s(t)$ and $D(t)$ are both greater than 0, from (44) we have $\eta_s E(t) + VC_e(t) < 0$ and $E(t) + VC_e(t) > 0$. Therefore, it must be $\eta_s E(t) < E(t) < 0$. Since $0 < \eta_s \leq 1$, we know it is impossible.

Fig. 2(b) illustrates the physical meaning of the above cases. The whole battery can be divided into five regions. In the upper two regions, the battery will not charge, while in the lower two regions, the battery will not discharge. If the SoC of the battery stays in the middle region, we can control the charging or discharging process of the simplified system by setting an appropriate electricity price between $C_{e,\min}$ and $C_{e,\max}$.

We use superscript i to represent the i th user. In each time slot, the amount of electricity bought from the grid for user i is $G_L^i(t) + G_S^i(t) = L_e^i(t) - D^i(t) + G_S^i(t)$. Then, the total electricity load in that time slot is $\sum L_e^i(t) - \sum D^i(t) + \sum G_S^i(t)$. Assume that the target load of the load shaping service is $L_t(t)$, at the beginning of each time slot, the control center obtains the total electricity demand of all users $\sum L_e^i(t)$ through two-way communications. The amount of load shaping service should be $L_t(t) - \sum L_e^i(t)$. Our objective is to set an appropriate RTP $C_e(t)$ so that the amount of demand response from the simplified battery system in time slot t ($\sum G_S^i(t) - \sum D^i(t)$) should be as close to the amount of load shaping service needed ($L_t(t) - \sum L_e^i(t)$) as possible. Therefore, we can formulate the following optimization problem:

$$\min_{D^i(t), G_S^i(t)} \left| \sum L_e^i(t) - \sum D^i(t) + \sum G_S^i(t) - L_t(t) \right| \quad (48)$$

where $L_e^i(t)$ and $L_t(t)$ are already known, and $D^i(t)$ and $G_S^i(t)$ are determined by solving the optimization problem (44).

This is a nested optimization problem. Since the objective function of the CHP system is nonconvex, we cannot use the existing bilevel programming to solve it. However, we can utilize some unique features of this problem to obtain the solution with a time complexity of $O(\log(n))$, where n is the number of RTPs $C_e(t)$ can be chosen from.

Notice that $G_S(t)$ is a nonincreasing function of $C_e(t)$, and $D(t)$ is a nondecreasing function of $C_e(t)$, thus $-\sum D^i(t) + \sum G_S^i(t)$ in (48) is a nonincreasing function of $C_e(t)$. To prove it, we first look at (44). If $C_e(t)$ increases, since $V > 0$, both $\eta_s E(t) + VC_e(t)$ and $E(t) + VC_e(t)$ will increase. Therefore, to minimize the objective function, $G_S(t)$ will remain the same when $\eta_s E(t) + VC_e(t) \leq 0$ or $G_S(t) = C_{\text{char}}$ when $\eta_s E(t) + VC_e(t) > 0$, $D(t)$ will remain the same when $E(t) + VC_e(t) \leq 0$, and $D(t) = 0$ when $E(t) + VC_e(t) > 0$.

Since $\sum L_e^i(t) - L_t(t)$ in (48) is already known at the beginning of each time slot, $\sum L_e^i(t) - \sum D^i(t) + \sum G_S^i(t) - L_t(t)$ in (48) is a nonincreasing function of $C_e(t)$ in each time slot. Therefore, we can find the appropriate RTP by using a binary search. Let $g(C_e(t)) = \sum L_e^i(t) - \sum D^i(t) + \sum G_S^i(t) - L_t(t)$ when the current electricity price is $C_e(t)$, the searching algorithm is described in Algorithm 1.

The parameter τ represents the minimum price resolution in our search. Lines 5–7 mean that we return the price range if $g(C_e(t))$ does not change within that range.

There are three points we need to notice. First, since the amount of charging or discharging of each simplified system is discrete, we cannot guarantee that the total load will be exactly the same as the target load. However, since all the variables in (48) are bounded, the difference between the two loads are also bounded. Second, the electrical price $C_e(t)$ obtained from Algorithm 1 may not be the only solution, because the value of $\sum L_e^i(t) - \sum D^i(t) + \sum G_S^i(t) - L_t(t)$ in (48) may remain the same even with different electricity price. For example, assuming there is only one simplified system, and we need it to discharge. Any electricity price which can make $E(t) + VC_e(t) > 0$ will reach this goal. Therefore, Algorithm 1 will also return the price range if found. With different electricity price within that range, the

Algorithm 1 Searching the Electricity Price

Require: $C_{e,\min}$, $C_{e,\max}$

- 1: $C_{e,\text{high}} = C_{e,\max}$
 - 2: $C_{e,\text{low}} = C_{e,\min}$
 - 3: $C_{e,\text{mid}} = (C_{e,\text{high}} + C_{e,\text{low}})/2$
 - 4: **while** $C_{e,\text{mid}} - C_{e,\min} > \tau$ OR $C_{e,\max} - C_{e,\text{mid}} > \tau$ **do**
 - 5: **if** $g(C_{e,\min}) = g(C_{e,\max})$ **then**
 - 6: **return** $C_{e,\text{low}}$ and $C_{e,\text{high}}$
 - 7: **end if**
 - 8: **if** $g(C_{e,\text{mid}}) < 0$ **then**
 - 9: $C_{e,\text{high}} = C_{e,\text{mid}}$
 - 10: **else**
 - 11: $C_{e,\text{low}} = C_{e,\text{mid}}$
 - 12: **end if**
 - 13: $C_{e,\text{mid}} = (C_{e,\text{high}} + C_{e,\text{low}})/2$
 - 14: **end while**
 - 15: **return** $\min\{g(C_{e,\max}), g(C_{e,\text{mid}}), g(C_{e,\min})\}$ and the corresponding C_e
-

operation decisions of each CHP will not change. Since the amount charged or discharged is fixed once the operation decision is made, the amount of demand response will be the same. Therefore, there will be no influence on the system states if the price is different within the range. However, this may lead to another problem: the utility company may always choose the highest price to maximize its profit. Therefore, how to design an appropriate mechanism to effectively restrict the behavior of the utility company is an interesting problem left for future research. Third, there are some tricks to simplify the calculation of $g(C_e(t))$. From the previous discussion of the behavior of the simplified system, the battery will remain idle if $\theta - VC_e(t)/\eta_s \leq B(t) \leq \theta - VC_e(t)$. In other words, given $B(t)$, the battery will remain idle if $C_e(t)$ changes between a certain range $[C_{e,l}(t), C_{e,h}(t)]$. If $C_e(t) < C_{e,l}(t)$, the battery will charge by C_{char} . On the other hand, if $C_e(t) > C_{e,h}(t)$, the battery will discharge by D_{max} . Therefore, with the value of $C_{e,l}(t)$ and $C_{e,h}(t)$ calculated beforehand, we can obtain the value of $-D^i(t) + G_S^i(t)$ in $O(1)$ time.

With respect to the CHP system, we have a more complicated model with a dependent hot water queue. However, the fundamental idea to find the RTP to coordinate all the CHP systems is similar. The higher the price is, the more likely the CHP system will sell the electricity to the grid and use the electricity stored in the battery. Let $V(C_e(t)) = \sum [G_S^i(t) + G_L^i(t) - (1 - r^i(t))\eta_{\text{co}} P_c^i(t)] - L_t(t)$ represent the difference between the total electricity demand and the target load. Then, $V(C_e(t))$ is a nonincreasing function of the RTP $C_e(t)$. The proof is straightforward and is omitted due to the space limit. Therefore, we can use a similar binary search algorithm to find the appropriate RTP. The algorithm is exactly the same as Algorithm 1 by replacing $g(C_e(t))$ with $V(C_e(t))$.

Notice that $L_e(t)$, $L_w(t)$, and $G_L(t)$ are not needed to solve Problem 3. Therefore, instead of solving the optimization problem for each CHP system in every time slot, we can build a lookup table with only three input parameters $C_e(t)$, $E(t)$, and $X(t)$ and five outputs $G_S(t)$, $P_c(t)$, $P_a(t)$, $r(t)$, and $D(t)$ for quick search, assuming homogeneous CHP systems.

TABLE II
AVERAGE COST WITH DIFFERENT QUEUE WEIGHTS

queue weight	0.2	0.6	2	6	10
water tank size (L)	170.9	165.7	388.4L	1036.6	1685.8
average cost per time slot (\$)	0.2000	0.2080	0.2104	0.2010	0.2035

VI. PERFORMANCE EVALUATION

In this section, the parameters for the CHP systems are the same as those in [18], except the electricity and hot water demand in each time slot, which will be specified later. The effectiveness of CHP systems on cost saving comparing to a benchmark algorithm has been demonstrated in [18], and is omitted due to the space limit. In addition, because of the key differences between the proposed system model and the system models in existing work discussed in Section II, only the load with fixed electricity price and that without CHP are illustrated to show the effectiveness of the proposed algorithm.

A. Influence of Different Queue Weights

As mentioned in Section IV, the required size of the water tank is related with the weight of the battery pack queue and the water tank queue. To explore the relationship between the queue weight w and the average cost, we assume that the real-time electricity price, and the electricity and hot water demand in each time slot are uniformly distributed between the corresponding minimum value and the maximum value. The value of V in Theorem 1 is set to 100. By running the simulation 50 times, we obtain the average cost for 100 time slots in Table II.¹

From Table II, we notice that the required water tank size first decreases and then increases with the increment of the queue weight (w). However, the fluctuation of the average cost in each time slot is quite small, less than 5%. The reason is that the average cost achieved by using the proposed algorithm is upper bounded by parameter V [18]. Therefore, we can minimize the size of the water tank by adjusting the queue weight while having almost the same average cost.

B. Load Shaping Using RTP

With battery packs and water tanks, CHP systems can be considered as energy buffers to provide load shaping services. In this simulation, we consider a scenario with 50 homogeneous CHP systems aiming to provide a 5-h load shaping service, consisting 20 time slots with a 15-min slot duration. The electricity and hot water demands in each time slot for each CHP system are uniformly distributed between 0 and its corresponding maximum value. We set the target load as a constant value which is 30 kW below the average electricity demand. The real-time electricity price used to coordinate all the CHP systems is obtained using the algorithm described in Section V. The load using CHP systems but with a constant electricity price, which is set to the middle of the maximum and minimum electricity price, is also simulated.

Fig. 3 shows the performance of the load shaping service. Since the CHP systems will generate electricity using

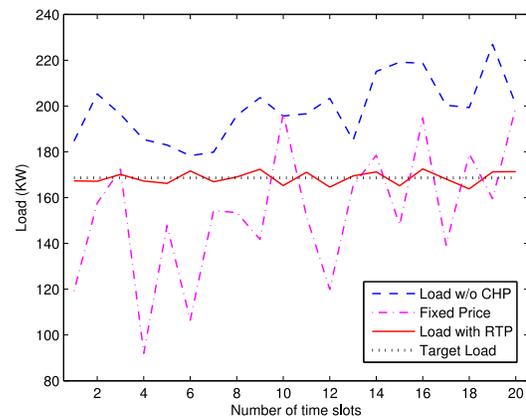


Fig. 3. Load with different price.

natural gas, the average load using a fixed price is lower than the average load without CHP systems. Although CHP systems can help users save money, the fluctuation of the total load with CHP systems and a fixed electricity price is higher than that without using CHP systems. This will add extra cost to the power company as more frequency regulation or spinning reservation services may be needed. By setting the RTP according to the proposed algorithm, we can reduce the fluctuation of the total load with CHP systems and make the total load close to the target load. Therefore, the combination of CHP systems and the RTP searching algorithm can help both users and the utility company reduce their cost.

VII. CONCLUSION

In this paper, we have proposed an approach to minimize the average energy cost for CHP systems. Different from our previous work [18], we assigned different weights to the battery pack queue and the water tank queue. Then, we discussed how to minimize the required size of the water tank by adjusting the queue weight while still maintaining a similar performance. We also considered selling the electricity stored in the battery to the power grid. On the other hand, we discussed how to set an appropriate RTP to coordinate all the CHP systems to provide load shaping service. Different from the existing works which mainly determine the RTP from the perspective of game theories, the proposed algorithm uses binary search to find the optimal RTP with a time complexity of $O(\log n)$. Extensive simulation shows that the use of CHP systems can reduce the cost of both users and the utility company with the proposed scheduling and pricing algorithms.

Although the proposed searching algorithm can help to find the optimal RTP, there may be multiple RTPs which can achieve the same effect. Therefore, it is possible that the utility company will always choose the maximum RTP to maximize its profit. How to effectively regulate the actions of the utility company is a problem left for future research. Second, since the target load will affect the RTP for all the CHP systems, and thus affect the profit of the utility company, how to select an appropriate target load is also an interesting problem needs further investigation. Third, in this paper, we assume that there are enough CHP systems to provide load shaping services. However, in the real world, we cannot make a finite number of CHP systems provide infinite load shaping services.

¹In this simulation, we set the capacity of the battery as fixed. If we fix the capacity of the water tank, we can obtain similar results, which is omitted due to the space limit.

Once all the batteries of CHP systems are low or high, they can no longer provide demand response. Therefore, we need to monitor the status of all the CHP systems and change the target load accordingly. This is also an important issue left for future research. Fourth, the cost effectiveness of CHP systems has been fully analyzed and proven [5], [20]. Therefore, in this paper, we assume that the CHP systems have already been purchased and their costs are not affected by the operation for simplicity. How to further extend the work considering the varying operation costs of CHP systems remains a further research issue. Finally, similar to many existing works, such as [15]–[17], in this paper, only one elastic appliance is considered to calculate the RTP. In real deployment, there may be multiple elastic appliances, therefore, the proposed algorithm needs to take their operation models into account as well to determine an appropriate RTP.

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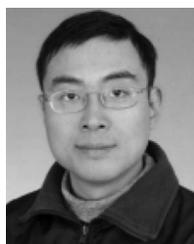
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