Chapter 5
APPLICATION OF TRANSFORM THEORY TO SYSTEMS
5.5.4 Aliasing

Victoria, BC, Canada
Email: aantoniou@ieee.org

July 10, 2018

Introduction

- When a continuous-time signal that contains frequencies outside the baseband $-\omega_s/2 < \omega < \omega_s/2$ is sampled, a phenomenon known as *aliasing* will arise whereby frequencies outside the baseband impersonate frequencies within the baseband.

Introduction

- When a continuous-time signal that contains frequencies outside the baseband $-\omega_s/2 < \omega < \omega_s/2$ is sampled, a phenomenon known as *aliasing* will arise whereby frequencies outside the baseband impersonate frequencies within the baseband.
- Aliasing is objectionable in practice and it must be prevented from occurring.

Introduction

- When a continuous-time signal that contains frequencies outside the baseband $-\omega_s/2 < \omega < \omega_s/2$ is sampled, a phenomenon known as *aliasing* will arise whereby frequencies outside the baseband impersonate frequencies within the baseband.

- Aliasing is objectionable in practice and it must be prevented from occurring.
- This presentation explores the nature of aliasing in DSP.

Introduction

- When a continuous-time signal that contains frequencies outside the baseband $-\omega_s/2 < \omega < \omega_s/2$ is sampled, a phenomenon known as *aliasing* will arise whereby frequencies outside the baseband impersonate frequencies within the baseband.

- Aliasing is objectionable in practice and it must be prevented from occurring.

- This presentation explores the nature of aliasing in DSP.

- Aliasing can occur in other types of systems where sampled signals are involved, for example, in videos and movies, as will be demonstrated.

Aliasing in DSP

- Consider a continuous-time sinusoidal signal with frequency $\omega + \omega_s$ rad/s, i.e.,

$$x(t) = \sin[(\omega + \omega_s)t]$$

where $0 < \omega < \omega_s/2$.

Aliasing in DSP

- Consider a continuous-time sinusoidal signal with frequency $\omega + \omega_s$ rad/s, i.e.,

$$x(t) = \sin[(\omega + \omega_s)t]$$

where $0 < \omega < \omega_s/2$.

- By sampling the signal using a sampling frequency of ω_s rad/s, a discrete-time sinusoidal signal with frequency $\omega + \omega_s$ rad/s, i.e.,

$$x(nT) = \sin[(\omega + \omega_s)nT]$$

will be generated.

...

$$x(nT) = \sin[(\omega + \omega_s)nT]$$

- Signal $x(nT)$ can be expressed as

$$\begin{aligned} x(nT) &= \sin\omega nT\cos\omega_s nT + \cos\omega nT\sin\omega_s nT \\ &= \sin\omega nT\cos\left(\frac{2\pi}{T}\cdot nT\right) + \cos\omega nT\sin\left(\frac{2\pi}{T}\cdot nT\right) \\ &= \sin\omega nT\cos 2n\pi + \cos\omega nT\sin 2n\pi \\ &= \sin\omega nT \end{aligned}$$

. . .

$$x(nT) = \sin[(\omega + \omega_s)nT]$$

- Signal $x(nT)$ can be expressed as

$$\begin{aligned} x(nT) &= \sin \omega nT \cos \omega_s nT + \cos \omega nT \sin \omega_s nT \\ &= \sin \omega nT \cos\left(\frac{2\pi}{T} \cdot nT\right) + \cos \omega nT \sin\left(\frac{2\pi}{T} \cdot nT\right) \\ &= \sin \omega nT \cos 2n\pi + \cos \omega nT \sin 2n\pi \\ &= \sin \omega nT \end{aligned}$$

- Evidently, sampling a sinusoidal signal of frequency $\omega + \omega_s$ will produce a discrete-time signal which is numerically identical to the discrete-time signal obtained by sampling a sinusoidal signal of frequency ω.

...

$$x(nT) = \sin[(\omega + \omega_s)nT]$$

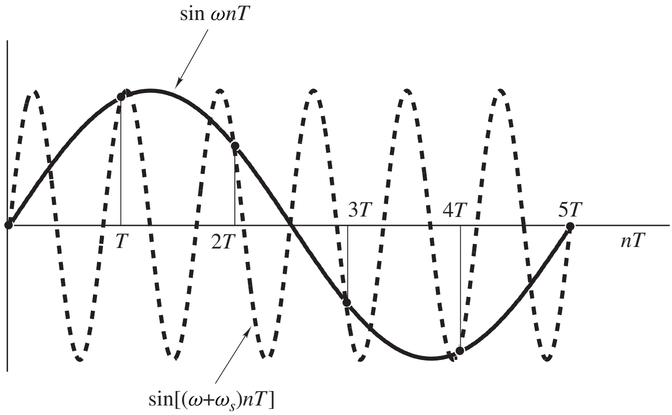
- Signal $x(nT)$ can be expressed as

$$\begin{aligned} x(nT) &= \sin\omega nT \cos\omega_s nT + \cos\omega nT \sin\omega_s nT \\ &= \sin\omega nT \cos\left(\frac{2\pi}{T}\cdot nT\right) + \cos\omega nT \sin\left(\frac{2\pi}{T}\cdot nT\right) \\ &= \sin\omega nT \cos 2n\pi + \cos\omega nT \sin 2n\pi \\ &= \sin\omega nT \end{aligned}$$

- Evidently, sampling a sinusoidal signal of frequency $\omega + \omega_s$ will produce a discrete-time signal which is numerically identical to the discrete-time signal obtained by sampling a sinusoidal signal of frequency ω.

- In effect, the sampled version of signal $\sin[(\omega + \omega_s)t]$ will be *aliased* to the sampled version of $\sin(\omega)t$ or frequency $\omega + \omega_s$ will be *aliased* to ω.

Aliasing in DSP *Cont'd*



Aliasing in DSP *Cont'd*

- Now consider a sinusoidal signal

$$x(nT) = \sin[(\omega_s - \omega)nT]$$

where $0 < \omega < \omega_s/2$.

Aliasing in DSP *Cont'd*

- Now consider a sinusoidal signal

$$x(nT) = \sin[(\omega_s - \omega)nT]$$

where $0 < \omega < \omega_s/2$.

- As before, we can write

$$\begin{aligned}
x(nT) &= \sin(\omega_s - \omega)nT \\
&= \sin\omega_s nT \cos\omega nT - \cos\omega_s nT \sin\omega nT \\
&= \sin\left(\frac{2\pi}{T}\cdot nT\right)\cos\omega nT - \cos\left(\frac{2\pi}{T}\cdot nT\right)\sin\omega nT \\
&= \sin 2n\pi \cos\omega nT - \cos 2n\pi \sin\omega nT \\
&= -\sin\omega nT = \sin(-\omega nT)
\end{aligned}$$

Aliasing in DSP *Cont'd*

- Now consider a sinusoidal signal

$$x(nT) = \sin[(\omega_s - \omega)nT]$$

where $0 < \omega < \omega_s/2$.

- As before, we can write

$$\begin{aligned} x(nT) &= \sin(\omega_s - \omega)nT \\ &= \sin \omega_s nT \cos \omega nT - \cos \omega_s nT \sin \omega nT \\ &= \sin\left(\frac{2\pi}{T} \cdot nT\right) \cos \omega nT - \cos\left(\frac{2\pi}{T} \cdot nT\right) \sin \omega nT \\ &= \sin 2n\pi \cos \omega nT - \cos 2n\pi \sin \omega nT \\ &= -\sin \omega nT = \sin(-\omega nT) \end{aligned}$$

- In this case, *frequency $\omega_s - \omega$ will be aliased to frequency $-\omega$.*

Aliasing in DSP *Cont'd*

- Aliasing has occurred in the sinusoidal signals considered because the frequencies of the signals were outside the baseband $-\omega_s/2 < \omega < \omega_s/2$.

Aliasing in DSP *Cont'd*

- Aliasing has occurred in the sinusoidal signals considered because the frequencies of the signals were outside the baseband $-\omega_s/2 < \omega < \omega_s/2$.

- If a signal has several frequency components outside the baseband, then *all* the frequencies beyond the Nyquist frequency will be aliased and a rather serious type of signal distortion will occur, which is known as *aliasing distortion*.

- Aliasing has occurred in the sinusoidal signals considered because the frequencies of the signals were outside the baseband $-\omega_s/2 < \omega < \omega_s/2$.

- If a signal has several frequency components outside the baseband, then *all* the frequencies beyond the Nyquist frequency will be aliased and a rather serious type of signal distortion will occur, which is known as *aliasing distortion*.

- Aliasing can be prevented by *filtering out* all frequency components outside the baseband using a lowpass filter.

Aliasing in DSP *Cont'd*

- Aliasing has occurred in the sinusoidal signals considered because the frequencies of the signals were outside the baseband $-\omega_s/2 < \omega < \omega_s/2$.

- If a signal has several frequency components outside the baseband, then *all* the frequencies beyond the Nyquist frequency will be aliased and a rather serious type of signal distortion will occur, which is known as *aliasing distortion*.

- Aliasing can be prevented by *filtering out* all frequency components outside the baseband using a lowpass filter.

- Filtering out the high-frequency content of the signal with a lowpass filter will, of course, distort the signal but the distortion introduced is less serious than aliasing distortion (see Chap. 6).

Aliasing at the Movies

- Aliasing can occur in other types of systems that rely on sampling.

Aliasing at the Movies

- Aliasing can occur in other types of systems that rely on sampling.
- A movie is a sequence of still photographs which constitutes a sampled signal.

  If the sampling rate (frames/second) is not fast enough to deal with the images projected, funny things can happen.

Aliasing at the Movies

- Aliasing can occur in other types of systems that rely on sampling.
- A movie is a sequence of still photographs which constitutes a sampled signal.

  If the sampling rate (frames/second) is not fast enough to deal with the images projected, funny things can happen.
- For example, as the cowboy wagon accelerates into the sunset, the wheels of the wagon appear to accelerate in the forward direction, then reverse, slow down, stop momentarily, and after that they accelerate again in the forward direction. This is what should be seen, it's not an illusion!

The slides that follow show a wagon wheel with a black dot.

- The number of wheel revolutions per second is analogous to frequency, say, ω.

The slides that follow show a wagon wheel with a black dot.

- The number of wheel revolutions per second is analogous to frequency, say, ω.
- The number of film frames per second, N, times the number of wheel revolutions per second is analogous to the sampling frequency ω_s, i.e.,

$$\omega_s = N\omega \quad \text{or} \quad \omega = \omega_s/N$$

Aliasing at the Movies *Cont'd*

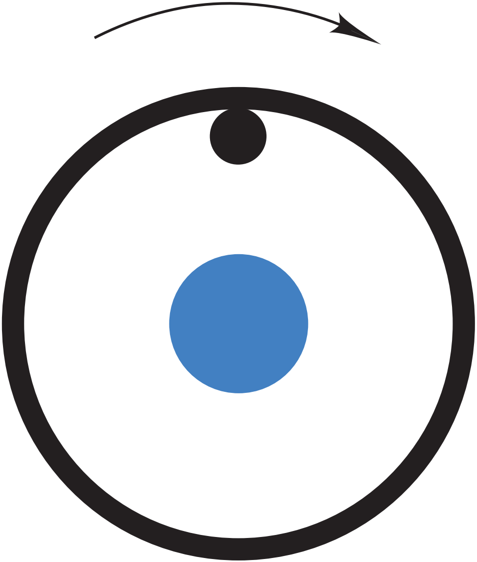
The slides that follow show a wagon wheel with a black dot.

- The number of wheel revolutions per second is analogous to frequency, say, ω.
- The number of film frames per second, N, times the number of wheel revolutions per second is analogous to the sampling frequency ω_s, i.e.,

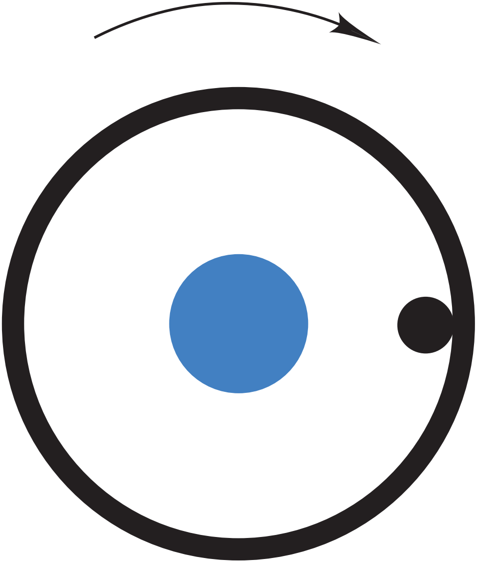
$$\omega_s = N\omega \quad \text{or} \quad \omega = \omega_s / N$$

- Rotation in the clockwise or counterclockwise direction is analogous to positive or negative frequency.

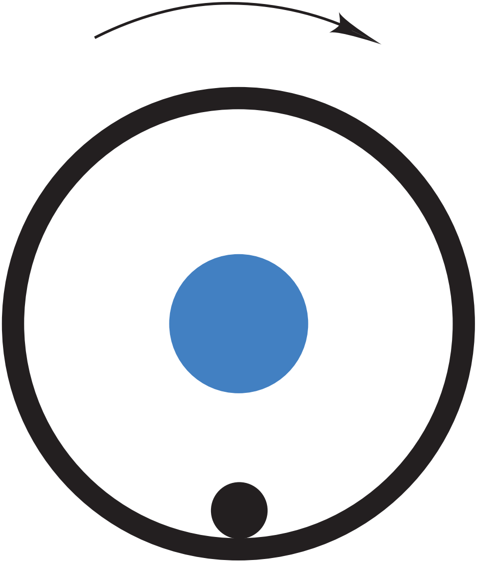
$\omega = \omega_s/4$, Frame #1



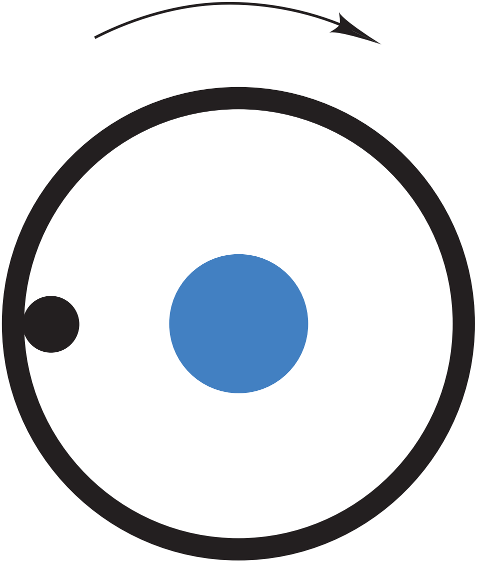
$\omega = \omega_s/4$, Frame #2



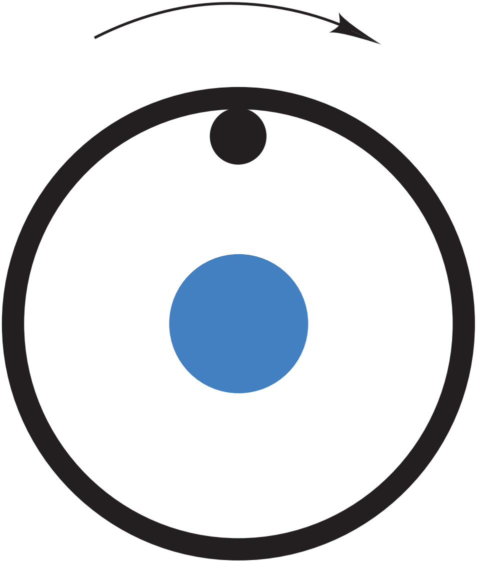
$\omega = \omega_s/4$, Frame #3



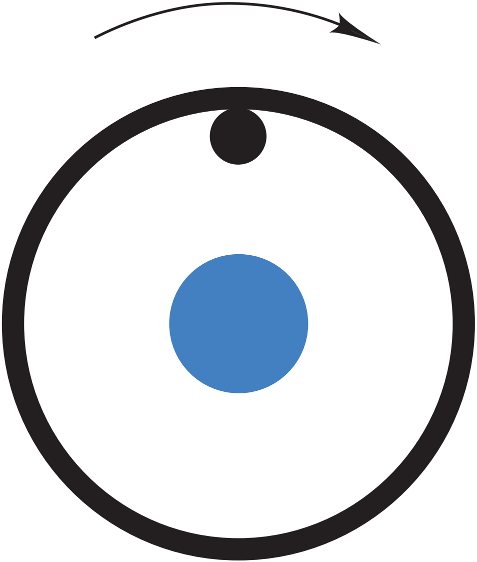
$\omega = \omega_s/4$, Frame #4



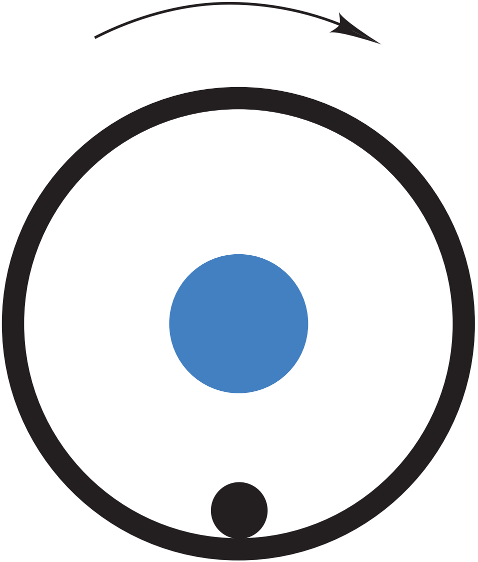
$\omega = \omega_s/4$, Frame #5



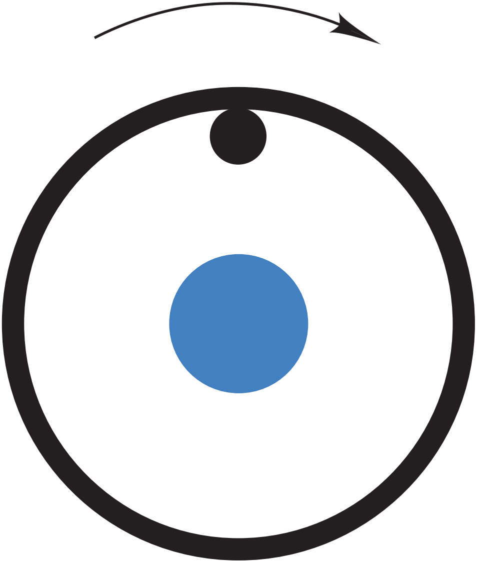
$\omega = \omega_s/2$, Frame #1



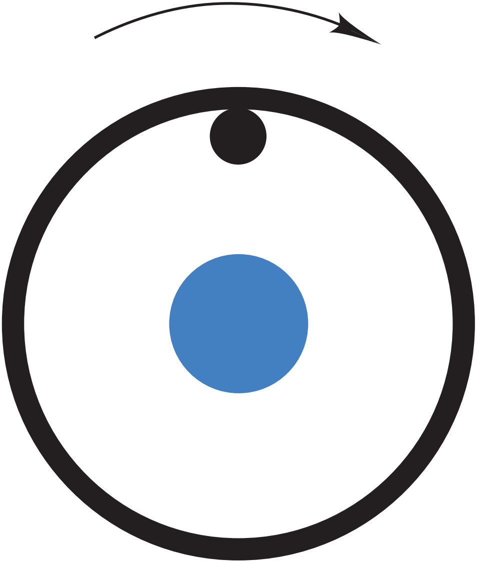
$\omega = \omega_s/2$, Frame #2



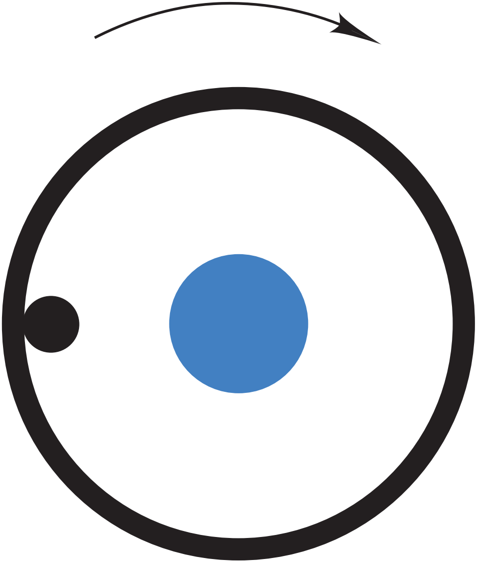
$\omega = \omega_s/2$, Frame #3



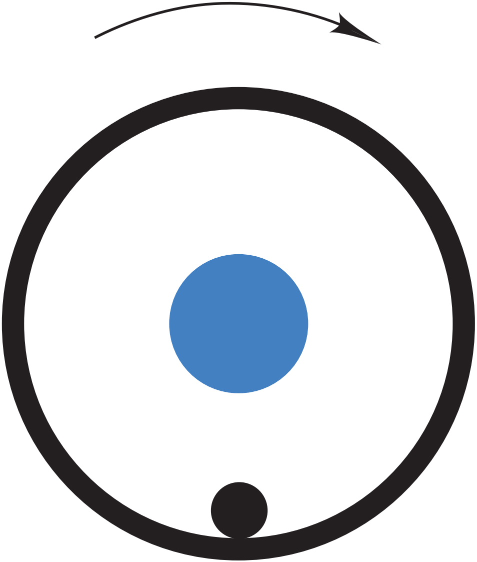
$\omega = 3\omega_s/4$, Frame #1



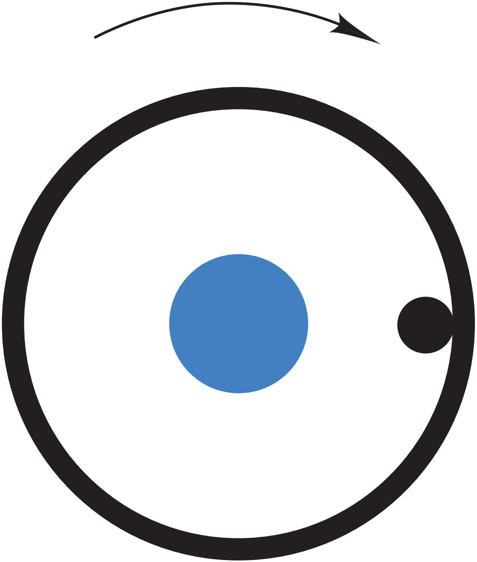
$\omega = 3\omega_s/4$, Frame #2



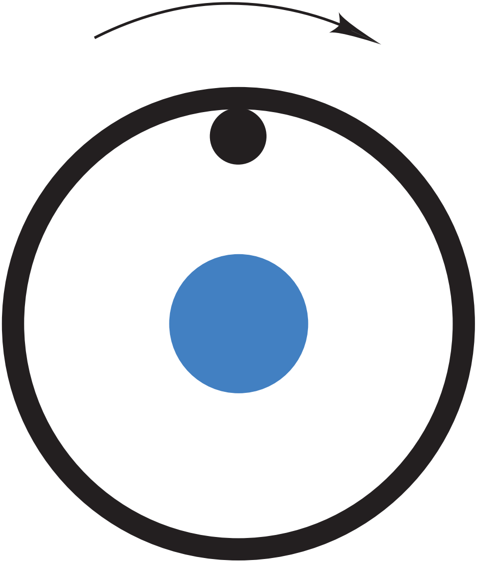
$\omega = 3\omega_s/4$, Frame #3



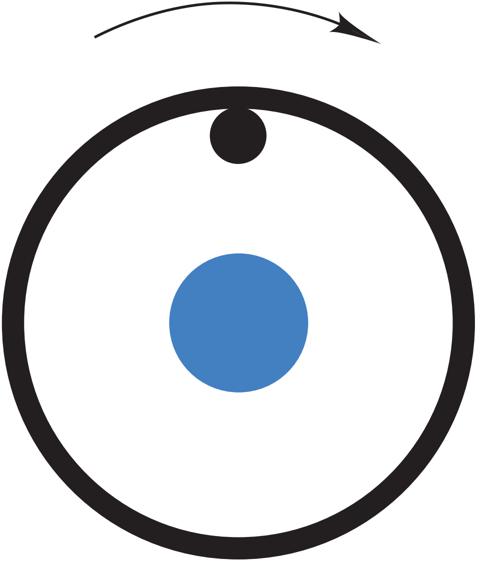
$\omega = 3\omega_s/4$, Frame #4



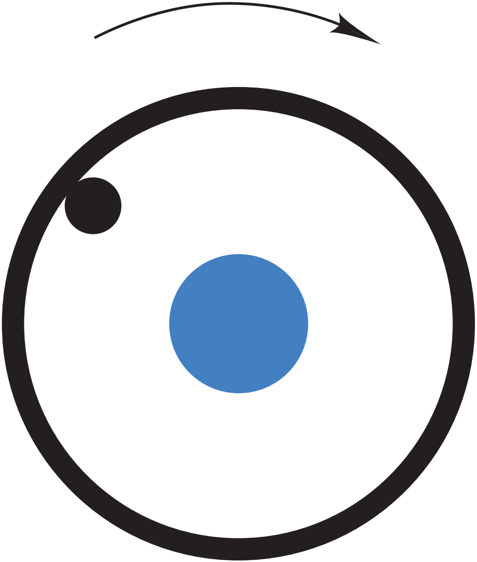
$\omega = 3\omega_s/4$, Frame #5



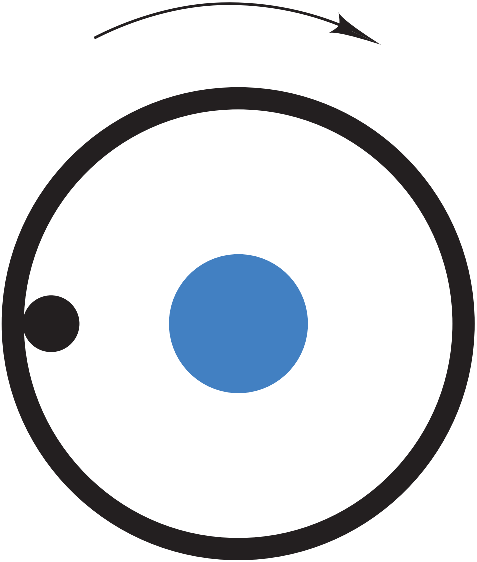
$\omega = 7\omega_s/8$, Frame #1



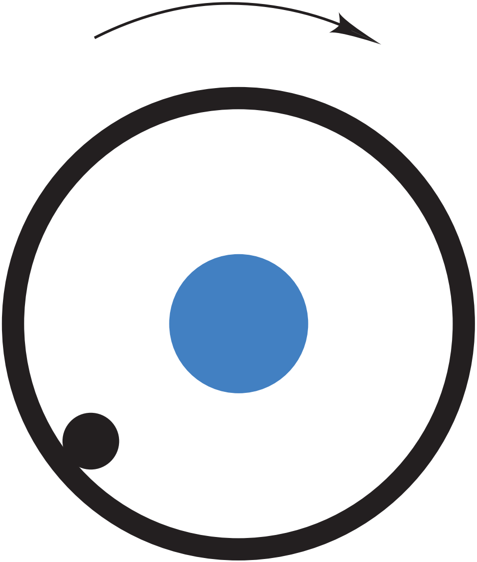
$\omega = 7\omega_s/8$, Frame #2



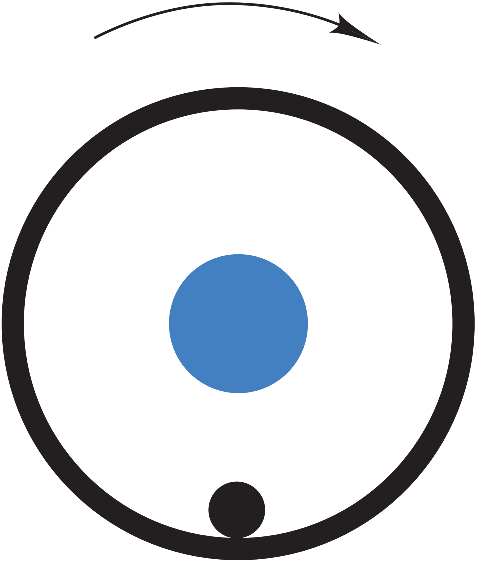
$\omega = 7\omega_s/8$, Frame #3



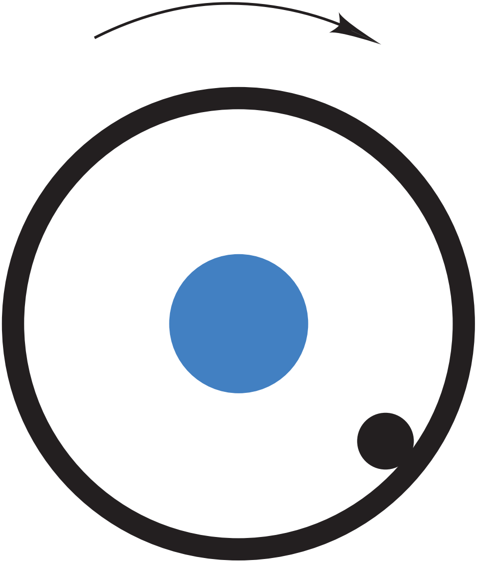
$\omega = 7\omega_s/8$, Frame #4



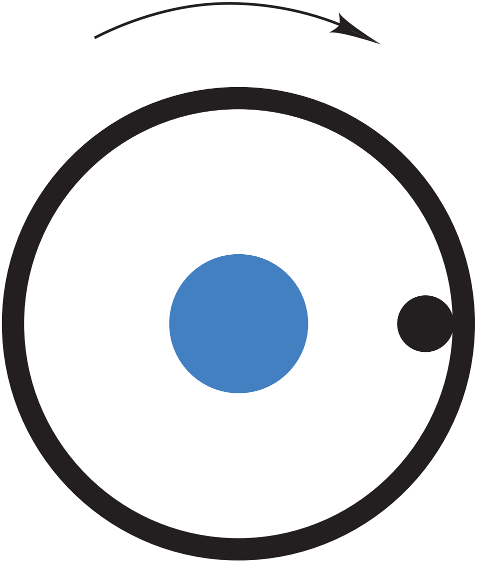
$\omega = 7\omega_s/8$, Frame #5



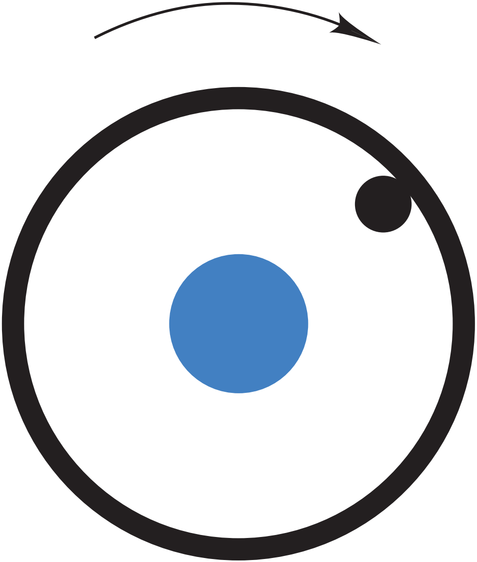
$\omega = 7\omega_s/8$, Frame #6



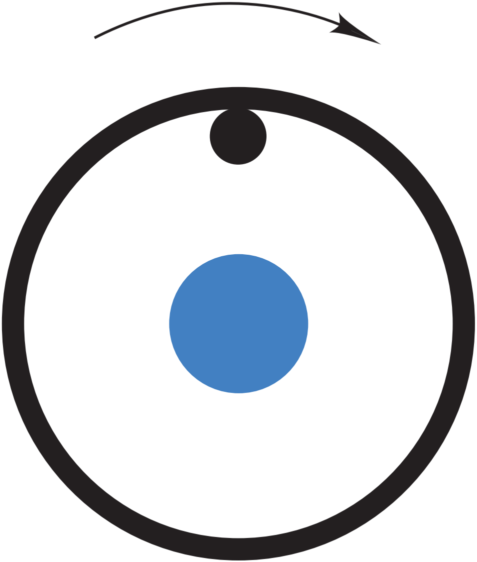
$\omega = 7\omega_s/8$, Frame #7



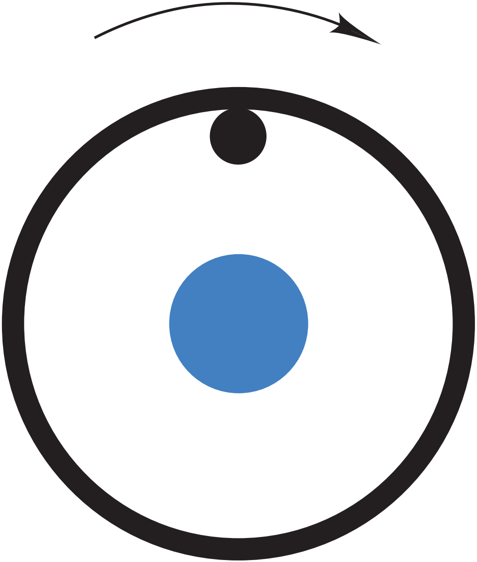
$\omega = 7\omega_s/8$, Frame #8



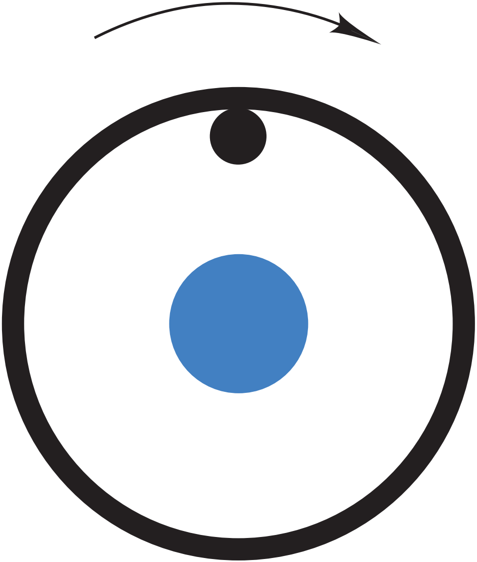
$\omega = 7\omega_s/8$, Frame #9



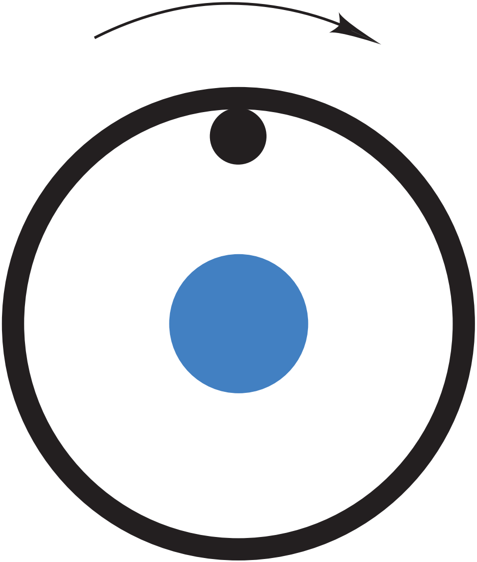
$\omega = \omega_s$, Frame #1

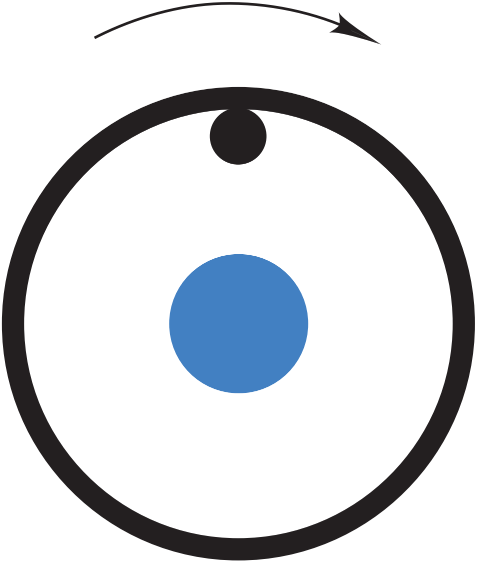


$\omega = \omega_s$, Frame #2

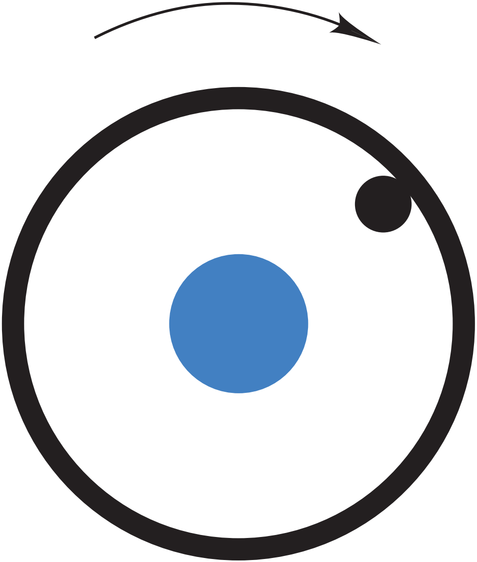


$\omega = \omega_s$, Frame #3

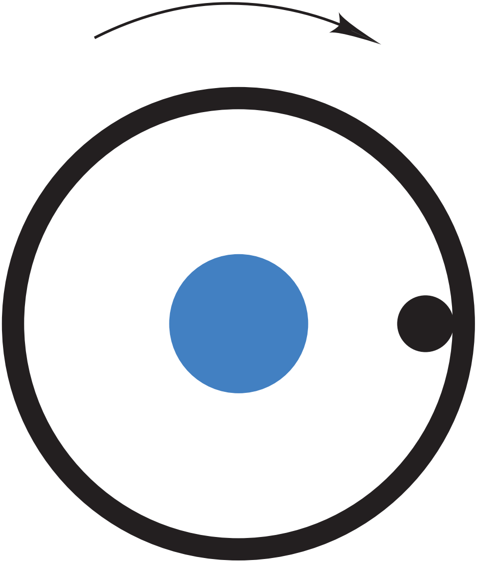




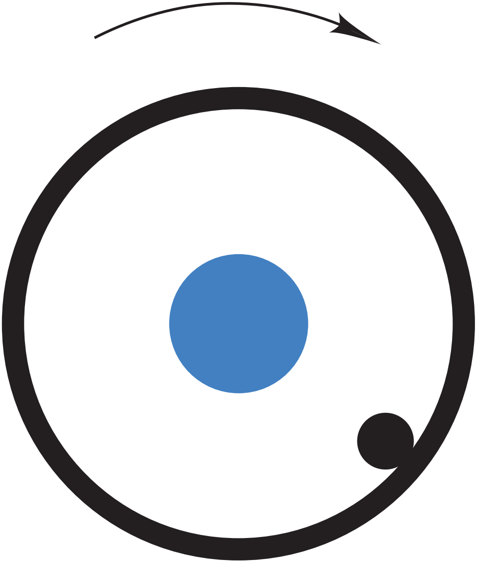
$\omega = 9\omega_s/8$, Frame #2



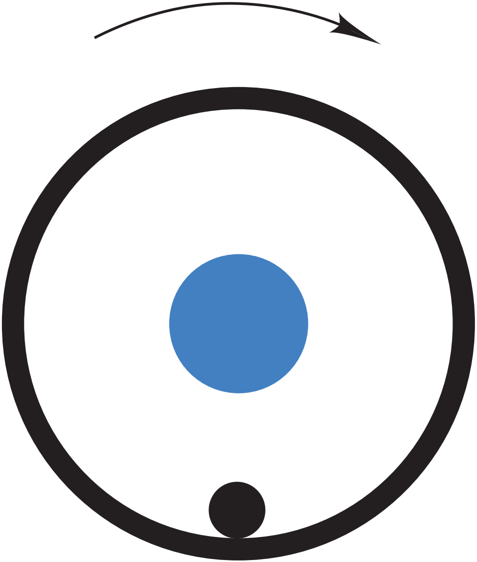
$\omega = 9\omega_s/8$, Frame #3



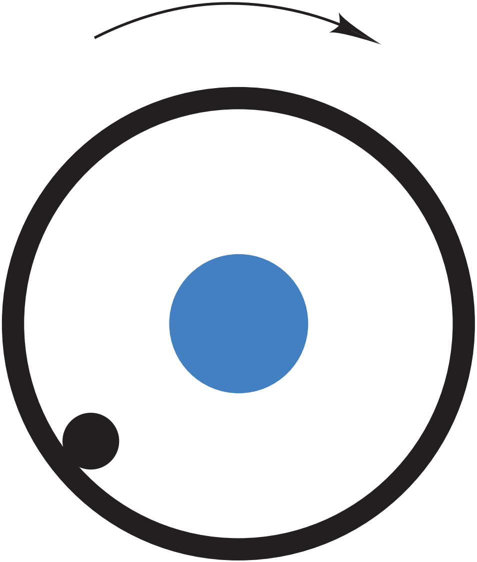
$\omega = 9\omega_s/8$, Frame #4



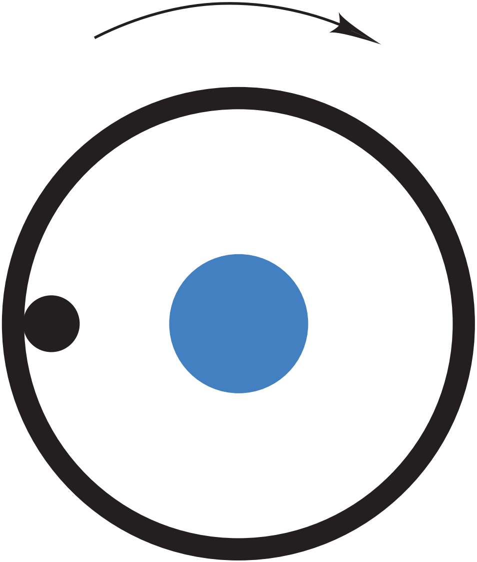
$\omega = 9\omega_s/8$, Frame #5



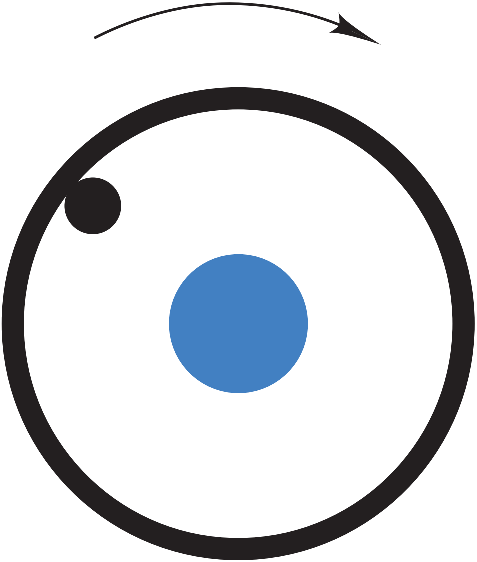
$\omega = 9\omega_s/8$, Frame #6



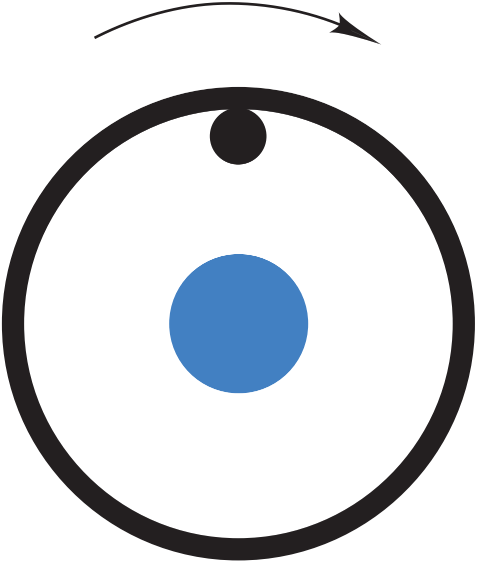
$\omega = 9\omega_s/8$, Frame #7



$\omega = 9\omega_s/8$, Frame #8



$\omega = 9\omega_s/8$, Frame #9



This slide concludes the presentation.
Thank you for your attention.