Chapter 9
REALIZATION
9.1 Introduction
9.2.1 Direct Realization
9.2.2 Direct Canonic Realization

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July 10, 2018

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- In direct methods, the problem is solved directly in the z domain.
- In indirect methods, a continuous-time transfer function is first obtained and then converted into a corresponding discrete-time transfer function.
- Nonrecursive filters are always designed through direct methods whereas recursive filters can be designed either through direct or indirect methods.

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- In closed-form methods, the problem is solved through a small number of design steps using a set of closed-form formulas.
- In iterative methods, an initial solution is assumed and through the application of optimization methods a series of progressively improved solutions are obtained until some design criterion is satisfied.

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  - yield precise designs,
  - require minimal computation effort, and so on.

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- As for approximation methods, realization methods can be classified as *direct* or *indirect*.
- In direct methods the filter structure is obtained directly from a given discrete-time transfer function whereas in indirect realizations it is obtained indirectly from an equivalent continuous-time transfer function.

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- The designer is usually interested in realizations that
  - are easy to implement in very-large-scale integrated (VLSI) circuit form,
  - require the minimum number of unit delays, adders, and multipliers,
  - are not seriously affected by the use of finite-precision arithmetic in the implementation, and so on.

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- A design will be approved to the extent that design imperfections do not violate the desired specifications.
- In digital filters and digital systems in general, most imperfections are caused by *numerical imprecision* of some form and studying the ways in which numerical imprecision will manifest itself needs to be undertaken.

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  - on the length of registers used to store numbers
  - the type of number system used (e.g., signed-magnitude, two's complement)
  - the type of arithmetic used (e.g., fixed-point or floating-point), etc.

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- When the transfer function coefficients are quantized, errors are introduced in the amplitude and phase responses of the filter, which are commonly referred to as *quantization errors*.

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- Errors introduced by the quantization of signals can be treated as *noise sources* and, as a consequence, they can have a dramatic effect on the processed signal.
- In short, the *effects of arithmetic errors* on the performance of the filter must be investigated and ways must be found to mitigate any problems associated with numerical imprecision.

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- A hardware implementation involves the conversion of the filter network into a dedicated piece of hardware.

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- In *real-time* applications, however, where data must be processed at a very high rate, e.g., in communication systems, a hardware implementation is mandatory.
- Often the best engineering solution might be partially in terms of software and partially in terms of hardware since software and hardware are highly exchangeable nowadays.

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- However, realization is the easiest to learn and for this reason it will be treated first.
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- See end of Chap. 9 for a discussion on the implementation of digital filters.

Realization

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- In direct methods, the transfer function is put in some form that enables the identification of an interconnection of elemental digital-filter subnetworks.
- Some of these methods are as follows:
  - Direct
  - Direct canonic
  - State-space
  - Lattice
  - Parallel
  - Cascade

- An arbitrary causal *nonrecursive* filter can be represented by the equation

$$Y(z) = N(z)X(z) = \left[\sum_{i=0}^{N} a_i z^{-i}\right] X(z)$$

Direct Realization

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- We can write

$$Y(z) = \left[a_0 + z^{-1}\sum_{i=1}^{N} a_i z^{-i+1}\right] X(z)$$

or $$Y(z) = [a_0 + z^{-1}N_1(z)]X(z)$$

where $$N_1(z) = \sum_{i=1}^{N} a_i z^{-i+1}$$

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- Therefore, transfer function $N(z)$ can be realized by using
  - a multiplier with a constant a_0
  - a unit delay, and
  - a network with a transfer function $N_1(z)$

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- The unit delay can be connected in cascade with the network for $N_1(z)$ to form $z^{-1}N_1(z)$ and the multiplier can be connected in parallel with the network for $z^{-1}N_1(z)$ to form $a_0 + z^{-1}N_1(z)$.

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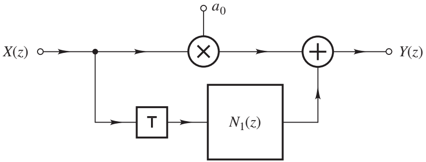
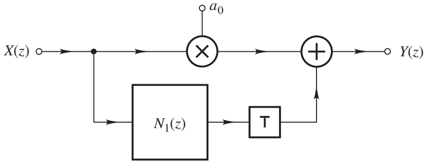
- There are two possible realizations as follows:





...

$$N_1(z) = \sum_{i=1}^{N} a_i z^{-i+1}$$

- Proceeding as before, we can now write

$$N_1(z) = a_1 + z^{-1} \sum_{i=2}^{N} a_i z^{-i+2} = a_1 + z^{-1} N_2(z)$$

where

$$N_2(z) = \sum_{i=2}^{N} a_i z^{-i+2}$$

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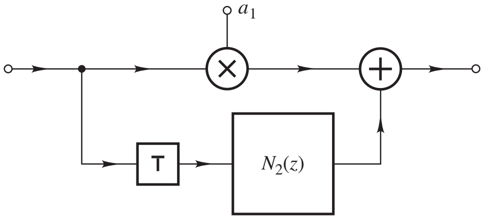
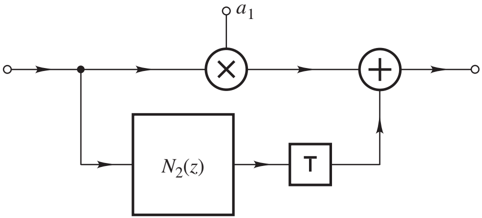
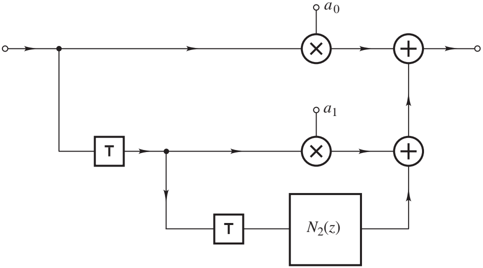
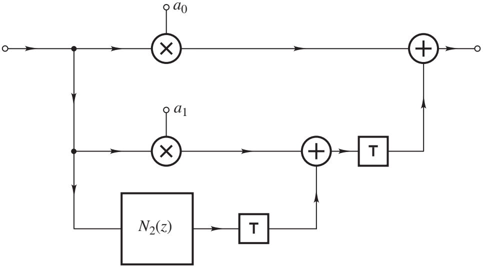

a_1
T
$N_2(z)$


a_1
$N_2(z)$
T

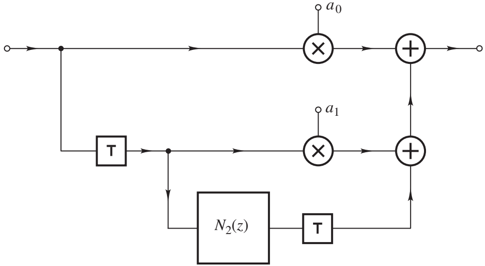
- Since there are two possible realizations for $N_1(z)$ it follows that there are four possible realizations for $N(z)$ as illustrated in the next two slides.

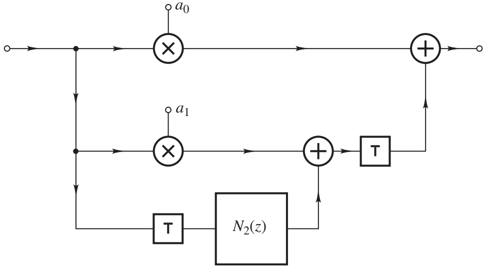
Direct Realization *Cont'd*





Direct Realization *Cont'd*





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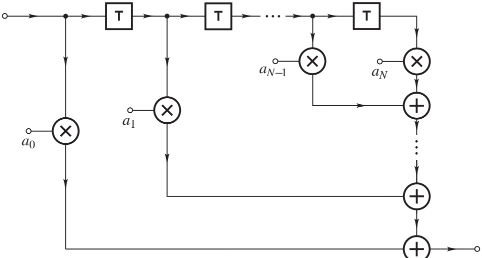
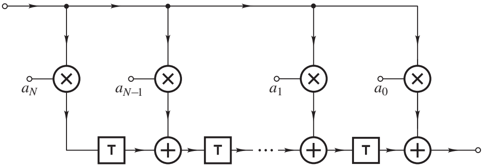
- Since there are N iterations in the procedure, and each iteration multiplies the number of distinct realizations by 2, a total of 2^N realizations are possible for $N(z)$.
- Three of the possible realizations are shown in the next two slides.

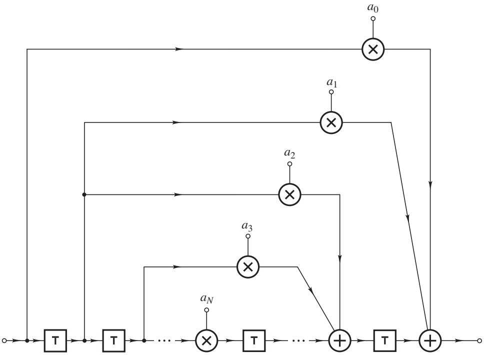
Direct Realization *Cont'd*



(*a*)

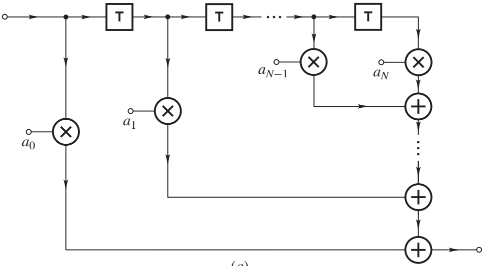


(*b*)



(c)

- Another interesting realization can be obtained from the first of the previous structures by noting that the outputs of multipliers a_0, a_1, . . . , a_N can be added in the reverse order without changing the output.



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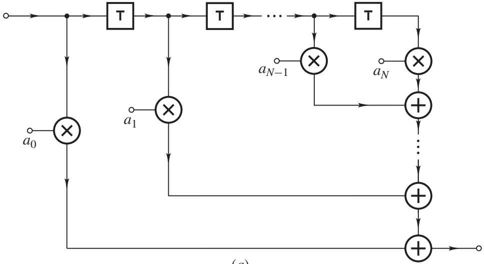

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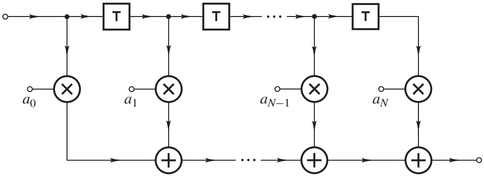

(*d*)

Direct Realization *Cont'd*

- The derived new structure is seen to be highly regular and it is, therefore, attractive for VLSI implementation (see Sec. 8.3).

- A causal *recursive* digital filter characterized by an Nth-order transfer function can be represented by the equation

$$\frac{Y(z)}{X(z)} = H(z) = \frac{N(z)}{D(z)} = \frac{N(z)}{1 + D'(z)}$$

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- We can write

$$Y(z) = N(z)X(z) - D'(z)Y(z)$$

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Direct Realization *Cont'd*

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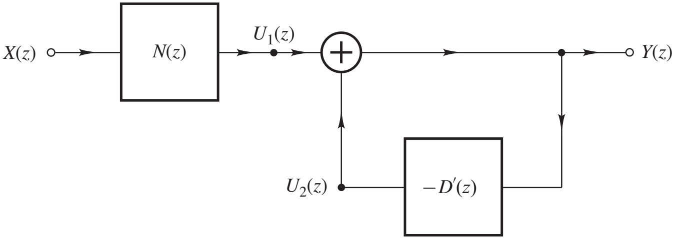
- Therefore, the recursive filter can be obtained by realizing $N(z)$ and $-D'(z)$ as nonrecursive filters and then connecting the resulting two nonrecursive filters as shown in the block diagram:



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the only difference between the two transfer functions is that

  - the coefficient for z^0 is zero in $-D'(z)$ and
  - its coefficients are the negatives of the coefficients of the denominator of the transfer function.

Example

Realize the transfer function

$$H(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}}$$

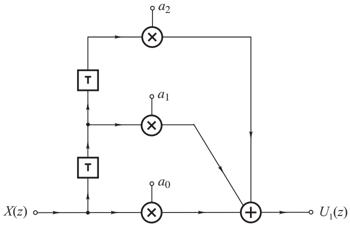
using the direct method.

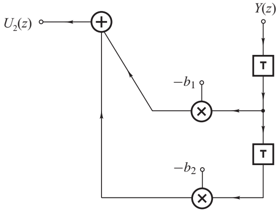
Solution The recursive structure can be obtained by realizing the nonrecursive transfer functions

$$N(z) = a_0 + a_1 z^{-1} + a_2 z^{-2} \quad \text{and} \quad -D'(z) = -b_1 z^{-1} - b_2 z^{-2}$$

Example *Cont'd*

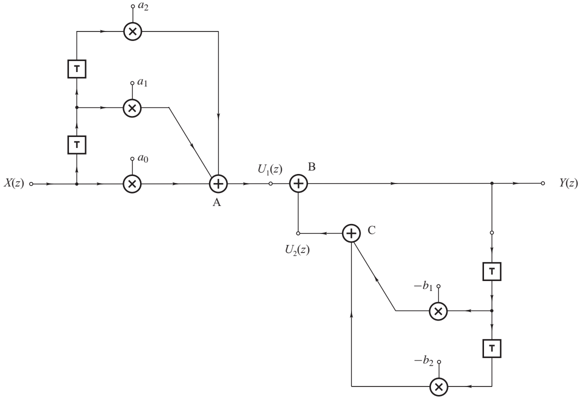
A pair of possible realizations for $N(z)$ and $-D'(z)$ are as shown:



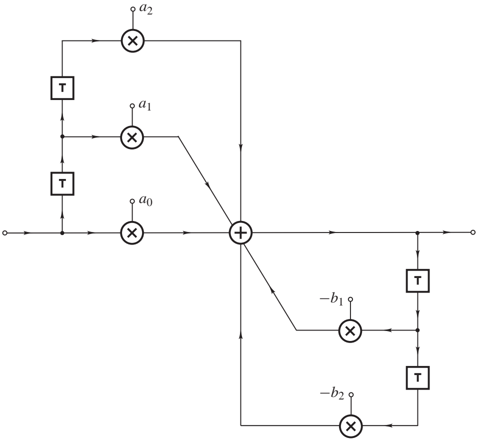


Example *Cont'd*

Connecting the realizations for $N(z)$ and $-D'(z)$ according to the block diagram, we get:



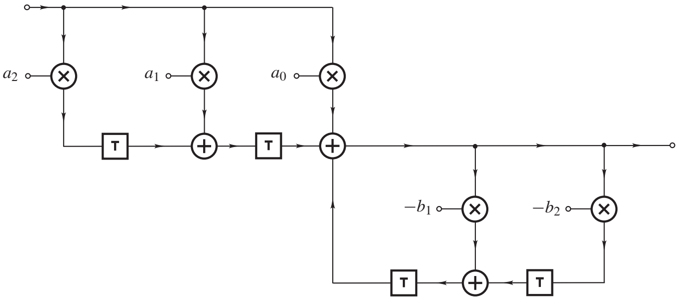
Now if we combine adders A, B, and C into a 5-input adder, we get the simplified structure shown:



(*a*)

Example *Cont'd*

Another possible direct realization that can be obtained using the same method is as follows:



(*b*)

Direct Canonic Realization

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- If the number of unit delays in an Nth-order digital-filter structure is N, then the structure is said to be *canonic*.
- The structures that can be obtained with the direct realization require $2N$ unit delays.
- However, one of the many possibilities can be rendered canonic through a simple technique, as will be shown.

- The equation

$$\frac{Y(z)}{X(z)} = H(z) = \frac{N(z)}{D(z)} = \frac{N(z)}{1 + D'(z)}$$

can be expressed

$$Y(z) = \frac{N(z)X(z)}{1 + D'(z)}$$

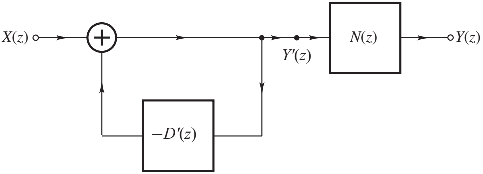
or

$$Y(z) = N(z)Y'(z) \quad \text{where} \quad Y'(z) = \frac{X(z)}{1 + D'(z)}$$

...

$$Y(z) = N(z)Y'(z) \quad \text{where} \quad Y'(z) = \frac{X(z)}{1 + D'(z)}$$

- These equations represent the system shown in the figure.



(a)

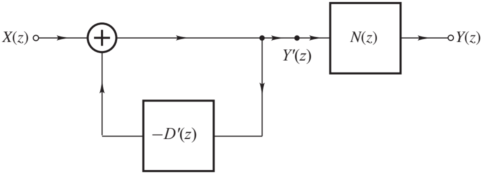
...

$$Y(z) = N(z)Y'(z) \quad \text{where} \quad Y'(z) = \frac{X(z)}{1 + D'(z)}$$

- These equations represent the system shown in the figure.
- Therefore, a digital-filter structure can be obtained by realizing the transfer functions

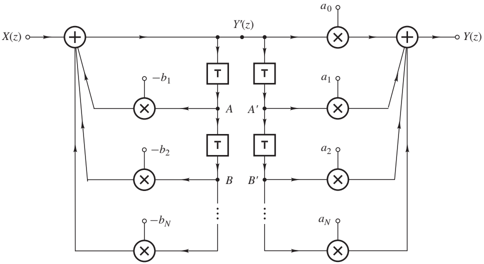
$$N(z) \quad \text{and} \quad \frac{1}{1 + D'(z)}$$

and then connecting the two realizations as shown:



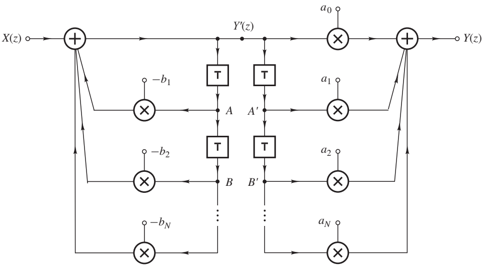
(a)

- Using the direct realization method, the configuration shown can be obtained.



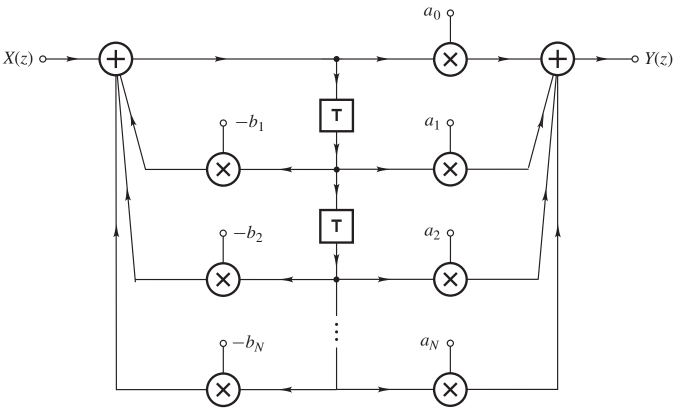
(b)

- Using the direct realization method, the configuration shown can be obtained.
- We note that the signals at nodes A', B' , . . . are exactly the same as the signals at nodes A, B, . . . and, therefore, nodes A, B, . . . can be connected to nodes A', B' , . . . and one set of unit delays can be eliminated.



(*b*)

Direct Canonic Realization *Cont'd*



This slide concludes the presentation.
Thank you for your attention.