

Chapter 5

THE APPLICATION OF THE Z TRANSFORM

5.5.4 Aliasing

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July 14, 2018

Introduction

- ◆ When a continuous-time signal that contains frequencies outside the baseband $-\omega_s/2 < \omega < \omega_s/2$ is sampled, a phenomenon known as *aliasing* will arise whereby frequencies outside the baseband impersonate frequencies within the baseband.

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- ◆ Aliasing is objectionable in practice and it must be prevented from occurring.
- ◆ This presentation explores the nature of aliasing in DSP.
- ◆ Aliasing can occur in other types of systems where sampled signals are involved, for example, in videos and movies, as will be demonstrated.

Aliasing in DSP

- ◆ Consider a continuous-time sinusoidal signal with frequency $\omega + \omega_s$ rad/s, i.e.,

$$x(t) = \sin[(\omega + \omega_s)t]$$

where $0 < \omega < \omega_s/2$.

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- ◆ By sampling the signal using a sampling frequency of ω_s rad/s, a discrete-time sinusoidal signal with frequency $\omega + \omega_s$ rad/s, i.e.,

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...

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◆ Signal $x(nT)$ can be expressed as

$$\begin{aligned}x(nT) &= \sin \omega nT \cos \omega_s nT + \cos \omega nT \sin \omega_s nT \\&= \sin \omega nT \cos \left(\frac{2\pi}{T} \cdot nT \right) + \cos \omega nT \sin \left(\frac{2\pi}{T} \cdot nT \right) \\&= \sin \omega nT \cos 2n\pi + \cos \omega nT \sin 2n\pi \\&= \sin \omega nT\end{aligned}$$

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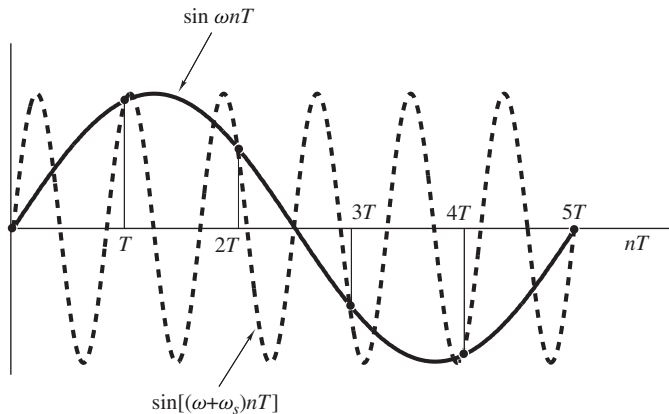
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- ◆ Evidently, sampling a sinusoidal signal of frequency $\omega + \omega_s$ will produce a discrete-time signal which is numerically identical to the discrete-time signal obtained by sampling a sinusoidal signal of frequency ω .
- ◆ In effect, the sampled version of signal $\sin[(\omega + \omega_s)t]$ will be *aliased* to the sampled version of $\sin(\omega)t$ or frequency $\omega + \omega_s$ will be *aliased* to ω .

Aliasing in DSP *Cont'd*



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- ◆ In this case, *frequency $\omega_s - \omega$ will be aliased to frequency $-\omega$.*

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- ◆ Aliasing can be prevented by *filtering out* all frequency components outside the baseband using a lowpass filter.
- ◆ Filtering out the high-frequency content of the signal with a lowpass filter will, of course, distort the signal but the distortion introduced is less serious than aliasing distortion (see Chap. 6).

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- ◆ For example, as the cowboy wagon accelerates into the sunset, the wheels of the wagon appear to accelerate in the forward direction, then reverse, slow down, stop momentarily, and after that they accelerate again in the forward direction. This is what should be seen, it's not an illusion!

Aliasing at the Movies *Cont'd*

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$$\omega_s = N\omega \quad \text{or} \quad \omega = \omega_s / N$$

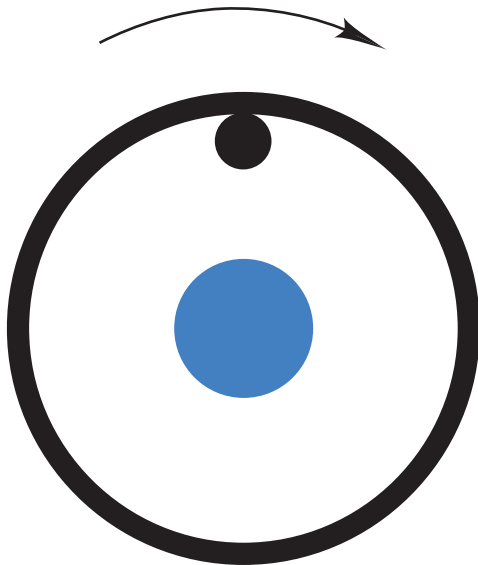
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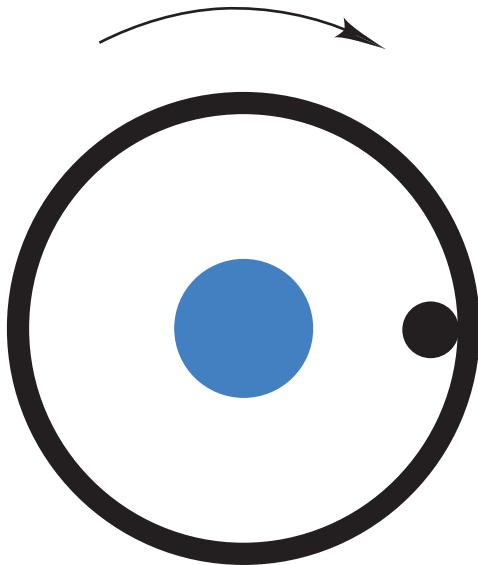
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- ◆ Rotation in the clockwise or counterclockwise direction is analogous to positive or negative frequency.

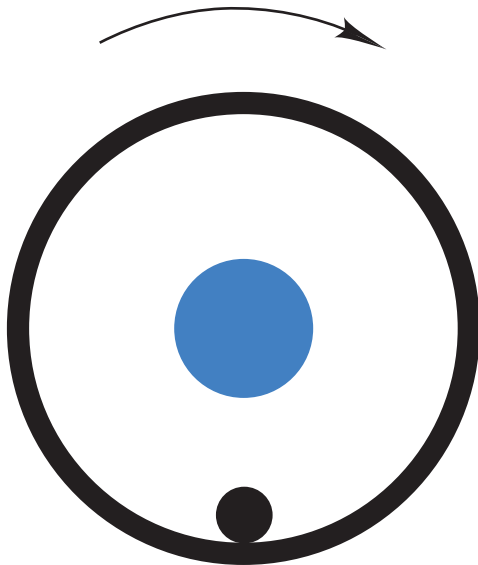
$\omega = \omega_s/4$, Frame #1



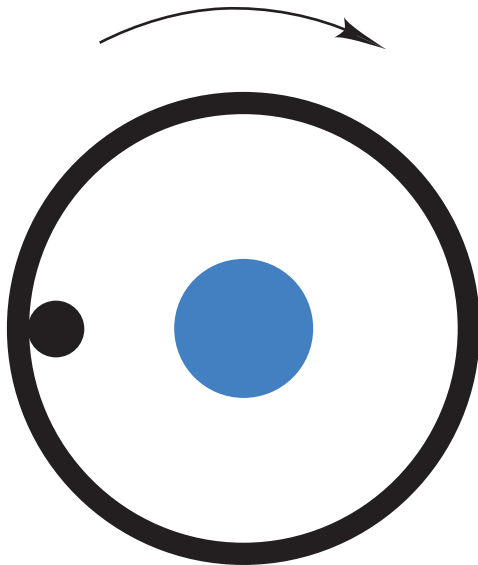
$\omega = \omega_s/4$, Frame #2



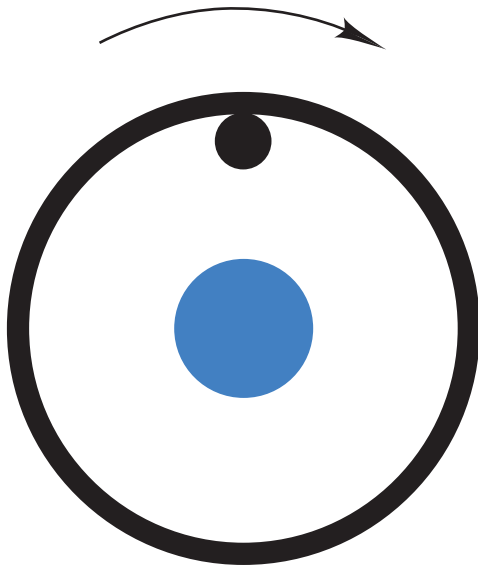
$\omega = \omega_s/4$, Frame #3



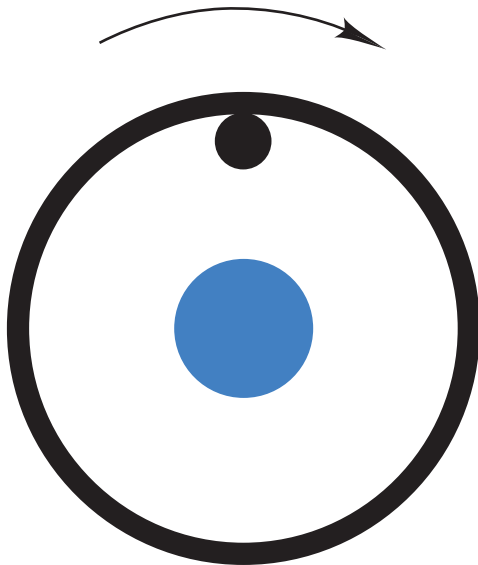
$\omega = \omega_s/4$, Frame #4



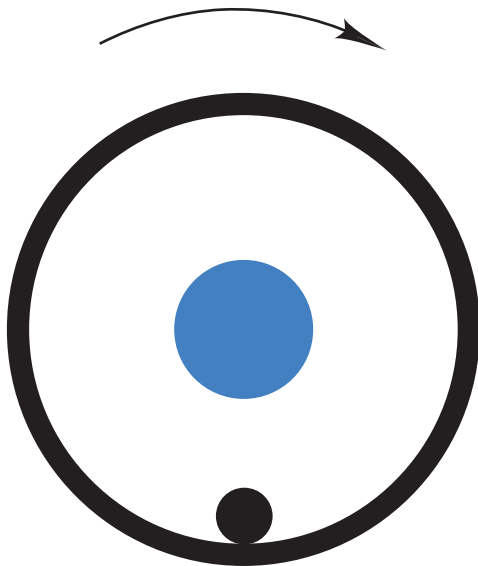
$\omega = \omega_s/4$, Frame #5



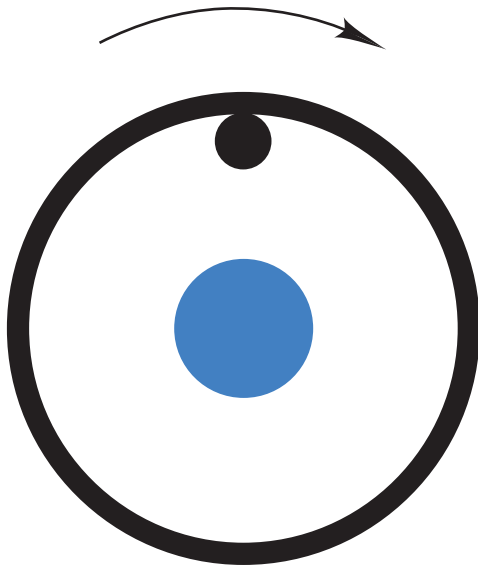
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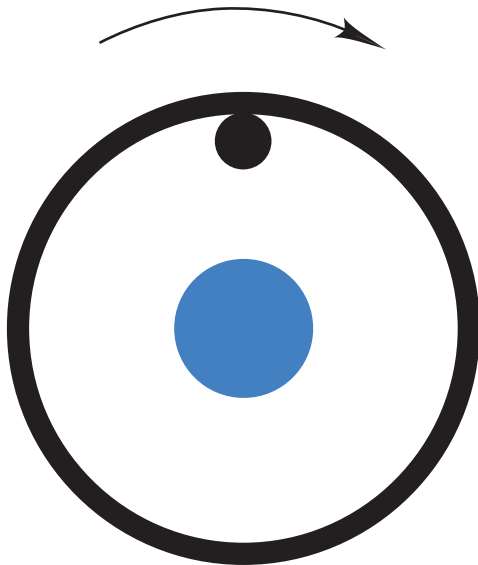
$\omega = \omega_s/2$, Frame #2



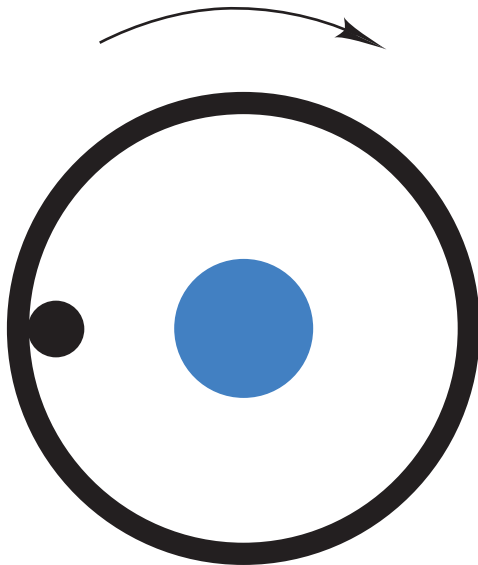
$\omega = \omega_s/2$, Frame #3



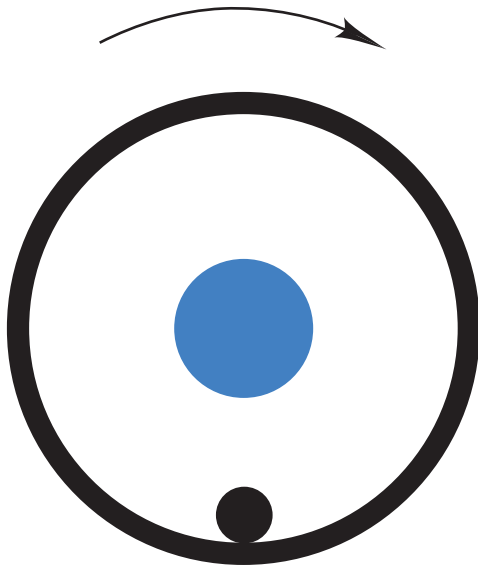
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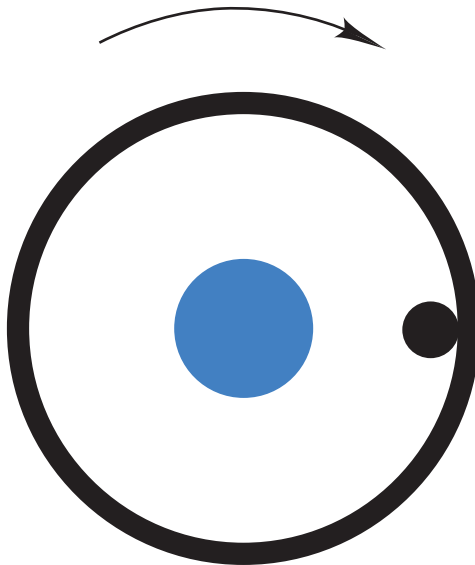
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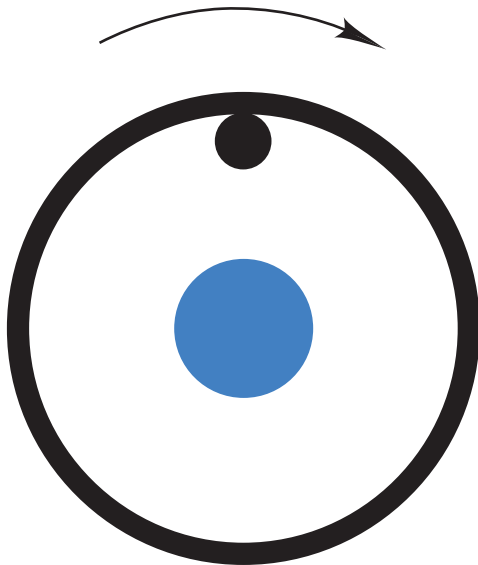
$\omega = 3\omega_s/4$, Frame #3



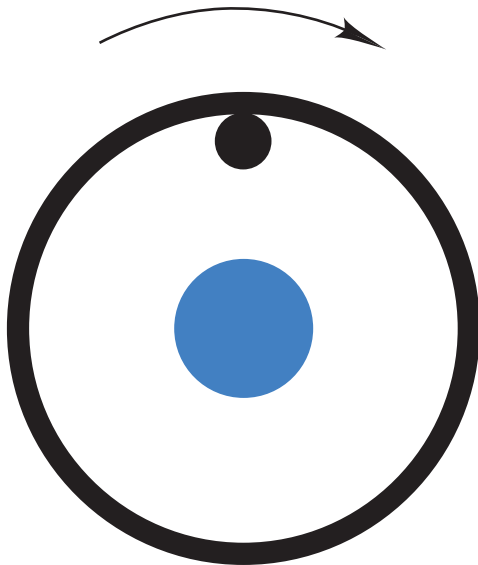
$\omega = 3\omega_s/4$, Frame #4



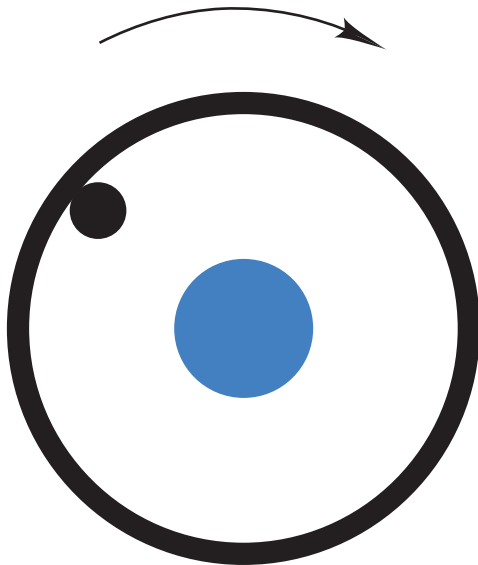
$\omega = 3\omega_s/4$, Frame #5



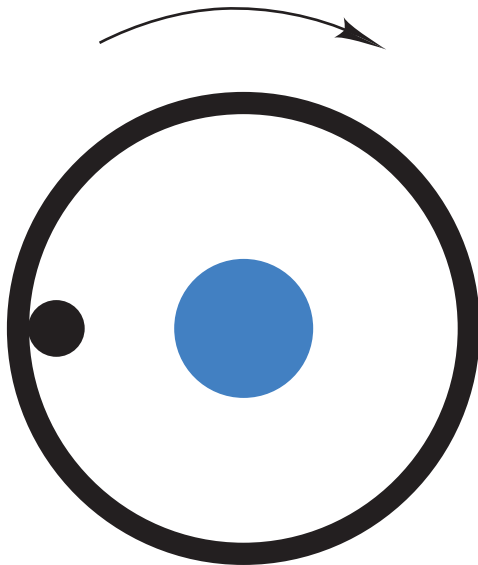
$\omega = 7\omega_s/8$, Frame #1



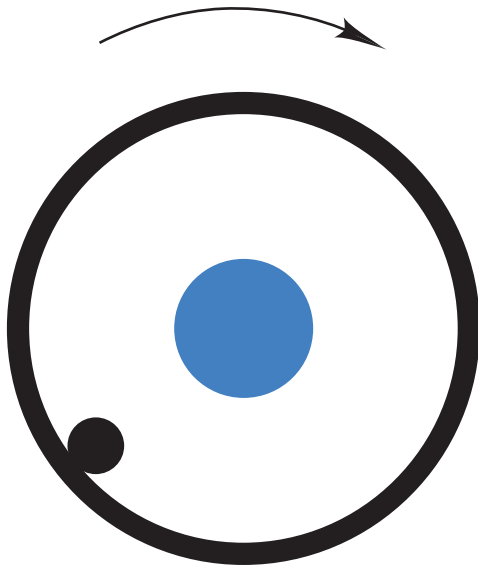
$\omega = 7\omega_s/8$, Frame #2



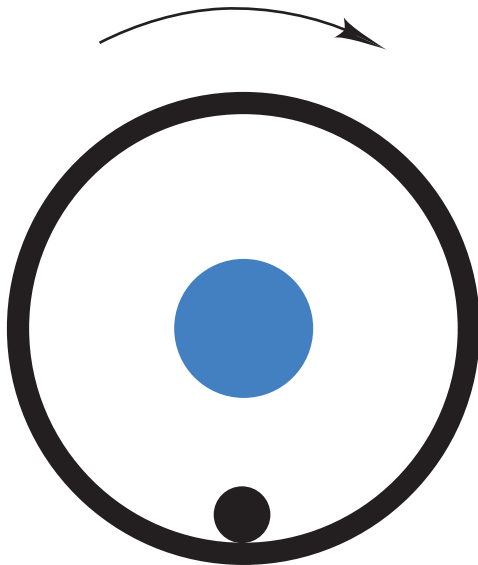
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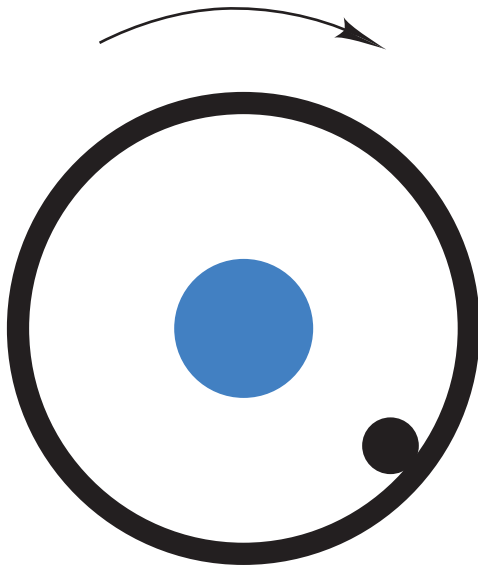
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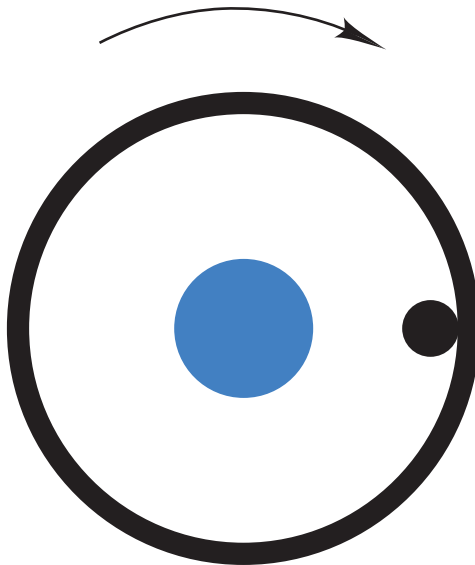
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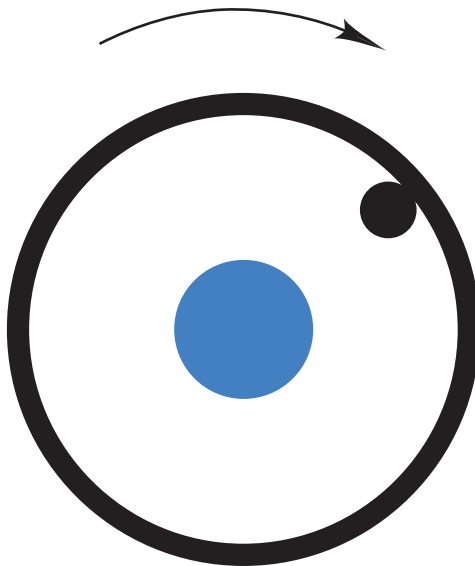
$\omega = 7\omega_s/8$, Frame #6



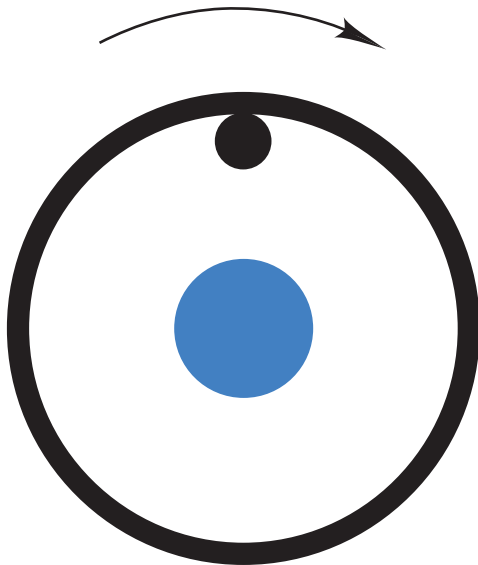
$\omega = 7\omega_s/8$, Frame #7



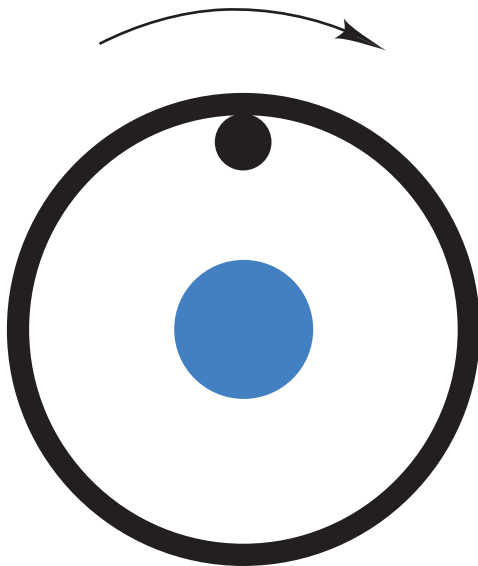
$\omega = 7\omega_s/8$, Frame #8



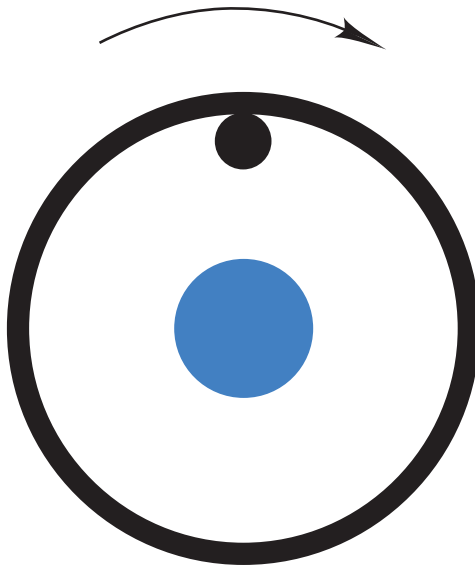
$\omega = 7\omega_s/8$, Frame #9



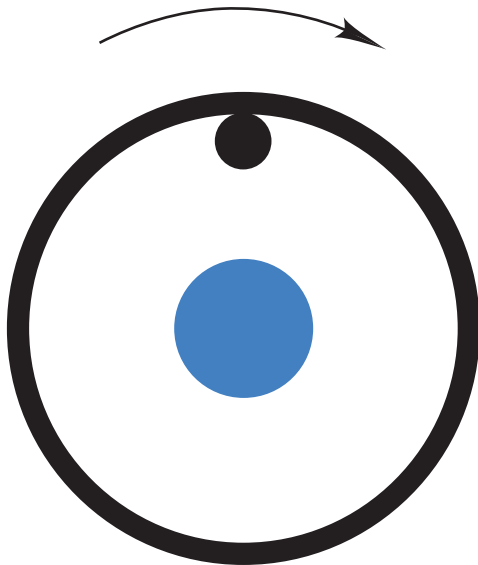
$\omega = \omega_s$, Frame #1



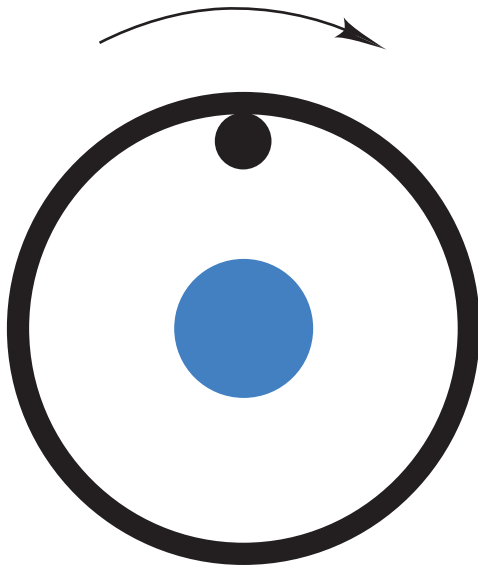
$\omega = \omega_s$, Frame #2



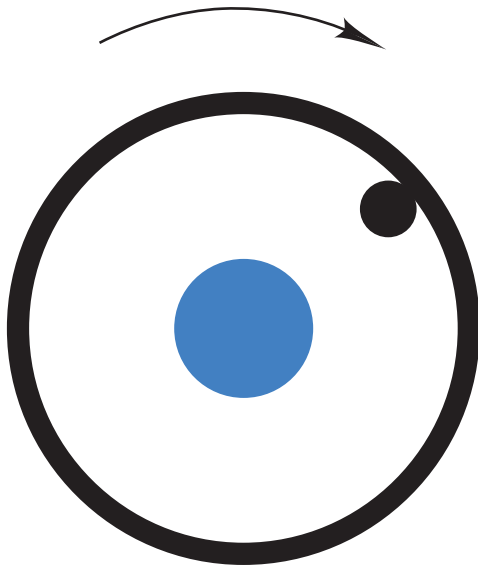
$\omega = \omega_s$, Frame #3



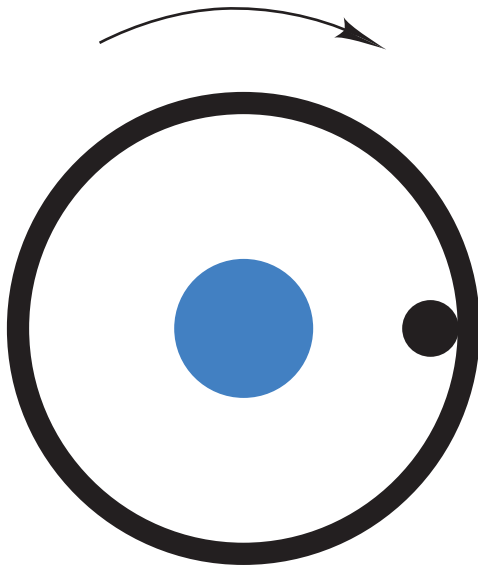
$\omega = 9\omega_s/8$, Frame #1



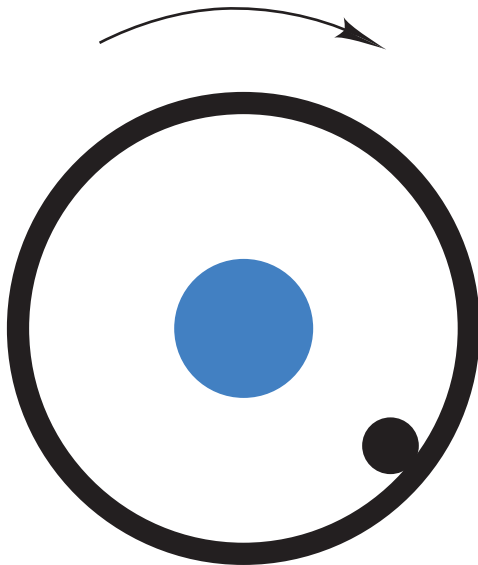
$\omega = 9\omega_s/8$, Frame #2



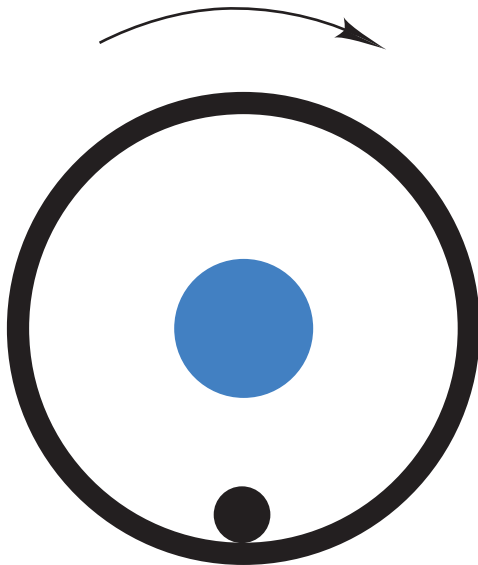
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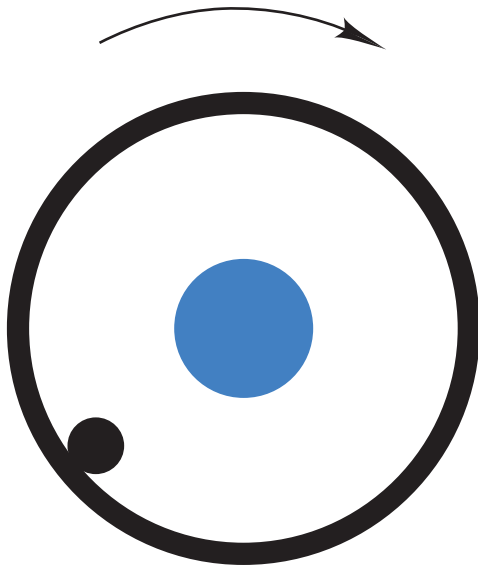
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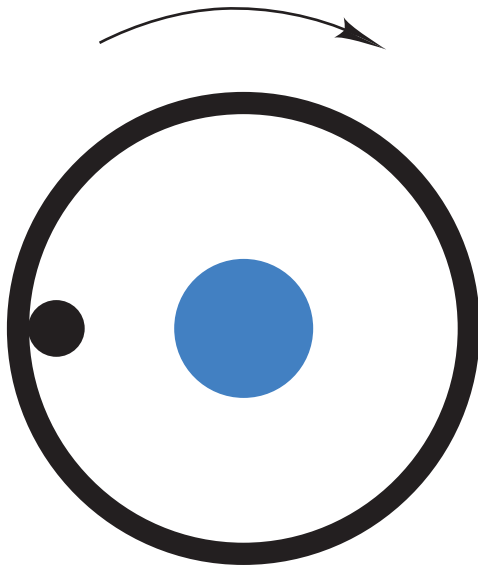
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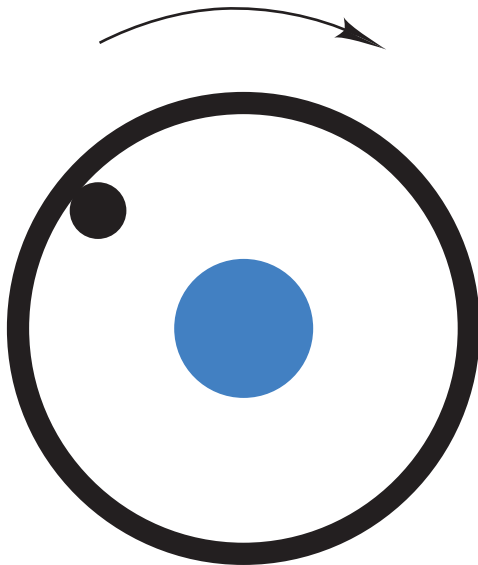
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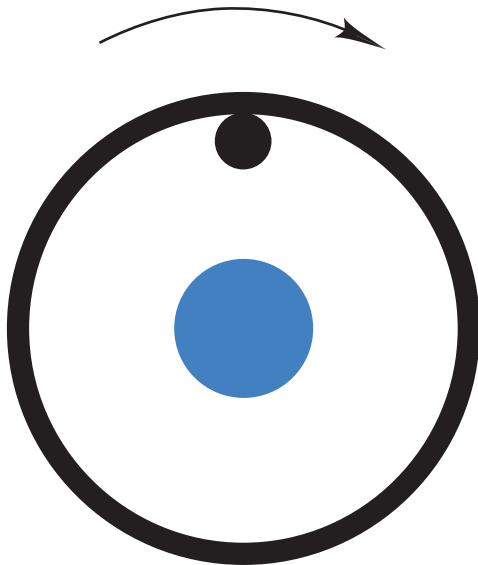
$\omega = 9\omega_s/8$, Frame #7



$\omega = 9\omega_s/8$, Frame #8



$\omega = 9\omega_s/8$, Frame #9



*This slide concludes the presentation.
Thank you for your attention.*