Chapter 6
THE SAMPLING PROCESS
6.9 Processing of Continuous-Time Signals Using Digital Filters
6.10 Practical A/D and D/A Converters

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Introduction

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- And by converting the processed impulse-modulated signal back to a continuous-time signal, a processed version of the continuous-time signal can be obtained.
- Thus *impulse-modulated filters can be used to process continuous-time signals*.

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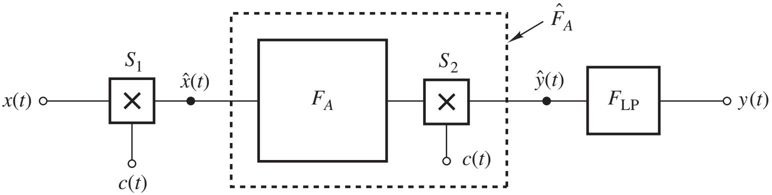
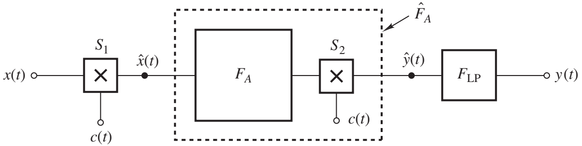
- The system is initially constructed using idealized A/D and D/A interfacing devices.
- Replacing the idealized A/D and D/A interfacing devices by practical ones tends to introduce certain imperfections.

  These imperfections are examined and methods for minimizing their effects are discussed.

- A discrete-time system that can be used to process continuous-time signals can be deduced by considering the filtering system shown:



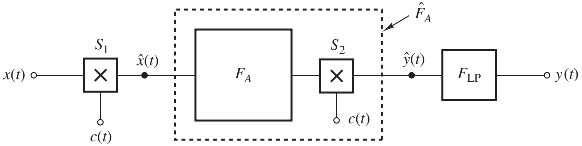
(a)



(*a*)

- F_A is an analog filter with a transfer function $H_A(s)$ and an impulse response

$$h_A(t) = \mathcal{L}^{-1} H_A(s)$$



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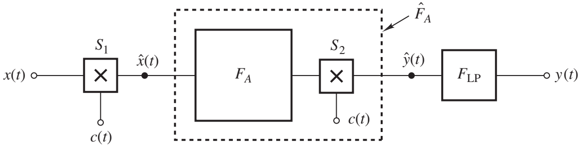
- F_{LP} is a lowpass filter with a frequency response

$$H_{LP}(j\omega) = \begin{cases} T^2 & \text{for } |\omega| < \omega_s/2 \\ 0 & \text{otherwise} \end{cases}$$



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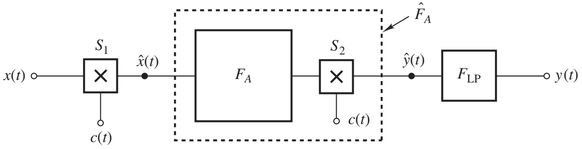
- Analog filter F_A along with impulse modulator S_2 constitute a so-called *impulse-modulated filter* designated as $\hat{F}_A$.

Impulse-Modulated Filter *Cont'd*



(*a*)

- Due to the presence of impulse modulator S_2, the impulse response of filter $\hat{F}_A$ will be an impulse modulated signal of the form

$$\hat{h}_A(t) = \sum_{n=0}^{\infty} h_A(nT)\delta(t - nT)$$

...

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- Applying Poisson's summation formula and then replacing $j\omega$ by s and e^{sT} by z, we get

$$\hat{H}_A(s) = H_D(z) = \frac{h_A(0+)}{2} + \frac{1}{T}\sum_{n=-\infty}^{\infty} H_A(s + jn\omega_s)$$

where

$$h_A(t) = \mathcal{L}^{-1}H_A(s), \quad h_A(0+) = \lim_{s\to\infty}[sH_A(s)]$$
$$H_D(z) = \mathcal{Z}h_A(nT), \quad z = e^{sT}$$

. . .

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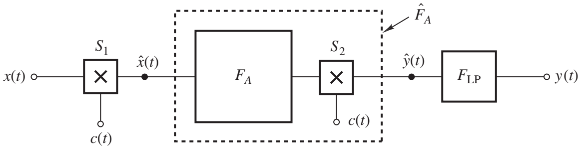

(*a*)

- Therefore, the impulse-modulated filter $\hat{F}_A$ can be represented by *a continuous-time transfer function* $\hat{H}_A(s)$ and *a discrete-time transfer function* $H_D(z)$.

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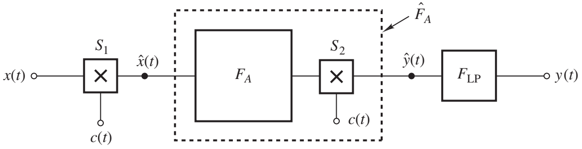
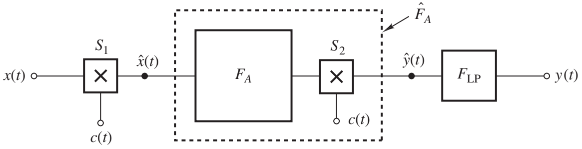
- To process continuous-time signals using digital filters.
- To design digital filters starting with analog filters.
- The processing of continuous-time signals using digital filters will be considered next.
- The design of digital filters on the basis of analog filters is considered in Chap. 11.



(a)

- The transfer function of the cascade arrangement of the impulse-modulated filter and the lowpass filter is the product of their individual transfer functions, i.e., $\hat{H}_A(s)H_{LP}(s)$.

Processing of Continuous-Time Signals



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- The transfer function of the cascade arrangement of the impulse-modulated filter and the lowpass filter is the product of their individual transfer functions, i.e., $\hat{H}_A(s)H_{LP}(s)$.
- Hence the Laplace transform of $y(t)$ can be obtained as

$$Y(s) = \hat{H}_A(s)H_{LP}(s)\hat{X}(s)$$

...

$$Y(s) = \hat{H}_A(s) H_{LP}(s) \hat{X}(s)$$

- Therefore, the frequency spectrum of the output signal is obtained as

$$Y(j\omega) = \hat{H}_A(j\omega) H_{LP}(j\omega) \hat{X}(j\omega)$$

where

$$\hat{H}_A(j\omega) = \frac{h_A(0+)}{2} + \frac{1}{T} \sum_{n=-\infty}^{\infty} H_A(j\omega + jn\omega_s)$$

$$H_{LP}(j\omega) = \begin{cases} T^2 & \text{for } |\omega| < \omega_s/2 \\ 0 & \text{otherwise} \end{cases}$$

$$\hat{X}(j\omega) = \frac{x(0+)}{2} + \frac{1}{T} \sum_{n=-\infty}^{\infty} X(j\omega + jn\omega_s)$$

- If we now assume that the input signal, $x(t)$, and the impulse response of the analog filter, $h_A(t)$, are bandlimited such that

$$x(0+) = 0 \quad \text{and} \quad X(j\omega) = H_A(j\omega) = 0 \quad \text{for} \ \ |\omega| \geq \omega_s/2$$

then no aliasing can occur in $\hat{X}(j\omega)$ or $\hat{H}_A(j\omega)$ and thus

$$\hat{X}(j\omega) = \frac{1}{T}X(j\omega) \quad \text{and} \quad \hat{H}_A(j\omega) = \frac{1}{T}H_A(j\omega) \quad \text{for} \ \ |\omega| < \frac{\omega_s}{2}$$

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- Substituting these results in

$$Y(j\omega) = \hat{H}_A(j\omega)H_{LP}(j\omega)\hat{X}(j\omega)$$

we get

$$Y(j\omega) = H_A(j\omega)X(j\omega)$$

and by letting $j\omega = s$, we have

$$Y(s) = H_A(s)X(s)$$

(See textbook for details.)

...

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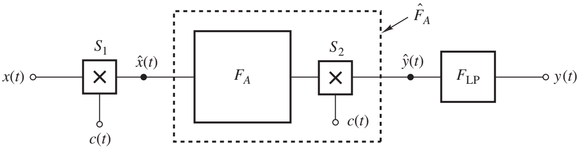
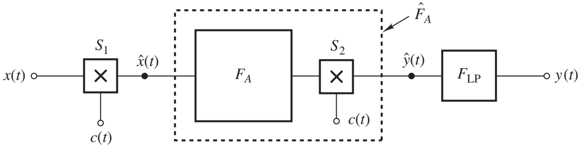


(*a*)

- This is rather interesting: Under the stated assumptions, *the filtering scheme shown behaves exactly like analog filter F_A* except that it uses several additional components, i.e., two impulse modulators and a lowpass analog filter.



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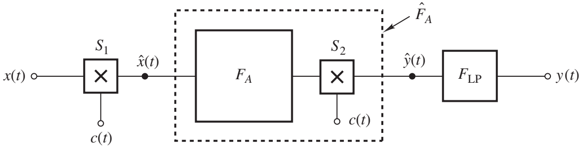
- However, something important has been achieved: Since an impulse-modulated filter *can be represented by a discrete-time transfer function, it can be implemented in the form of a digital filter.*



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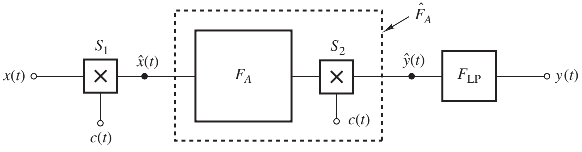
- However, something important has been achieved: Since an impulse-modulated filter *can be represented by a discrete-time transfer function, it can be implemented in the form of a digital filter.*

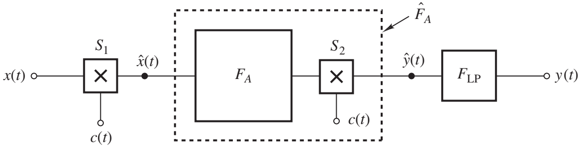
- By replacing the impulse-modulated filter by a digital filter, a filtering scheme can be obtained that can be used to process continuous-time signals, which is quite nice.



(*a*)

- Since the signals in impulse-modulated filters are analog signals and those in digital filters are digital signals in binary form, suitable interfacing devices have to be used.

Processing of Continuous-Time Signals *Cont'd*



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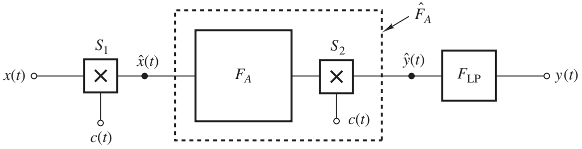
- Since the signals in impulse-modulated filters are analog signals and those in digital filters are digital signals in binary form, suitable interfacing devices have to be used.
- At the output of impulse modulator S_1, we need to add an A/D converter and at the output of the digital filter we need to add a D/A converter.



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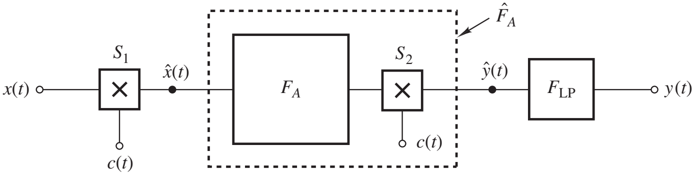
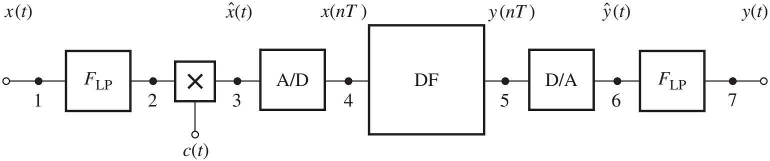
- At the output of impulse modulator S_1, we need to add an A/D converter and at the output of the digital filter we need to add a D/A converter.
- We must also add a lowpass filter at the input to ensure that the input signal is bandlimited in order to prevent aliasing.



S_1
$x(t)$
$c(t)$
$\hat{x}(t)$
F_A
S_2
$c(t)$
$\hat{F}_A$
$\hat{y}(t)$
F_{LP}
$y(t)$

(*a*)



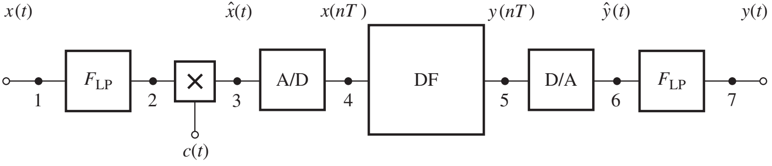
$x(t)$
$\hat{x}(t)$
$x(nT)$
$y(nT)$
$\hat{y}(t)$
$y(t)$
1
F_{LP}
2
$c(t)$
3
A/D
4
DF
5
D/A
6
F_{LP}
7

(*b*)

The DSP system shown is used to process the periodic signal given by

$$x(t) = \begin{cases} \sin \omega_0 t & \text{for } 0 \le t \le T_0/2 \\ 0 & \text{for } -T_0/2 \le t \le 0 \end{cases}$$

where $\omega_0 = 2\pi/T_0$.



(b)

Example *Cont'd*

The lowpass filters are characterized by

$$H_{LP}(j\omega) = \begin{cases} 1 & \text{for } 0 \leq |\omega| < 6\omega_0 \\ 0 & \text{otherwise} \end{cases}$$

The digital filter is a bandpass filter with a baseband frequency response

$$H_D(e^{j\omega T}) = \begin{cases} T & \text{for } 0.95\omega_0 < |\omega| < 1.05\omega_0 \\ 0 & \text{otherwise} \end{cases}$$

Assuming that $\omega_s = 12\omega_0$, find the time- and frequency-domain representations of the signals at nodes $1, 2, \ldots, 7$.

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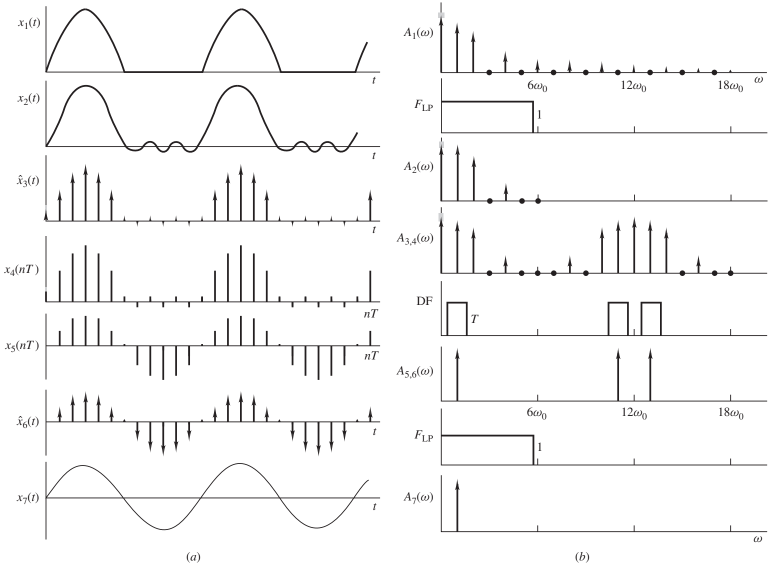
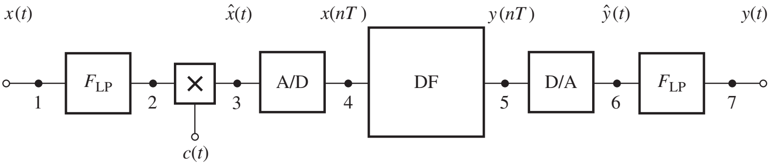
Solution The time- and frequency-domain representations of the signals are illustrated in the next slide. See textbook for the formulas.

Example *Cont'd*



Practical Considerations – Input Interface



(b)

- The input interface consists of an impulse modulator followed by a special type of A/D converter that will sense the strengths of a series of impulses and produce a series of binary numbers.

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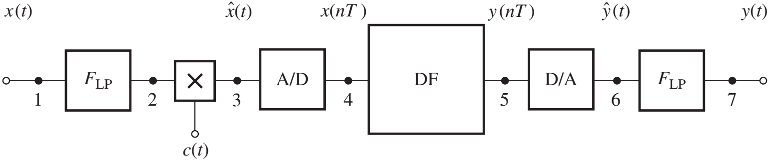
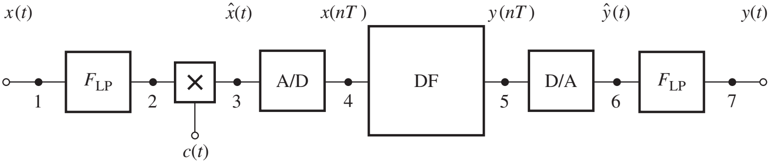


(b)

- The input interface consists of an impulse modulator followed by a special type of A/D converter that will sense the strengths of a series of impulses and produce a series of binary numbers.

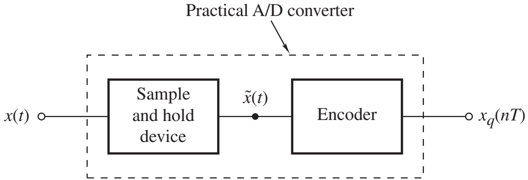
- Since the strengths of the impulses are equal to the amplitude values of the input signal at the sampling instants, a much more practical input interface can be constructed by using a sample-and-hold circuit followed by an encoder.

• • •

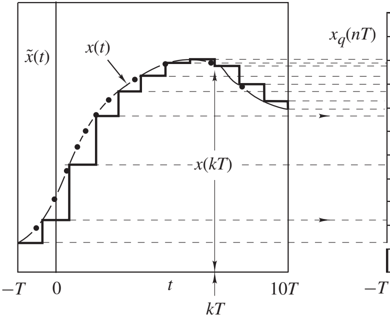


(*b*)

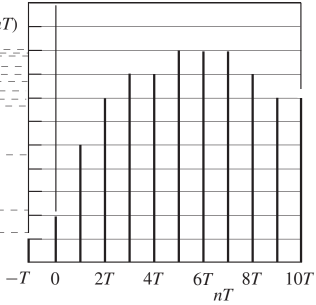
- Recall that a signal must be quantized before it can be converted into a binary signal.
- Therefore, quantization error is introduced.


Practical A/D converter
$x(t)$
Sample and hold device
$\tilde{x}(t)$
Encoder
$x_q(nT)$

(*a*)

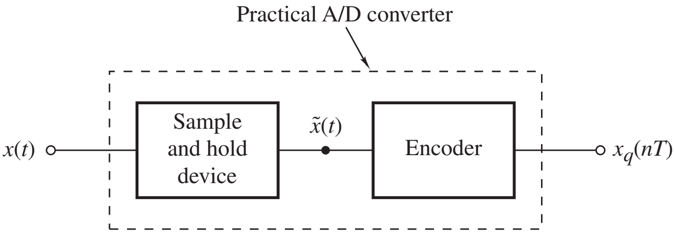

$\tilde{x}(t)$
$x(t)$
$x(kT)$
$-T$
0
t
$10T$
kT

(*b*)

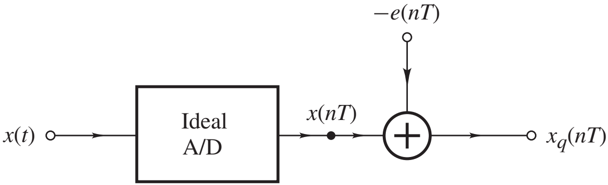

$x_q(nT)$
$-T$
0
$2T$
$4T$
$6T$
$8T$
$10T$
nT

(*c*)

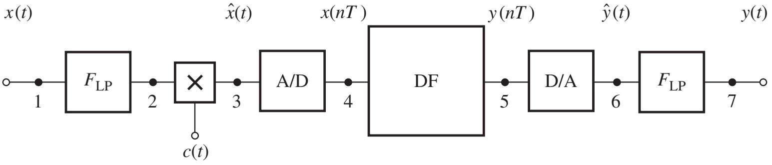
Input Interface – Model



(a)



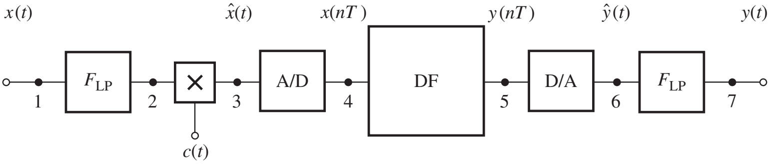
(d)



(b)

- The DSP system will operate correctly only if the output of the D/A converter is an impulse-modulated signal which is a sequence of analog impulses.

Practical Considerations – Output Interface



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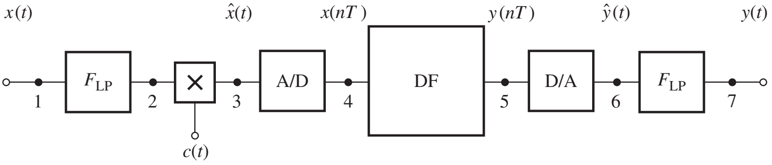
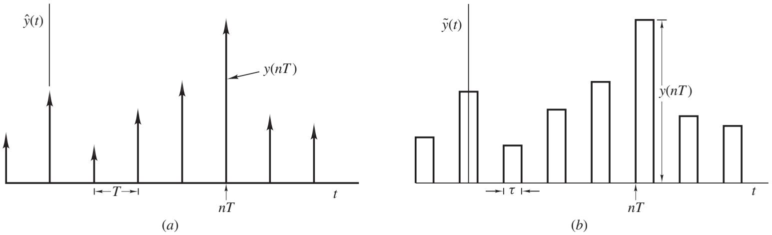
- The DSP system will operate correctly only if the output of the D/A converter is an impulse-modulated signal which is a sequence of analog impulses.
- Recall that analog impulses are supposed to be very thin and very tall pulses.

Practical Considerations – Output Interface

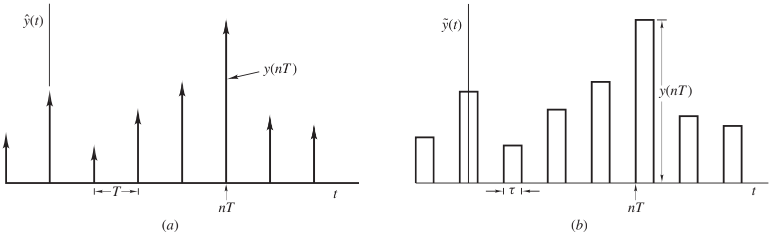


(b)

- The DSP system will operate correctly only if the output of the D/A converter is an impulse-modulated signal which is a sequence of analog impulses.
- Recall that analog impulses are supposed to be very thin and very tall pulses.
- However, practical D/A converters will produce pulses that are neither particularly tall nor particularly thin, and this causes a somewhat serious problem.



- The output of the D/A converter is in theory an impulse-modulated signal as shown in figure (*a*) but in practice it assumes the form shown in figure (*b*).



- The output of the D/A converter is in theory an impulse-modulated signal as shown in figure (*a*) but in practice it assumes the form shown in figure (*b*).
- Such a waveform can be represented by the equation

$$\tilde{y}(t) = \sum_{n=-\infty}^{\infty} y(nT)p_\tau(t - nT)$$

...

$$\tilde{y}(t) = \sum_{n=-\infty}^{\infty} y(nT)p_\tau(t - nT)$$

- From the table of Fourier transforms,

$$\mathcal{F}p_\tau(t) = \frac{2\sin(\omega\tau/2)}{\omega}$$

Output Interface *Cont'd*

...

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- By using the time-shifting theorem, we obtain

$$\mathcal{F}p_\tau(t-nT) = \frac{2\sin(\omega\tau/2)}{\omega}e^{-j\omega nT}$$

...

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$$\mathcal{F}p_\tau(t - nT) = \frac{2\sin(\omega\tau/2)}{\omega}e^{-j\omega nT}$$

- Hence the Fourier transform of $\tilde{y}(t)$ can be obtained as

$$\tilde{Y}(j\omega) = \sum_{n=-\infty}^{\infty} y(nT)\mathcal{F}p_\tau(t - nT) = \frac{2\sin(\omega\tau/2)}{\omega}\sum_{n=-\infty}^{\infty} y(nT)e^{-j\omega nT}$$

Output Interface *Cont'd*

. . .

$$\tilde{Y}(j\omega) = \frac{2\sin(\omega\tau/2)}{\omega} \sum_{n=-\infty}^{\infty} y(nT)e^{-j\omega nT}$$

- Alternatively,

$$\tilde{Y}(j\omega) = H_p(j\omega)\hat{Y}(j\omega)$$

where

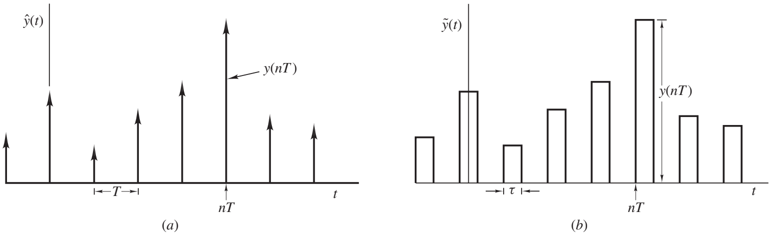
$$H_p(j\omega) = \frac{\tau\sin(\omega\tau/2)}{\omega\tau/2} \quad \text{and} \quad \hat{Y}(j\omega) = \sum_{n=-\infty}^{\infty} y(nT)e^{-j\omega nT}$$

- Since

$$\hat{Y}(j\omega) = \sum_{n=-\infty}^{\infty} y(nT)e^{-j\omega nT} = \mathcal{F} \sum_{n=-\infty}^{\infty} y(nT)\delta(t-nT) = \mathcal{F}\hat{y}(t)$$

it follows that $\hat{Y}(j\omega)$ is the frequency spectrum of the impulse-modulated signal that should appear at the output of an ideal D/A converter.

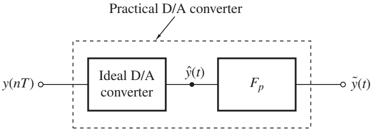
Output Interface *Cont'd*



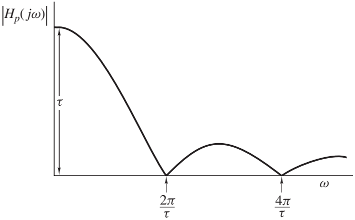
- Therefore, we conclude that the frequency spectrum of the output of a practical D/A converter (the signal shown in figure (b) can be regarded as a corrupted version of the spectrum of the output of an ideal D/A converter (the signal shown in figure (a), and it is given by

$$\tilde{Y}(j\omega) = H_p(j\omega)\hat{Y}(j\omega) \quad \text{where} \quad H_p(j\omega) = \frac{\tau \sin(\omega\tau/2)}{\omega\tau/2}$$

- In effect, *a practical D/A converter can be modelled in terms of an ideal D/A converter followed by a parasitic filter F_p*, as shown in figure (c):



(c)

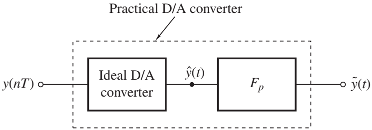


(d)

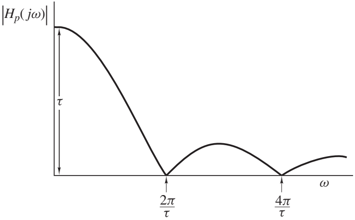
- The amplitude response of the parasitic filter is given by

$$|H_p(j\omega)| = \left| \frac{\tau \sin(\omega\tau/2)}{\omega\tau/2} \right|$$

and is illustrated in figure (d).



(c)



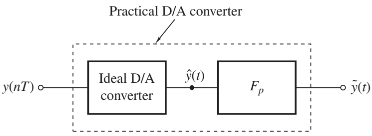
(d)

- The amplitude response of the parasitic filter is given by

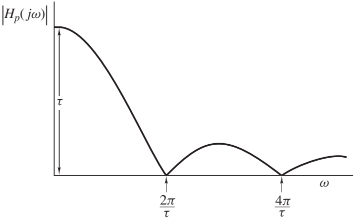
$$|H_p(j\omega)| = \left|\frac{\tau \sin(\omega\tau/2)}{\omega\tau/2}\right|$$

and is illustrated in figure (d).

- It tends to distort the amplitude response of the digital filter by introducing amplitude distortion, often referred to as *sinc distortion.*

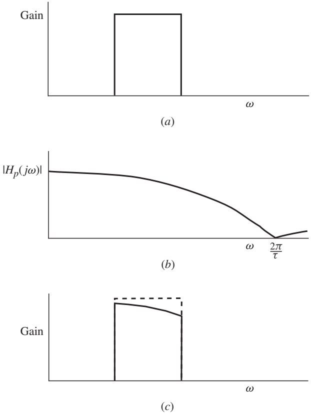


(c)

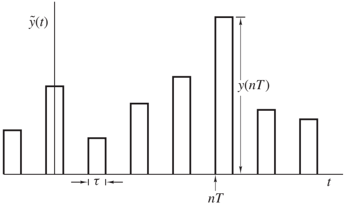


(d)

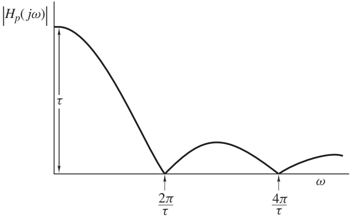
- The effect of sinc distortion on the response of a bandpass digital filter is illustrated below.



- Sinc distortion can be reduced by reducing the width of the pulses τ.



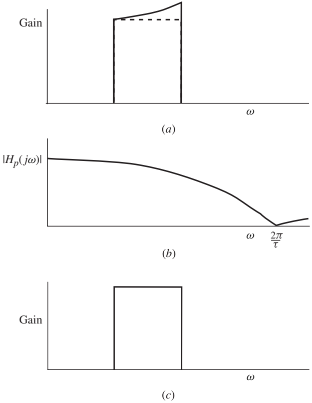
(b)



(d)

Output Interface *Cont'd*

- Another way to reduce sinc distortion is to design the digital filter with predistorted amplitude response as shown so as to compensate for the sinc distortion.



- See Example 6.6 for detailed calculations on the effects of sinc distortion in the case where the digital filter is a bandpass filter.

This slide concludes the presentation.
Thank you for your attention.