

LECTURE #5: ELECTRICAL PROPERTIES OF SOLIDS

Basic question: How current conduction takes place in the different solids?

The answers are linked to:

- Relationship between charge flow and current
- How charged carriers are formed
- Effects of a voltage gradient and a temperature gradient on the charged carriers
- Conduction properties of metals, semiconductors and insulators

Conduction in Solids

- Current conduction in a solid is linked to the movement of charges, i.e., the flow of the electrons and the holes
- A hole may be viewed as a missing electron (holes are only important in semiconductors)

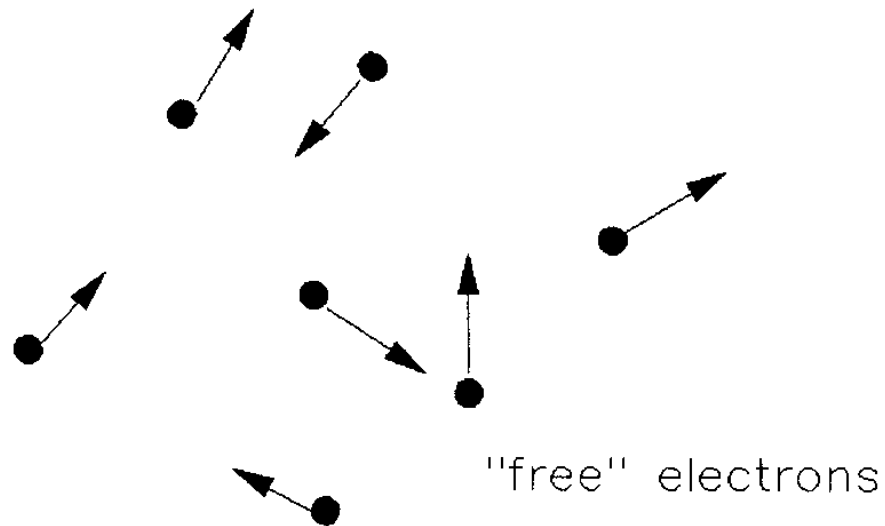


Fig.II–1: Random motion of "free" electrons

- Electrons and holes move in the presence of an electric potential gradient - they are referred to as the charged carriers or simply carriers

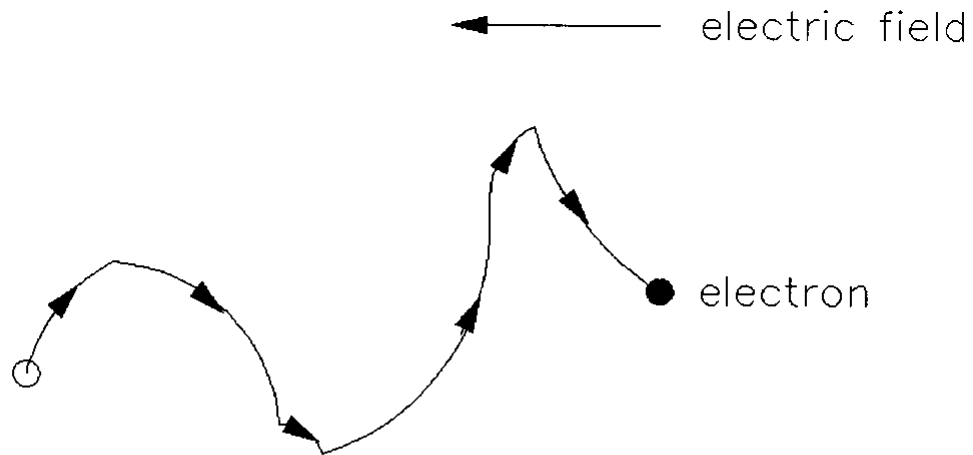


Fig.II—3: Trajectory of an electron under an applied electric field

- Under the same potential gradient, electrons and holes move in opposite directions
- Current is defined as the charge flow per unit time – it has the unit of Coulomb/s or Ampere

Mathematical formulation of the conduction process

Ohm's law defines the relationship between a current and the potential difference. It is given by:

$$V = IR \quad (8)$$

where V is the potential difference, I is the current, and R is the resistance of the sample.

For a rectangular sample, resistance can be expressed as:

$$R = \rho L/A_{cs} \quad (9)$$

where ρ is the resistivity of the sample, L is the length, and A_{cs} is the cross-sectional area.

Furthermore,

$$\rho = 1/(n q \mu_n + p q \mu_p) \quad (10)$$

where n is the electron density, q is the electron charge, μ_n is the electron mobility, p is the hole density, and μ_p is the hole mobility.

Mobility is usually a constant and is defined as: $\mu = |v|/E$, where $|v|$ is the magnitude of the carrier velocity, and E is the electric field.

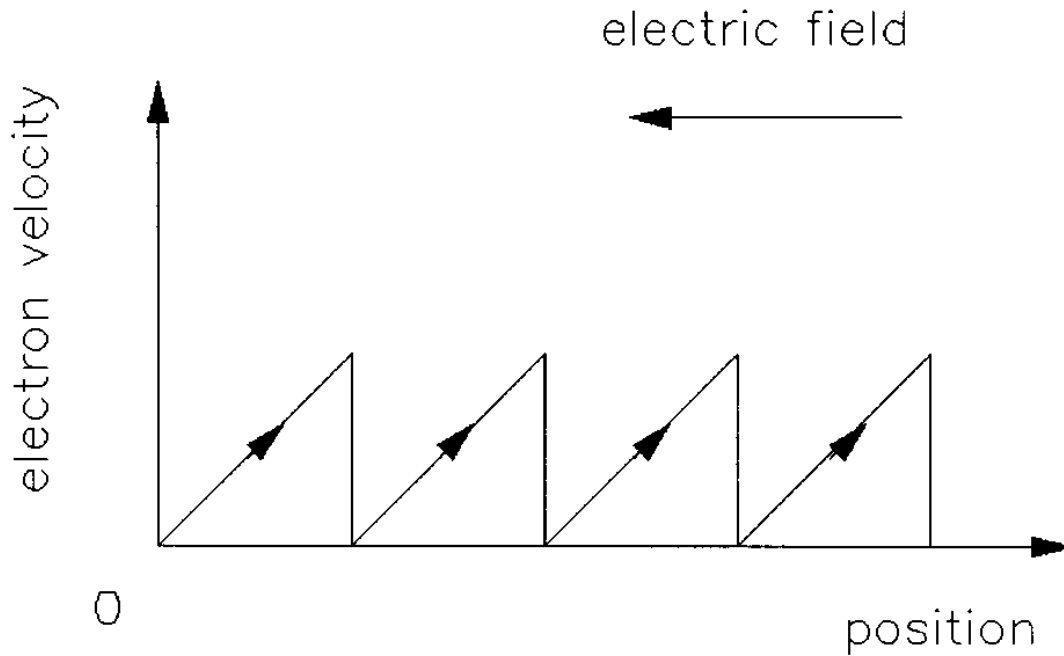


Fig.II–4: Electron velocity in an electric field

The conductivity (σ) of a solid is the inverse of ρ and it is given by:

$$\sigma = n q \mu_n + p q \mu_p \quad (11)$$

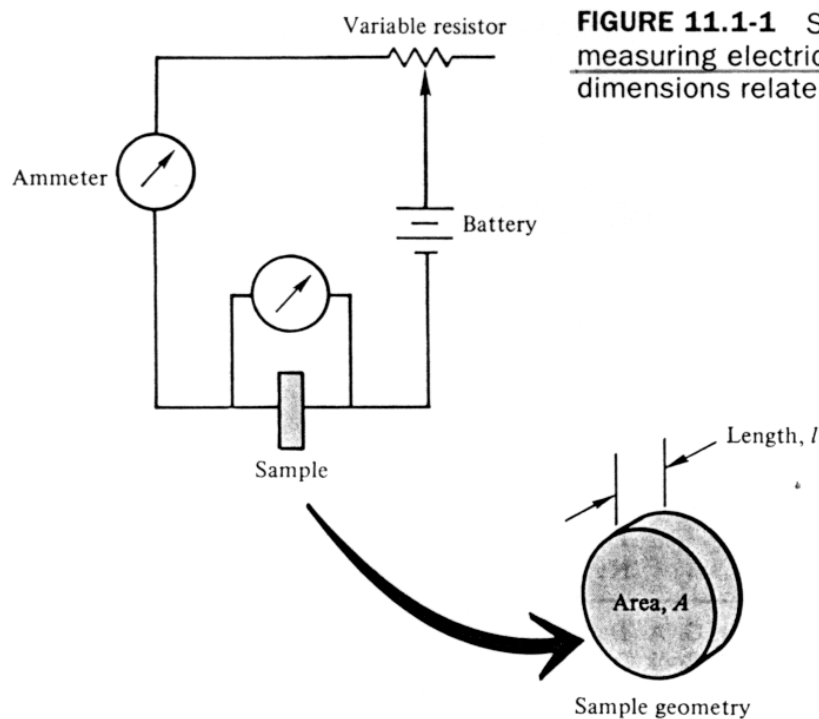


FIGURE 11.1-1 Schematic of a circuit for measuring electrical conductivity. Sample dimensions relate to Equation 11.1-2.

TABLE 11.1-1 Electrical Conductivities of Some Materials at Room Temperature

Conducting Range	Material	Conductivity, σ ($\Omega^{-1} \cdot \text{m}^{-1}$)
<u>Conductors</u>	Aluminum (annealed)	35.36×10^6
	Copper (annealed standard)	58.00×10^6
	Iron (99.99 + %)	10.30×10^6
	Steel (wire)	$5.71\text{--}9.35 \times 10^6$
	Germanium (high purity)	2.0
<u>Semiconductors</u>	Silicon (high purity)	0.40×10^{-3}
	Lead Sulfide (high purity)	38.4
	Aluminum oxide	$10^{-10}\text{--}10^{-12}$
<u>Insulators</u>	Borosilicate glass	10^{-13}
	Polyethylene	$10^{-13}\text{--}10^{-15}$
	Nylon 66	$10^{-12}\text{--}10^{-13}$

Source: Data from C. A. Harper, ed., *Handbook of Materials and Processes for Electronics*, McGraw-Hill Book Company, New York, 1970; and J. K. Stanley, *Electrical and Magnetic Properties of Metals*, American Society for Metals, Metals Park, Ohio, 1963.

Relationship between Energy Bands (levels) and Carrier Densities

- Energy levels will transform into energy bands as many atoms come together to form a solid
- Energy bands are in essence very closely-spaced energy levels

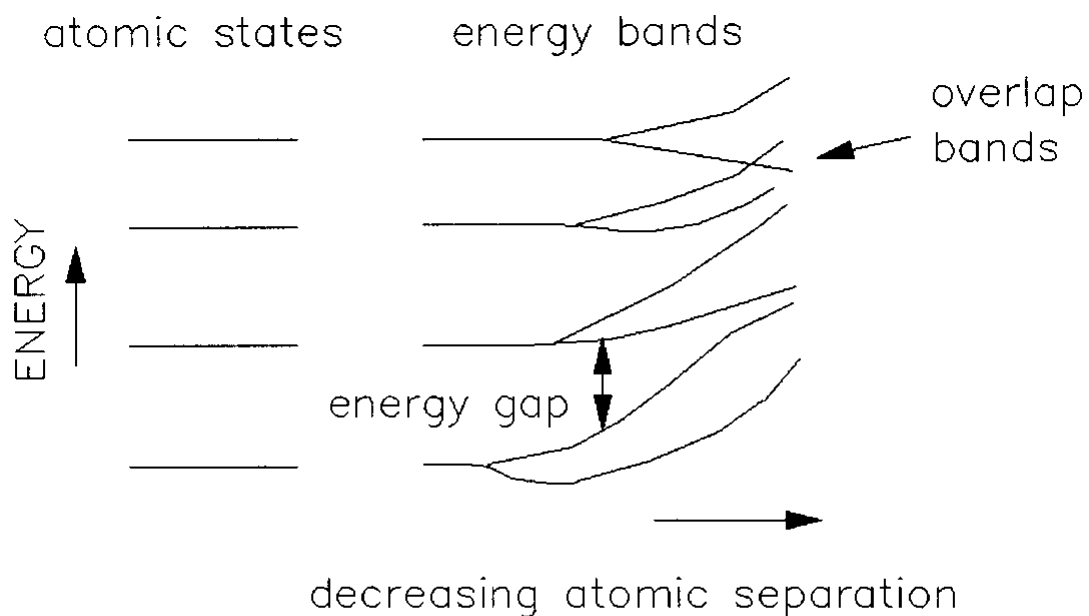


Fig.II-7:

Diagram showing atomic states and energy bands

- In between the energy bands, there are energy gaps
- there will be no electrons in these energy gaps

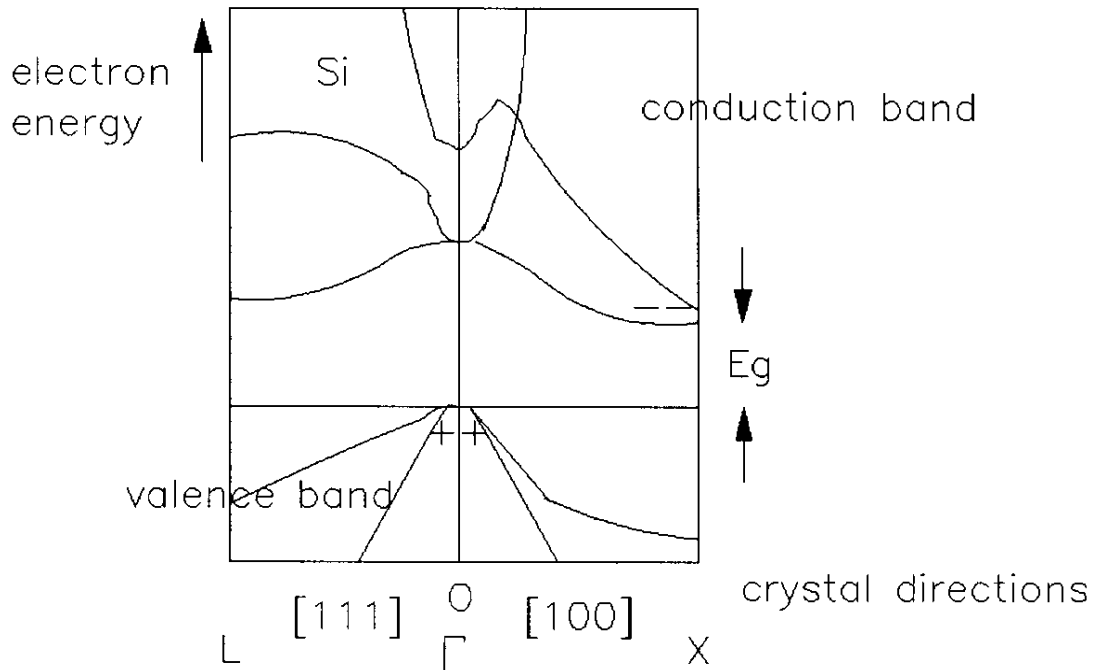


Fig.II–8: Energy band diagram of a typical solid

- At very low temperature, the energy bands are filled with electrons up to a certain energy value – above which the bands are empty
- Highest filled energy band is called the valence band and lowest unfilled (or partially-filled) band is called the conduction band
- Electrons and holes can only conduct a current in a partially-filled energy band – thus, electrons and holes in filled band do not contribute to current conduction
- Empty bands can be partially-filled as temperature increases

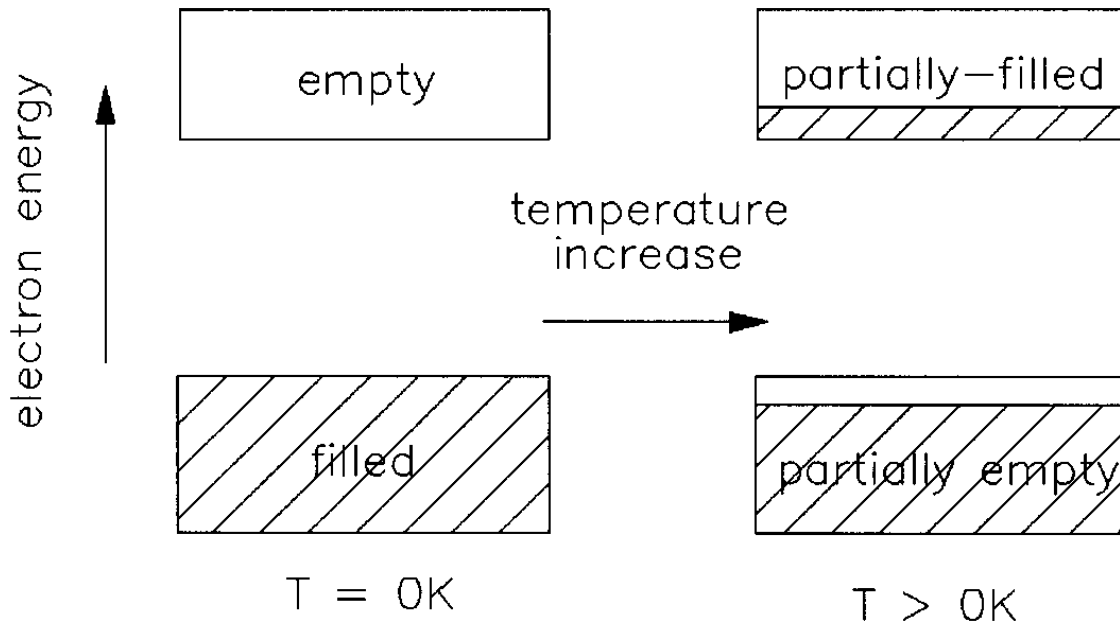


Fig.II–10: Energy bands in semiconductors

Statistics on the Filling of the Energy Bands

The probability of finding an electron in a solid with an energy E is governed by the Fermi-Dirac function $f(E)$. It is given by:

$$f(E) = 1/\{1 + \exp((E - E_F)/kT)\} \quad (12)$$

where E_F is the Fermi-level, k is the Boltzmann constant and T is the absolute temperature

For a given E_F , $f(E)$ is small when $E \gg E_F$ and equal to 1 when $E - E_F = 0$. When $E = E_F$, $f(E)$ is 0.5.

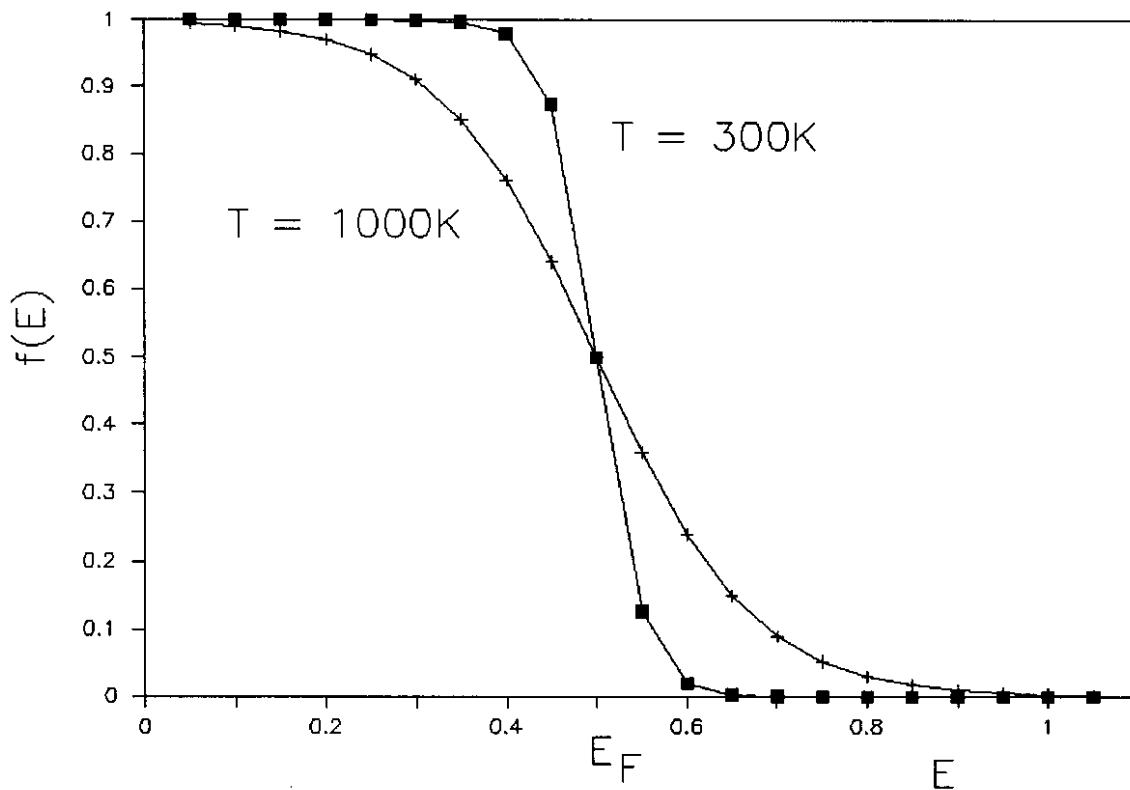


Fig.II-5: Fermi-Dirac distributions

Thus, the probability of finding an electron in a solid with high energy (a large E) is relatively small.

Computation of the Electron and Hole Densities:

The electron and hole densities are given by:

$$n = N_C f(E_C)$$

$$p = N_V [1 - f(E_V)] \quad (13)$$

where N_C and N_V are the effective densities of states in the conduction band and in the valence band, respectively, and

E_C and E_V are the respective energies at the conduction band edge and the valence band edge.

When $|E_C - E_F|$ and $|E_F - E_V|$ are $\gg kT$, it can be shown that:

$$n = N_C \times \exp(-(E_C - E_F)/kT)$$

$$p = N_V \times \exp(-(E_F - E_V)/kT) \quad (14)$$

The location of the Fermi-level therefore determines the electron and hole densities.

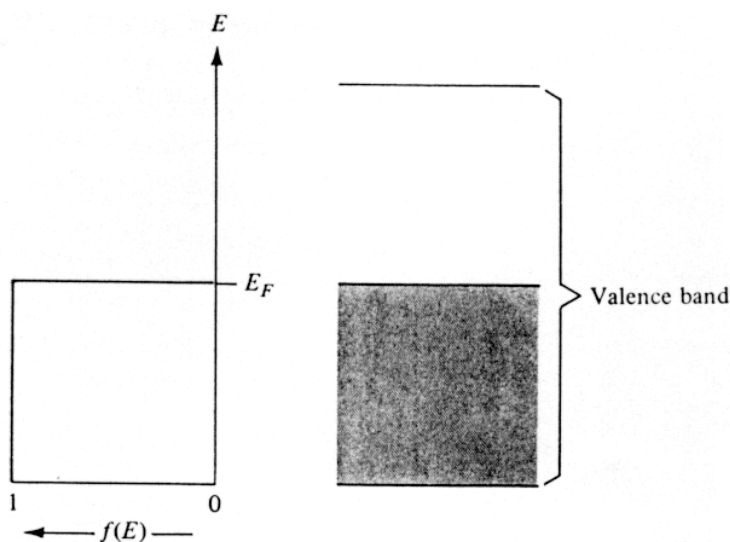


FIGURE 11.2-4 The Fermi function, $f(E)$, describes the relative filling of energy levels. At 0 K, all energy levels are completely filled up to the Fermi level, E_F , and completely empty above E_F .

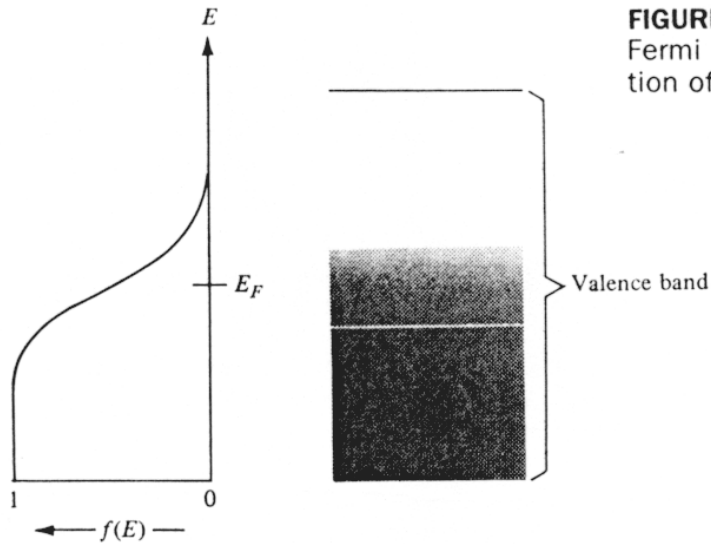


FIGURE 11.2-5 At $T > 0$ K, the Fermi function, $f(E)$, indicates promotion of some electrons above E_F .

Current Conduction in the Different Solids

A. Metals

- Have overlapping energy bands and a large number of “free” electrons in the partially-filled conduction bands
- Conductivity is of the order of 10^6 S ($\text{ohm}^{-1}\text{-m}^{-1}$)
- Electron density is around $10^{27} / \text{m}^3$
- Normally no holes are present
- Conduction properties are relatively unaffected by temperature

For convenience, metal resistivity is given as:

$$\rho_M = \rho_{RT} (1 + \alpha (T - T_{RT})) \quad (15)$$

where ρ_{RT} is the room temperature resistivity, α is its temperature coefficient and T_{RT} is the room temperature.

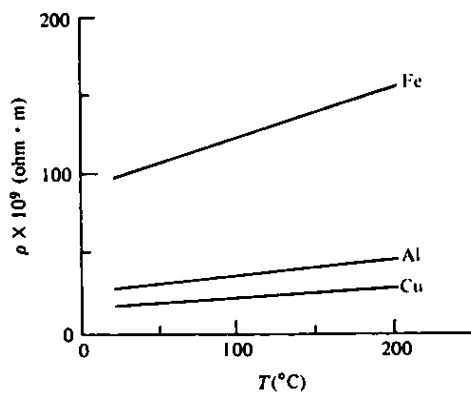


FIGURE 11.3-2 Variation in electrical resistivity with temperature for the same metals shown in Figure 11.3-1. The linearity of these data defines the temperature coefficient of resistivity, α .

TABLE 11.3-1 Resistivities and Temperature Coefficients of Resistivity for Some Metallic Conductors

Material	Resistivity at 20°C ρ_{20} ($\Omega \cdot \text{m}$)	Temperature Coefficient of Resistivity at 20°C α ($^{\circ}\text{C}^{-1}$)
Aluminum (annealed)	28.28×10^{-9}	0.0039
Copper (annealed standard)	17.24×10^{-9}	0.00393
Gold	24.4×10^{-9}	0.0034
Iron (99.99 + %)	97.1×10^{-9}	0.00651
Lead (99.73 + %)	206.48×10^{-9}	0.00336
Magnesium (99.80%)	44.6×10^{-9}	0.01784
Mercury	958×10^{-9}	0.00089
Nickel (99.95% + Co)	68.4×10^{-9}	0.0069
Nichrome (66% Ni + Cr and Fe)	1000×10^{-9}	0.0004
Platinum (99.99%)	106×10^{-9}	0.003923
Silver (99.78%)	15.9×10^{-9}	0.0041
Steel (wire)	$107\text{--}175 \times 10^{-9}$	0.006–0.0036
Tungsten	55.1×10^{-9}	0.0045
Zinc	59.16×10^{-9}	0.00419

Source: Data from J. K. Stanley, *Electrical and Magnetic Properties of Metals*, American Society for Metals, Metals Park, Ohio, 1963.

For a metal with a given fraction of impurities added to it, the resistivity is given by:

$$\rho_M = \rho_0 (1 + \beta y) \quad (16)$$

where ρ_0 is the resistivity of the pure metal, β is an impurity coefficient, and y is the fraction of impurities present.

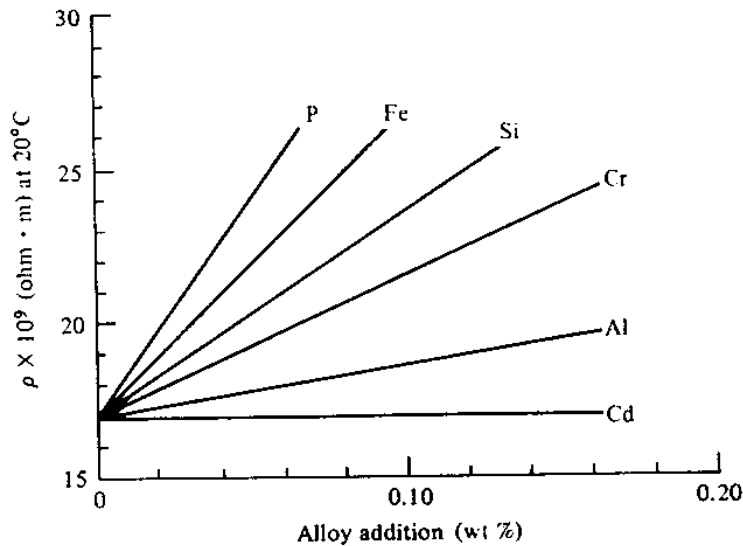


FIGURE 11.3-3 Variation in electrical resistivity with composition for various copper alloys with small levels of elemental additions. Note that all data are at a fixed temperature (20°C). (From J. K. Stanley, *Electrical and Magnetic Properties of Metals*, American Society for Metals, Metals Park, Ohio, 1963.)

Metal	σ_0 (S/m)	α_T (/K)	Impurity	β (/wt%)
Fe	1.03×10^7	0.00652		
Au	4.09×10^7	0.00340		
Cu	5.80×10^7	0.00393	Fe	5.41
			Cr	2.47
			Al	0.88

Table I: Values of σ_0 and α_T for some metals and β for Cu in the presence of common impurities.

Applications

Metals are most frequently used as lead wires, interconnections and in ground planes. Sometimes, metal junctions are used in thermocouples.

Thermoelectric Effect

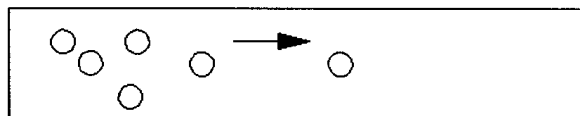
When there is a temperature difference across a metal wire, a voltage is developed across the wire as the electrons move away from the hotter junction. This results in a potential gradient across the wire given by:

$$\Delta V = K_S \Delta T \quad (17)$$

where K_S is the Seebeck coefficient and ΔT is the temperature difference.

INITIAL:

LOWER T



movement of electrons towards the cold junction

STEADY-STATE:

LOWER T



negative
potential

Fig.11aa: Thermoelectric effect in a metals

This is known as the thermoelectric effect and is the principle behind the construction of thermocouples.

Thermocouples

A thermocouple is built by connecting 2 different metal wires A and B in series to form two junctions (i.e., A-B-A or B-A-B). One of the junctions (the cold junction) is usually put in an ice bath (at 0°C), while the hot junction is placed at the point where the temperature is to be measured.

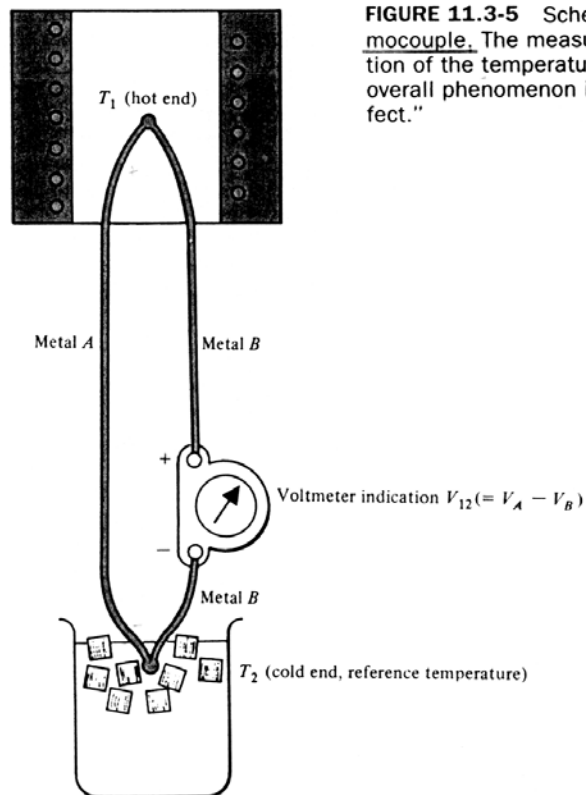


FIGURE 11.3-5 Schematic illustration of a thermocouple. The measured voltage, V_{12} , is a function of the temperature difference, $T_1 - T_2$. The overall phenomenon is termed the "Seebeck effect."

The voltage generated is given by:

$$\Delta V_{AB} = (K_{SA} - K_{SB}) T_h \quad (18)$$

where K_{SA} and K_{SB} are the respective Seebeck coefficients and T_h is the unknown temperature.

<u>Metal A</u>	<u>Metal B</u>	<u>$K_{SA} - K_{SB}$ (mV/°C)</u>
Chromel	Alumel	0.040
Platinum	Platinum/rhodium	0.006
Copper	Constantin	0.039

Table II-7: Seebeck coefficients for the common thermocouples (the cold junction is at 0 °C).

TABLE 11.3-2 Common Thermocouple Systems

Type	Common Name	Positive Element*	Negative Element*	Recommended Service Environment(s)	Maximum Service Temp. (°C)
B	Platinum–rhodium/platinum–rhodium	70 Pt–30 Rh	94 Pt–6 Rh	Oxidizing Vacuum Inert	1700
E	Chromel/constantan	90 Ni–9 Cr	44 Ni–55 Cu	Oxidizing	870
J	Iron/constantan	Fe	44 Ni–55 Cu	Oxidizing Reducing	760
K	Chromel/alumel	90 Ni–9 Cr	94 Ni–Al, Mn, Fe, Si, Co	Oxidizing	1260
R	Platinum/platinum–rhodium	87 Pt–13 Rh	Pt	Oxidizing Inert	1480
S	Platinum/platinum–rhodium	90 Pt–10 Rh	Pt	Oxidizing Inert	1480
T	Copper/constantan	Cu	44 Ni–55 Cu	Oxidizing Reducing	370

Source: Data from *Metals Handbook*, 9th ed., Vol. 3, American Society for Metals, Metals Park, Ohio, 1980.
*Alloy compositions expressed as weight percents.

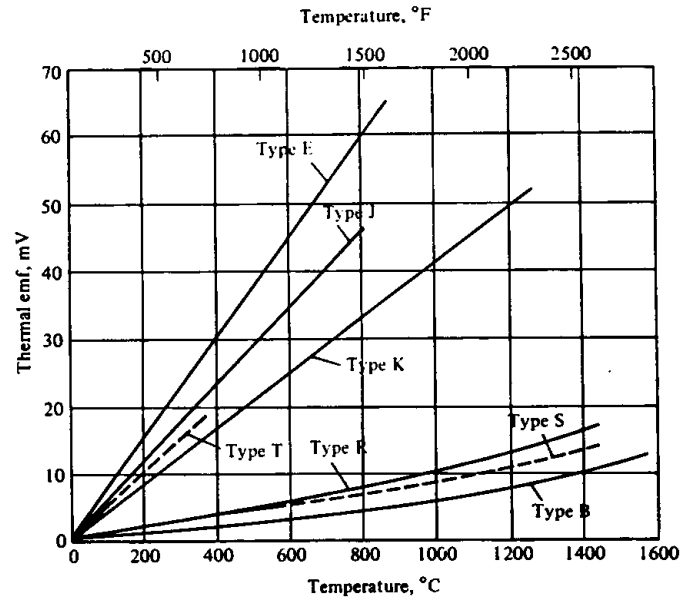


FIGURE 11.3-6 Plot of thermocouple electromotive force ($= V_{12}$ in Figure 11.3-5) as a function of temperature for some common thermocouple systems listed in Table 11.3-2. (From *Metals Handbook*, 9th ed., Vol. 3, American Society for Metals, Metals Park, Ohio, 1980.)

B. Semiconductors

- These are insulators at low temperature since the valence band is completely-filled and the conduction band is completely empty
- Electrons are excited to the conduction band at room temperature and “free” electrons and holes are formed

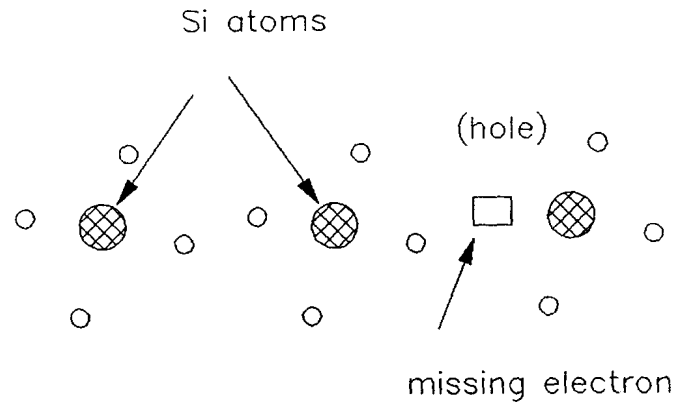


Fig.II-a: An array of Silicon atoms with a missing electron (hole)

- In a pure semiconductor, the electron density and the hole density are exactly equal - usually $10^{12-16} / \text{m}^3$ (in a doped semiconductor, these values are different)
- Conduction takes place when holes are filled by electrons released from nearby bonds

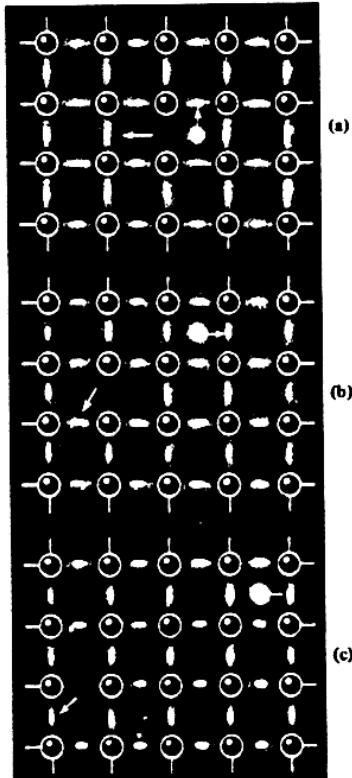


FIGURE 11.5-1 Creation and motion of a conduction electron and an electron hole in a semiconductor.

(a) An electron breaks away from the covalent bond, leaving a vacant bonding state, or a hole. The electron is now free to move in an electric field. In terms of the band model, the electron has gone from the valence band to the conduction band, leaving a hole in the valence band. The electron is shown moving upward, and the hole to the left. (b) The conduction electron will now move to the right, the hole, down to the left. (c) The motions of (b) have been completed; the hole and electron continue to move outward. (From R. M. Rose, L. A. Shepard, and J. Wulff, *The Structures and Properties of Materials*, Vol. 4: *Electronic Properties*, John Wiley & Sons, Inc., New York, 1966.)

a) Pure/intrinsic semiconductor

- Electron density and the hole density are both equal to the intrinsic carrier density, n_i
- Based on the law of mass action, the product of the electron density and the hole density should be equal to the square of the intrinsic carrier density (i.e., $n \times p = n_i^2$)

The intrinsic carrier density is given by:

$$n_i = \sqrt{N_C N_V} \exp(- (E_C - E_V)/2kT) \quad (19)$$

where $E_C - E_V = E_g$, the energy gap.

The intrinsic conductivity (σ_I) of a semiconductor is given by:

$$\ln(\sigma_I) = \ln(\sigma') - E_g/2kT \quad (20)$$

where $\sigma' = q \sqrt{N_c N_v} (\mu_n + \mu_p)$

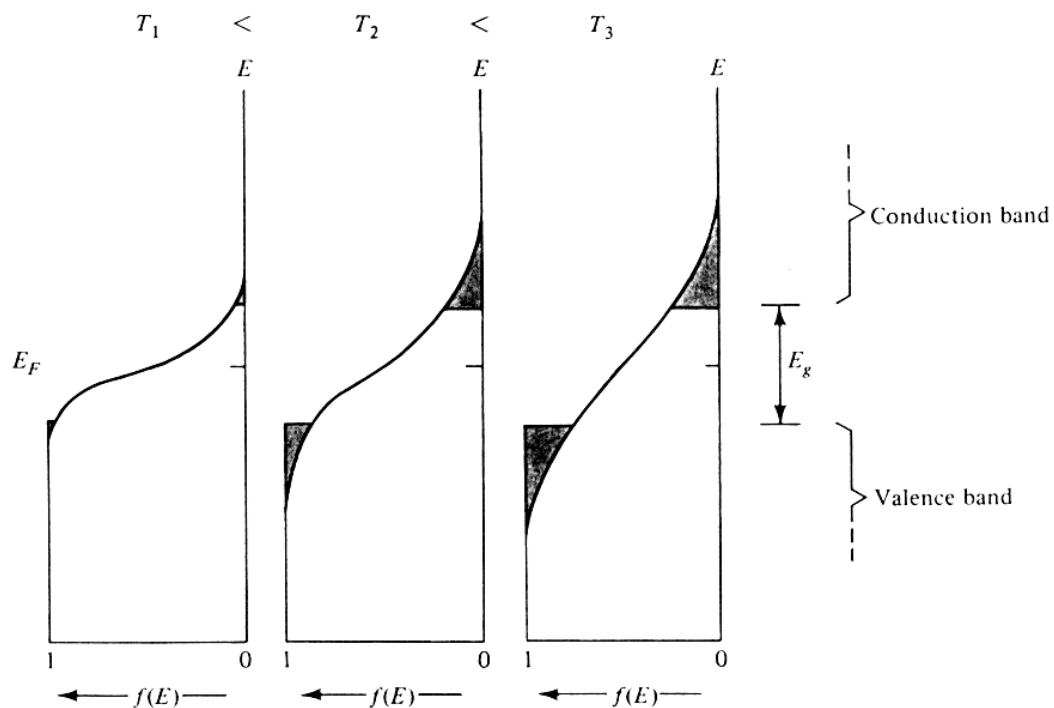


FIGURE 12.1-2 Schematic illustration of how increasing temperature increases overlap of the Fermi function, $f(E)$, with the conduction and valence bands giving increasing numbers of charge carriers. (Note also Figure 11.2-8.)

Thus, the intrinsic conductivity of a semiconductor increases with increasing temperature.

TABLE 12.6-1 Electrical Properties for Some Intrinsic Semiconductors at Room Temperature (300 K)

Group	Semiconductor	E_g (eV)	μ_e [m ² /(V · s)]	μ_h [m ² /(V · s)]	$n_i (= n_h)$ (m ⁻³)
IVA	Si	1.107	0.140	0.038	14×10^{15}
	Ge	0.66	0.364	0.190	23×10^{18}
III-V	AlSb	1.6	0.090	0.040	—
	GaP	2.25	0.030	0.015	—
	GaAs	1.47	0.720	0.020	1.4×10^{12}
	GaSb	0.68	0.500	0.100	—
	InP	1.27	0.460	0.010	—
	InAs	0.36	3.300	0.045	—
	InSb	0.17	8.000	0.045	13.5×10^{21}
II-VI	ZnSe	2.67	0.053	0.002	—
	ZnTe	2.26	0.053	0.090	—
	CdS	2.59	0.034	0.002	—
	CdTe	1.50	0.070	0.007	—
	HgTe	0.025	2.200	0.016	—

Source: Data from W. R. Runyan and S. B. Watelski, in *Handbook of Materials and Processes for Electronics*, C. A. Harper, ed., McGraw-Hill Book Company, New York, 1970.

b) Doped/extrinsic semiconductor

- Electronic properties depend on dopants - dopants are normally either Group III or Group V elements

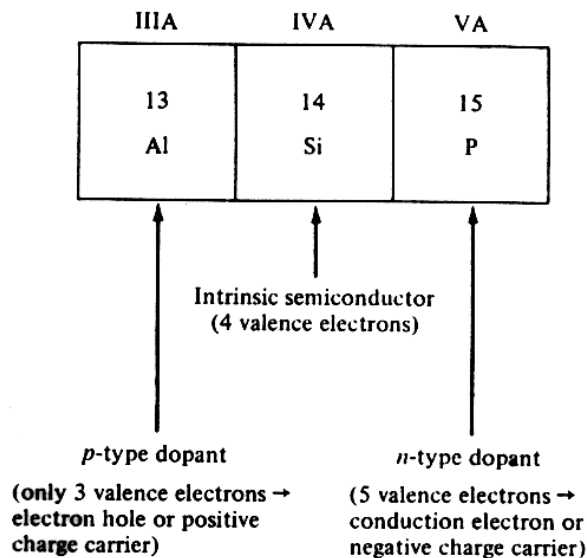


FIGURE 12.2-1 Small section of the periodic table of elements. Silicon, in group IVA, is an intrinsic semiconductor. Adding a small amount of phosphorus from group VA provides extra electrons (not needed for bonding to Si atoms). As a result, phosphorous is an n -type dopant (i.e., an addition producing negative charge carriers). Similarly, aluminum, from group IIIA, is a p -type dopant in that it has a deficiency of valence electrons leading to positive charge carriers (electron holes).

- A dopant that “provides” an additional electron is known as a donor; one that “provides” an additional hole is known as an acceptor

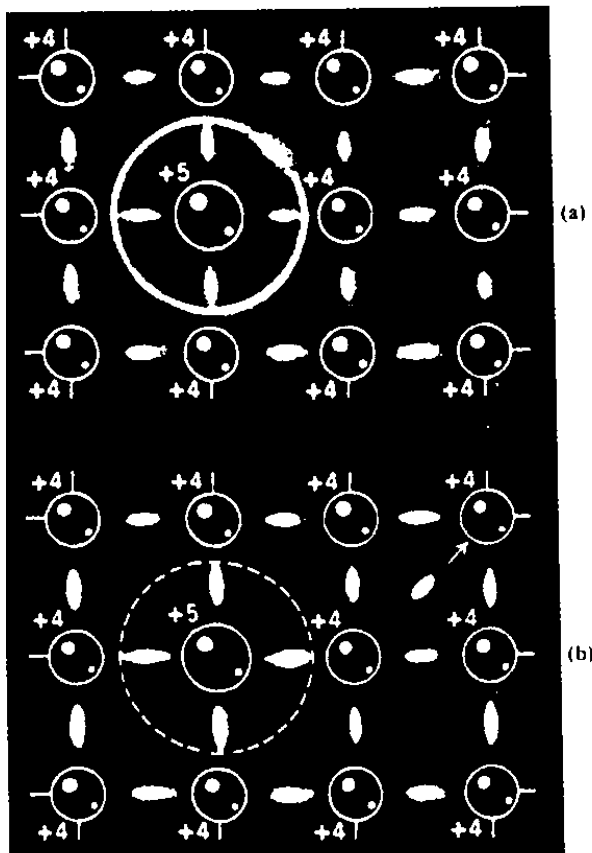


FIGURE 12.2-4 Schematic of the production of a conduction electron in an n -type semiconductor. (a) The extra electron associated with the group VA atom can (b) easily break away, becoming a conduction electron and leaving behind an empty donor state associated with the impurity atom. This can be contrasted with the similar figure for intrinsic material in Figure 11.5-1. (From R. M. Rose, L. A. Shepard, and J. Wulff, *The Structure and Properties of Materials*, Vol. 4: *Electronic Properties*, John Wiley & Sons, Inc., New York, 1966.)

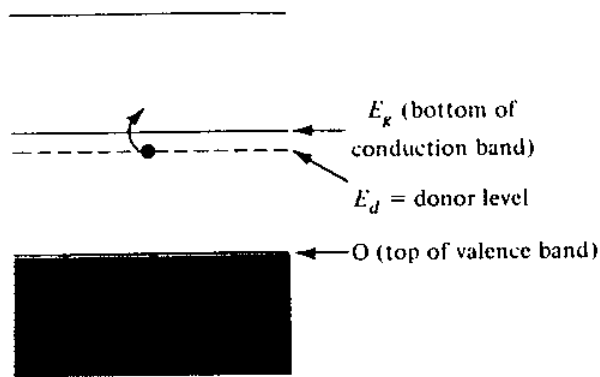


FIGURE 12.2-2 Energy band structure of an n -type semiconductor. The “extra electron” from the group VA dopant produces a donor level (E_d) near the conduction band. This provides relatively easy production of conduction electrons. This figure can be contrasted with the energy band structure of an intrinsic semiconductor in Figure 11.2-8.

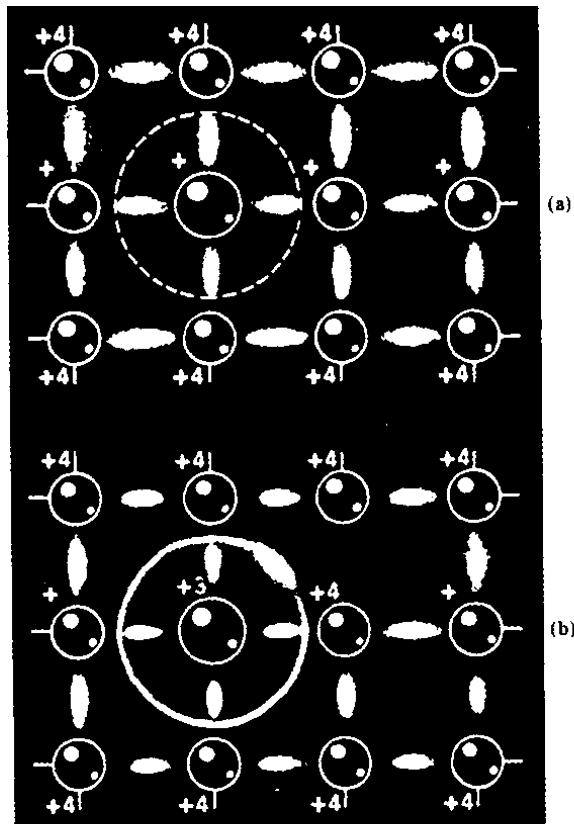


FIGURE 12.2-9 Schematic of the production of an electron hole in a *p*-type semiconductor. (a) The deficiency in valence electrons for the group IIIA atom creates an empty state, or electron hole, orbiting about the acceptor atom. (b) The electron hole becomes a positive charge carrier as it leaves the acceptor atom behind with a filled acceptor state. (The motion of electron holes, of course, is due to the cooperative motion of electrons.) (From R. M. Rose, L. A. Shepard, and J. Wulff, *The Structure and Properties of Materials*, Vol. 4: *Electronic Properties*, John Wiley & Sons, Inc., New York, 1966.)

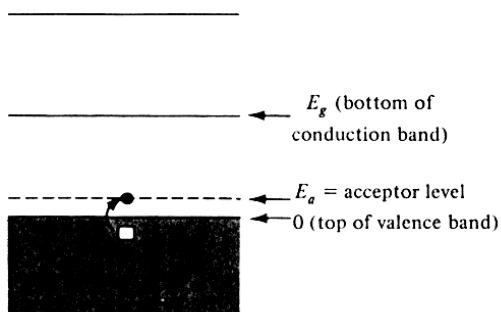


FIGURE 12.2-7 Energy band structure of a *p*-type semiconductor. The deficiency of valence electrons in the group IIIA dopant produces an acceptor level (E_a) near the valence band. Electron holes are produced as a result of thermal promotion over this relatively small energy barrier.

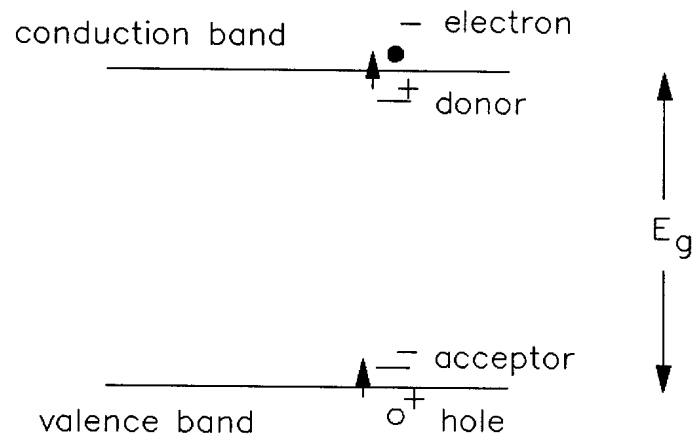


Fig.II-23:
Location of donor and acceptor energy states

TABLE 12.6-2 Impurity Energy Levels for Extrinsic Semiconductors

Semiconductor	Dopant	$E_x - E_d$ (eV)	E_a (eV)
Si	P	0.044	—
	As	0.049	—
	Sb	0.039	—
	Bi	0.069	—
	B	—	0.045
	Al	—	0.057
	Ga	—	0.065
	In	—	0.16
	Tl	—	0.26
Ge	P	0.012	—
	As	0.013	—
	Sb	0.096	—
	B	—	0.01
	Al	—	0.01
	Ga	—	0.01
	In	—	0.011
	Tl	—	0.01
GaAs	Se	0.005	—
	Te	0.003	—
	Zn	—	0.024
	Cd	—	0.021

Source: Data from W. R. Runyan and S. B. Watelski, in *Handbook of Materials and Processes for Electronics*, C. A. Harper, ed., McGraw-Hill Book Company, New York, 1970.

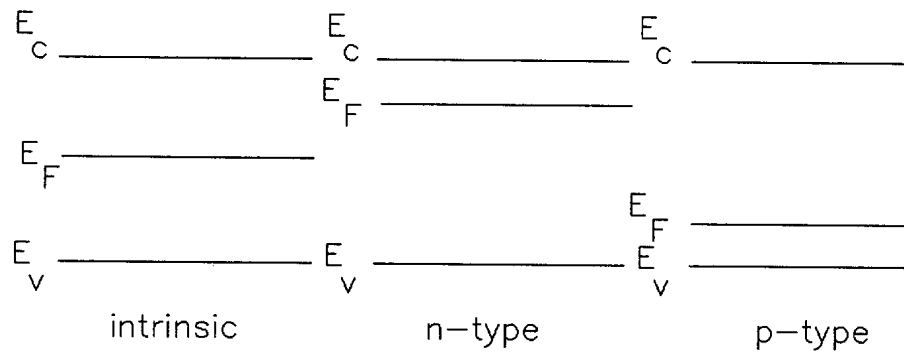


Fig.II-24: Different positions of the Fermi-level

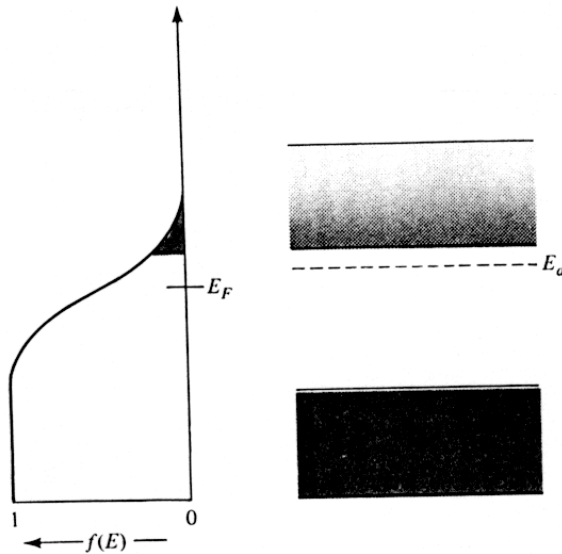


FIGURE 12.2-3 Comparison of the Fermi function, $f(E)$, with the energy band structure for an *n*-type semiconductor. The extra electrons shift the Fermi level (E_F) upward compared to Figure 11.2-8 (where it was in the middle of the band gap for an intrinsic semiconductor).

- Usually either donors/acceptors are added to a solid - but not both
- A semiconductor is N-type when the electron density is much greater than the hole density, and P-type when the hole density is much greater than the electron density
- Electrons are the majority carriers in N-type semiconductors (holes are the minority carriers)
- Holes are the majority carriers in P-type semiconductors (electrons are the minority carriers)
- A highly-doped semiconductor is called a degenerate semiconductor

For an N-type semiconductor, $N_D \gg n_i$, $n = N_D$. Thus, $p = n_i^2/N_D$.

Similarly, for a P-type semiconductor, $N_A \gg n_i$, $p = N_A$ and $n = n_i^2/N_A$.

Semiconductor	E_g (eV)	ϵ_s ($\times \epsilon_0$)	μ_n ($\times 10^{-1} \text{m}^2/\text{V.s}$)	μ_p ($\times 10^{-1} \text{m}^2/\text{V.s}$)	N_c ($\times 10^{25}/\text{m}^3$)	N_v ($\times 10^{25}/\text{m}^3$)	n_i ($\times 10^{16}/\text{m}^3$)
Si	1.08-1.12	11.9	1.45	0.45	2.8	1.04	1.45
Ge	0.66	16.0	3.90	1.90	1.04	0.6	2.4×10^3
GaP	2.26	10.0	0.30	0.15			
GaAs	1.42-1.43	13.1	8.50	0.40	0.047	0.7	1.8×10^{-4}
InP	1.35	12.1	4.00	0.60			
InSb	0.18	18.0	100	1.70			

Table II-8: Physical properties of the important semiconductors at 300 K (ϵ_0 is the permittivity of vacuum).

c) Temperature effect

- Temperature increase will raise the intrinsic carrier density but not (usually) the carriers arising from the dopant - at some point, an extrinsic semiconductor should become intrinsic as temperature rises
- Lowering the temperature will decrease the “activation” of dopants (this is called freeze-out) and the carrier densities will decrease

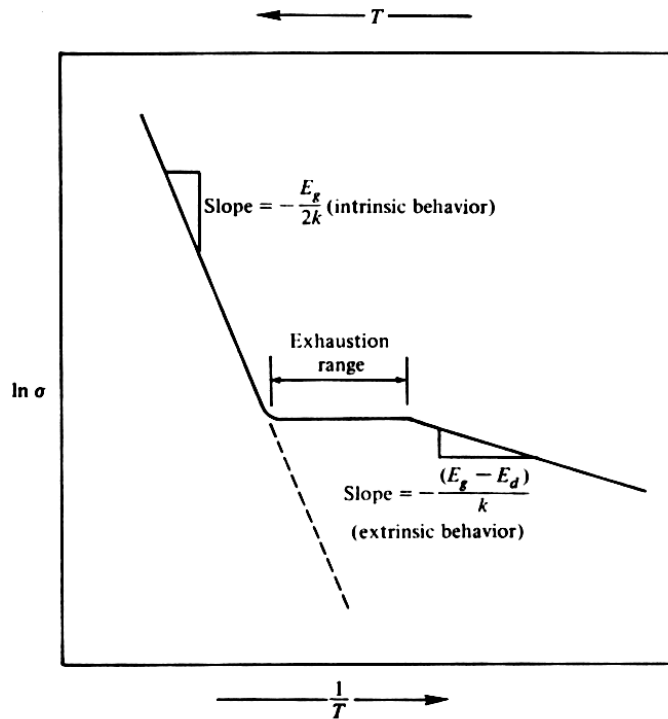


FIGURE 12.2-6 Arrhenius plot of electrical conductivity for an n -type semiconductor over a wider temperature range than shown in Figure 12.2-5. At low temperatures (high $1/T$), the material is extrinsic. At high temperatures (low $1/T$), the material is intrinsic. In between is the exhaustion range, in which all "extra electrons" have been promoted to the conduction band.

d) Structural effects

- Polycrystalline and amorphous semiconductors normally have inferior physical properties
- Main advantage of these materials is low-cost

e) Diffusion currents

- Relatively low carrier densities in semiconductors can allow carrier density gradients to be established - giving rise to diffusion currents

- Diffusion currents exist independent of the drift currents (which arise from an electrical potential gradient)
- Diffusivity and mobility are related by the Einstein relation (i.e., $\mu = D/kT$)

Mathematical Formulation of the Diffusion Currents:

Based on Fick's 1st law, the carrier flux is given by:

$$F_x = -D \frac{d(C)}{dx} = -D_n \frac{d(n)}{dx} - D_p \frac{d(p)}{dx} \quad (21)$$

where D_n and D_p are the electron and hole diffusivities, respectively.

Since electric current is: $I = (+/-) q F_x A_{cs}$, we have:

$$I = q A_{cs} (D_n \frac{d(n)}{dx} - D_p \frac{d(p)}{dx}) \quad (22)$$

The diffusion currents therefore depend exclusively on the carrier density gradients.

Applications

Semiconductors are widely used in devices and circuits when fabricated in the form of P-N junctions and transistors. They are also used as sensors such as magnetic field sensors.

A. Hall Effect

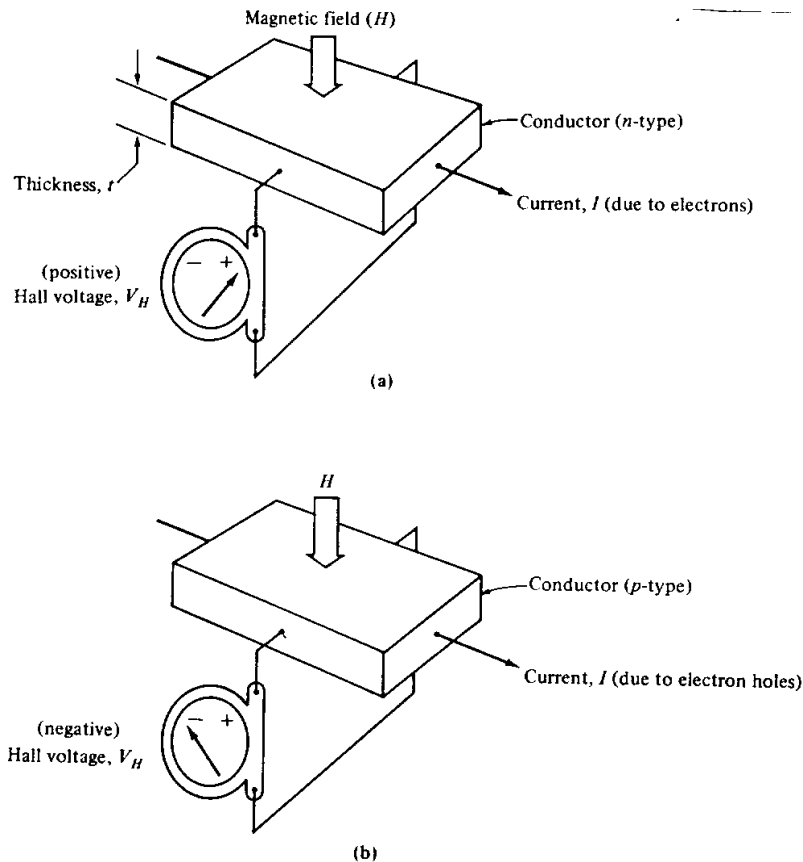


FIGURE 12.2-11 The application of a magnetic field (with field strength, H), perpendicular to a current, I , causes a sideways deflection of charge carriers and a resulting voltage, V_H . This phenomenon is known as the Hall effect. The Hall voltage is given by Equation 12.2-5. For (a) an *n*-type semiconductor, the Hall voltage is positive. For (b) a *p*-type semiconductor, the Hall voltage is negative.

Hall Effect relates the (majority) carrier density in a semiconductor to an applied magnetic field. The experimental setup involves passing a known current I longitudinally along a rectangular semiconductor block and applying a magnetic field (intensity) B in the vertical direction. This will result in a transverse voltage V_H (Hall voltage) due to the orthogonal movement of the carriers in orthogonal electric and magnetic fields.

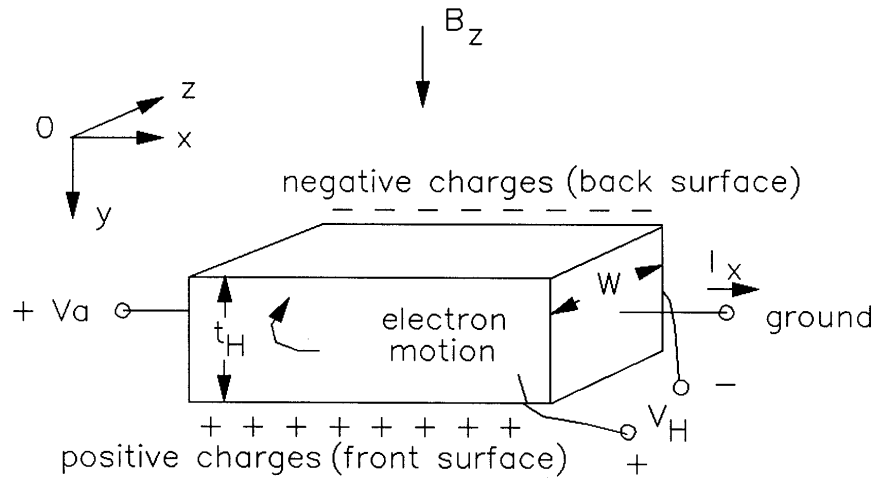


Fig.II-15: Hall effect showing charge accumulation

It can be shown that:

$$V_H = - R_H I B / t_H \quad (23)$$

where the R_H (Hall constant) = $- 1/(q n)$ for an N-type semiconductor and $= 1/(q p)$ for a P-type semiconductor. t_H is the sample thickness.

In the above equation, V_H , I , B and t_H can be measured to determine R_H .

If the (majority) carrier density of the semiconductor is already known, then Hall effect can be used to determine any unknown magnetic field (intensity).

B. P-N Junction

A P-N junction is formed when a P-type semiconductor is electrically alloyed to an N-type semiconductor. After the cross-diffusion of the (majority) carriers, a potential barrier

should be formed. Such a P-N junction has rectification properties, i.e., current can flow easily in one direction (from the P side to the N side) but not in the opposite direction due to the existence of the barrier.

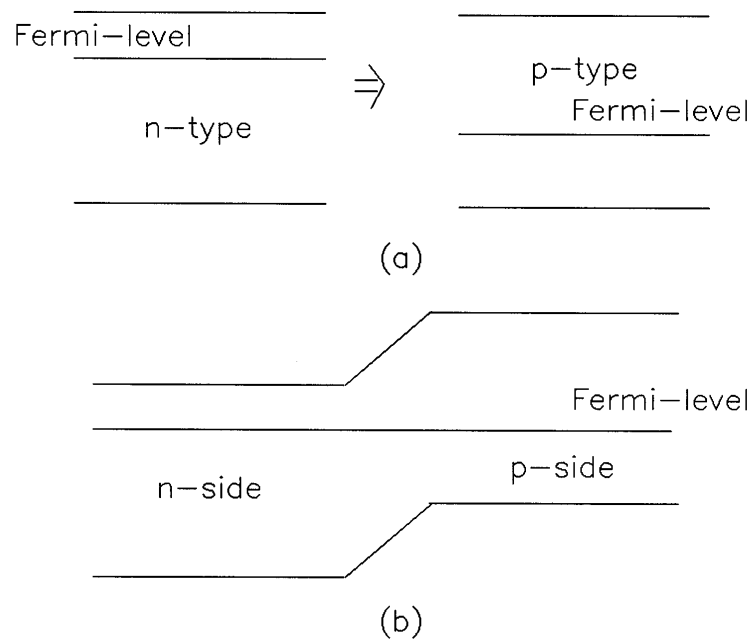


Fig.III-4: Alignment of the Fermi-levels in a p-n junction

The simplest use of a P-N junction would be to use it as a (current) switch.

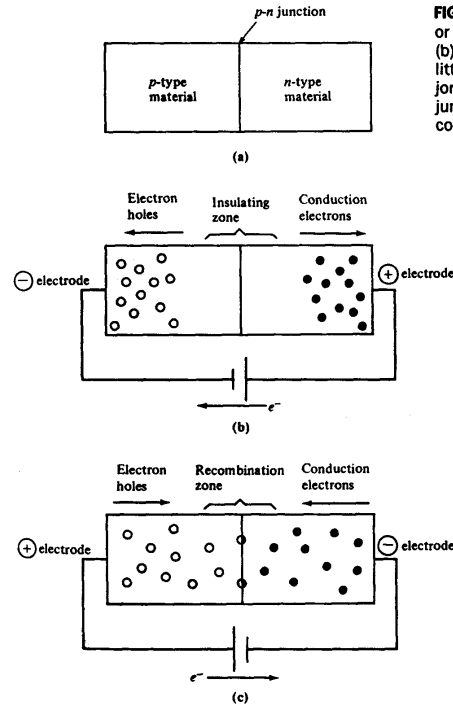


FIGURE 12.5-1 (a) A solid-state rectifier, or diode, contains a single *p-n* junction. (b) In reverse bias, polarization occurs and little current flows. (c) In forward bias, majority carriers in each region flow toward the junction, where they are continuously recombined.

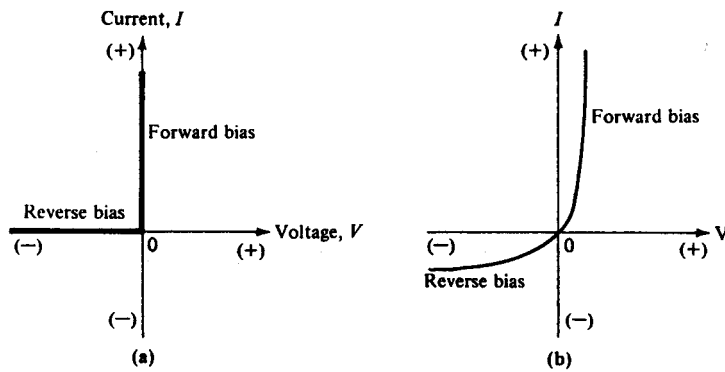


FIGURE 12.5-2 Current flow as a function of voltage in (a) an ideal rectifier and (b) an actual device such as that shown in Figure 12.5-1.

C. Transistors

A bipolar transistor is formed when two back-to-back P-N junctions are connected together to form a 3-terminal device. When properly bias, current injected from one P-N junction (input) can overshoot to the other P-N junction (output). This injection current when delivered to a load can result in voltage amplification across the output.

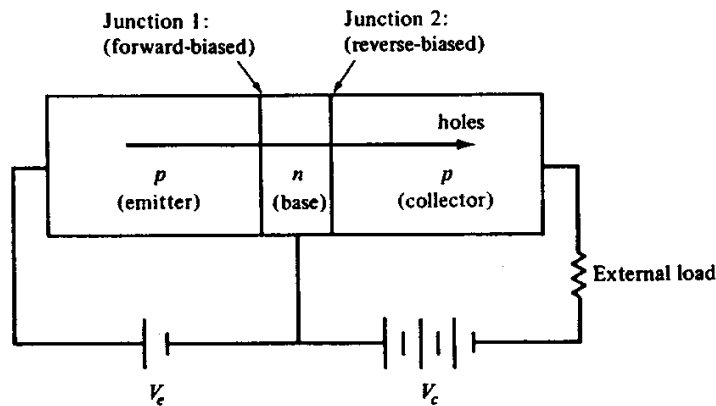


FIGURE 12.5-4 Schematic of a transistor (a *p-n-p* “sandwich”). The overshoot of electron holes across the base (*n*-type region) is an exponential function of the emitter voltage, V_e . Because the collector current (I_c) is similarly an exponential function of V_e , this device serves as an amplifier. An *n-p-n* transistor functions similarly except that electrons rather than holes are the overall current source.

A MOS (metal-oxide-semiconductor) transistor is essentially a gate-controlled current source. When a current passes under the metal gate (insulated from the substrate by the oxide), the magnitude of this current (output) can change depending on the electric field strength. Through changing the gate bias (input), the current can be turned on or off. In this manner, the MOS transistor acts as a switch or a logic gate.

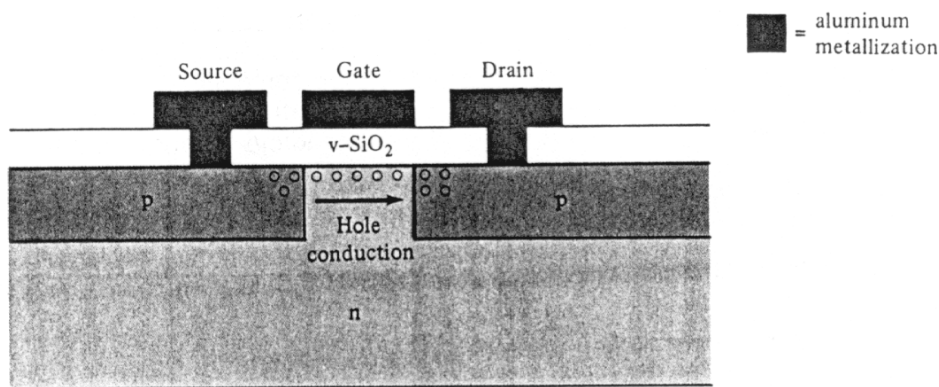


FIGURE 12.5-5 Schematic of a *field-effect transistor (FET)*. A negative voltage applied to the *gate* produces a field under the vitreous silica layer and a resulting *p*-type conductive channel between the *source* and the *drain*.

Integrated circuits rely on the placement of a large number of transistors on a semiconductor die (substrate). Whole circuits or even an entire electronic system can be manufactured in this manner. A single semiconductor (wafer) may contain many identical dies.

C. Insulators

- Known to be poor current conductors (commonly called dielectrics) due to lack of “free” carriers
- Many are transparent to visible light because of the large energy gaps
- Become easily polarized (i.e., positive and negative charges can be separated)
- Polarization effect is often used to increase the Dielectric strength or charge storage in capacitors
- Conduct an ac current under ac bias

Mathematical Formulation of dielectric properties

- Simplest way to examine the dielectric properties of an insulator is to make use of a parallel-plate capacitor
- Properties are measured in terms of the permittivity or the dielectric constant

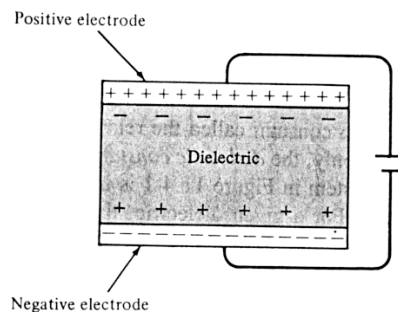


FIGURE 11.4-1 A parallel-plate capacitor involves an insulator (*dielectric*) between two metal electrodes. The charge density build-up at the capacitor surface is related to the *dielectric constant* of the material, as indicated by Equation 11.4-3.

For a parallel-plate capacitor, the capacitance C is given by:

$$C = Q/V \quad (24)$$

where Q is the amount of charge residing on the plate and V is the applied voltage.

Capacitance can be expressed as:

$$C = \varepsilon A_{cs}/d \quad (25)$$

where ε is the permittivity, and d is the plate separation.

A measure of the capacitance in the presence and in the absence of the insulator provides a means to determine the relative permittivity ($= \varepsilon_i/\varepsilon_0$) of the insulator.

In general:

$$\varepsilon_i = K_D \varepsilon_0 \quad (26)$$

where K_D is the dielectric constant, and $\varepsilon_0 (= 8.85 \times 10^{-12} \text{ F/m})$ is the free space permittivity.

It can be shown that (approximately):

$$K_D = (1 + N \alpha_p/\varepsilon_0) \quad (27)$$

where N is the dipole density in the insulator, and α_p is the polarizability (a measure of the dipole strength).

Sources of dipoles

a) Permanent dipoles

- Related to charge separation in the molecules or groups of ions
- Dielectric properties are linked to the orientation of the dipoles
- Respond to dc or low-frequency electric field

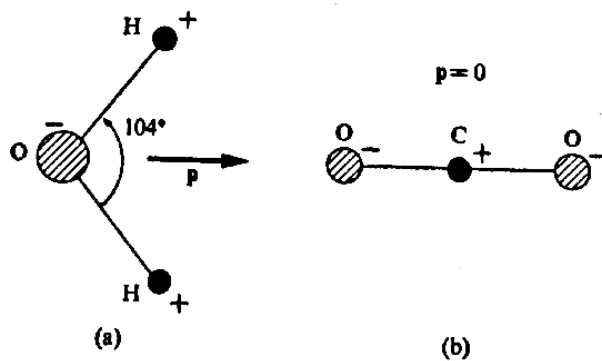


Fig. 8.7 (a) The water molecule and its permanent moment. $p \approx 1.9$ debye units (1 debye = 10^{29} coul · m). (b) CO₂ molecule.

b) Ionic dipoles

- Dielectric properties are linked to the vibrations or oscillations of the positive and negative ions
- Exist only in ionic insulators
- Respond to higher frequency electric field

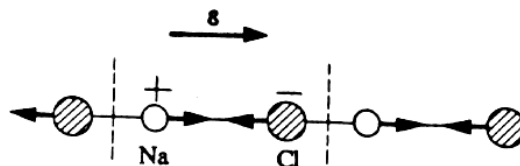


Fig. 8.8 Ionic polarization in NaCl. The field displaces the ions Na⁺ and Cl⁻ in opposite directions, changing the length of the bond.

c) Electronic dipoles

- Dielectric properties are linked to the polarization of the electrons surrounding the ion cores
- Exist in all solids
- Respond to very high-frequency electric field

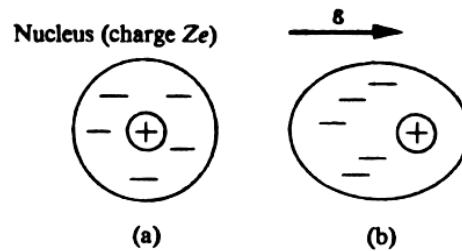


Fig. 8.9 Electronic polarization. (a) Unpolarized atom. (b) Atom polarized as a result of the field.

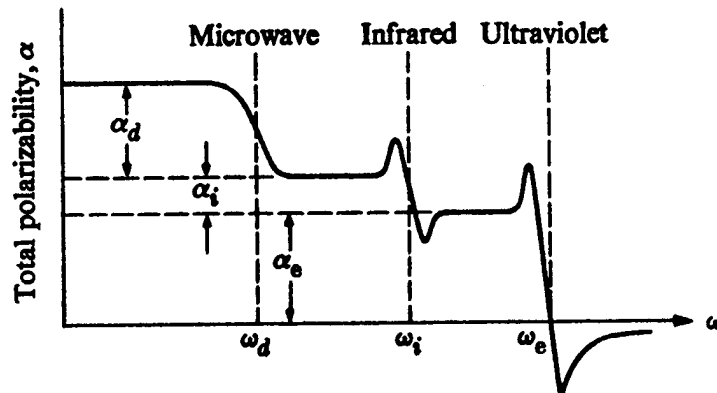


Fig. 8.10 Total polarizability α versus frequency ω for a dipolar substance.

<u>Material</u>	ϵ_r
Polymers	1-2
Water	80
Ceramics, glasses	4-10
Ferrites	10^4

Table II-10: Values of the relative permittivity or dielectric constant of the common insulators.

Current leakage in insulators

- Mostly due to ion movement
- Could be due to charge hopping between the ions
- Normally insignificant due to the small magnitude

Losses in insulators:

- Related to relaxation and damping of the dipoles
- Expressed in terms of a loss factor
which is given by:

$$\tan(\delta) = \text{energy loss per cycle} / (2\pi \times \text{maximum energy stored})$$

(28)

<u>Materials</u>	<u>$\tan(\delta)$</u>
Polymers	10^{-4}
Water	0.1
Ceramics, glasses	10^{-3}
Ferrites	10^{-2}

Table II-11: Values of $\tan(\delta)$ in different insulators.

D. Ferroelectrics

- Insulators with permanent dipoles appearing in the form of domains
- It is possible to align the domains in an electric field (poling)
- Removal of the electric field leaves a remanent polarization
- Domains will disappear above a critical temperature
- Often used in opto-electronic devices - an example being the BaTiO_3 crystal

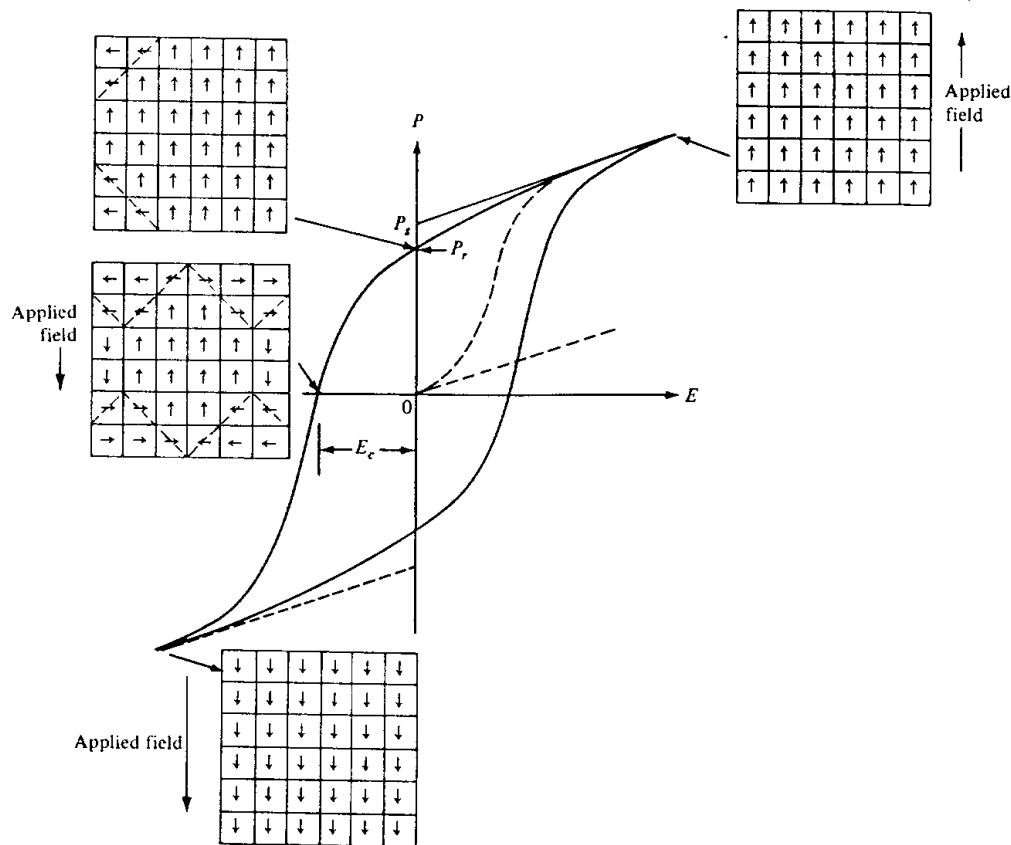
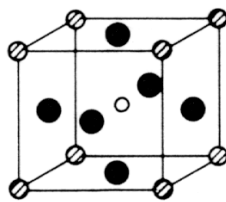
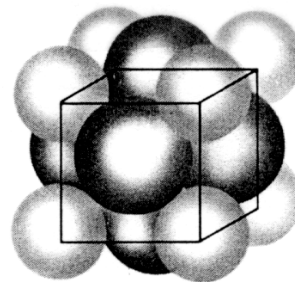


FIGURE 11.4-5 A ferroelectric hysteresis loop is the result of an alternating electric field. A dashed line indicates the initial spontaneous polarization illustrated in Figure 11.4-4. Saturation polarization (P_s) is the result of maximum domain growth (extrapolated back to zero field). Upon actual field removal, some remanent polarization (P_r) remains. A coercive field (E_c) is required to reach zero polarization (equal volumes of opposing domains).

- Ti^{4+} : at the body center
- ⊗ Ca^{2+} : at corners
- O^{2-} : at face centers



(a)



(b)

Structure: perovskite (CaTiO_3)-type
 Bravais lattice: simple cubic
 Ions/unit cell: 1 Ca^{2+} + 1 Ti^{4+} + 3 O^{2-}
 Typical ceramics: CaTiO_3 and BaTiO_3

FIGURE 3.4-7 Perovskite (CaTiO_3) unit cell showing (a) ion positions and (b) full-size ions.

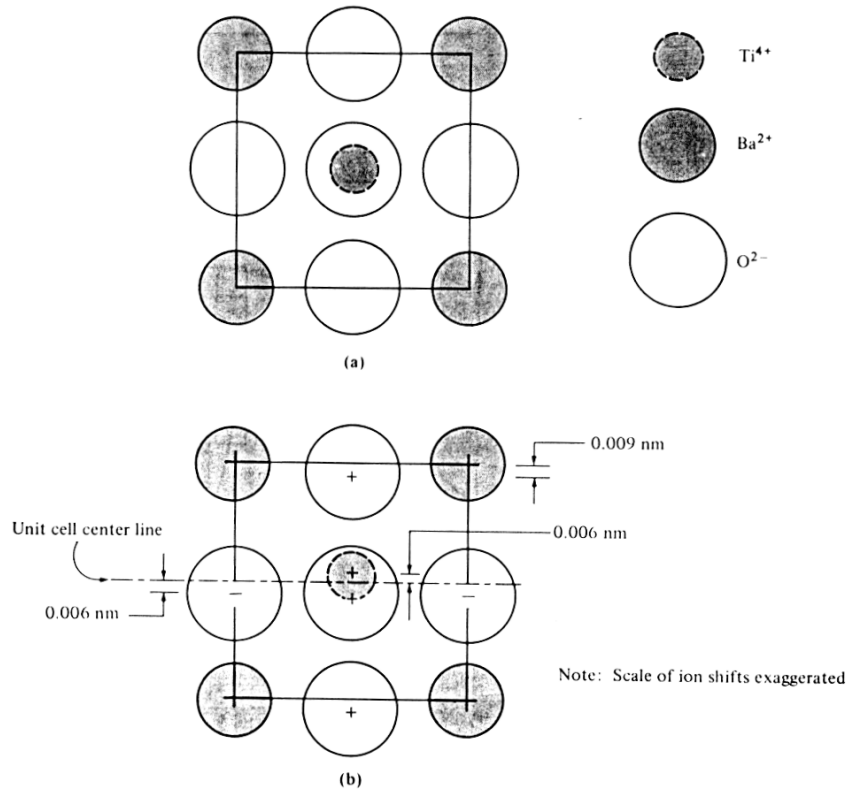


FIGURE 11.4-2 (a) Front view of the cubic BaTiO_3 structure. This can be compared with Figure 3.4-7. (b) Below 120°C , a tetragonal modification of the structure occurs. The net result is an upward shift of cations and a downward shift of anions.

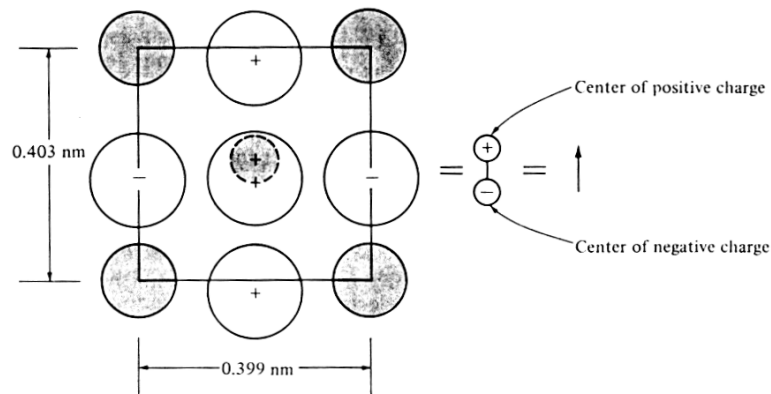


FIGURE 11.4-3 The tetragonal unit cell shown in Figure 11.4-2(b) is equivalent to an electrical dipole (with magnitude equal to charge times distance of separation).

E. Piezoelectric

- Exhibit a potential difference when under stress (applied pressure)
- Process is reversible (for instance, a strain of 10^{-7} requires $\Delta E = 100 \text{ kV/m}$)

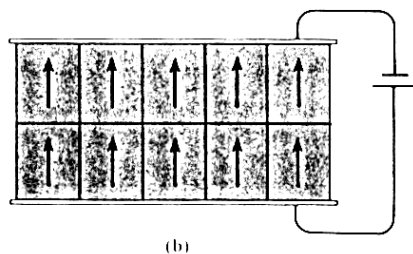
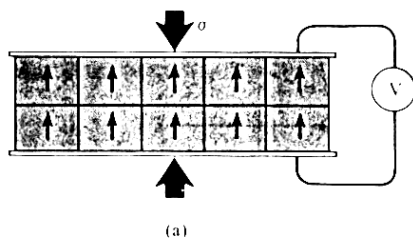
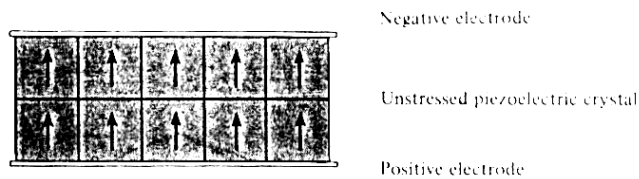
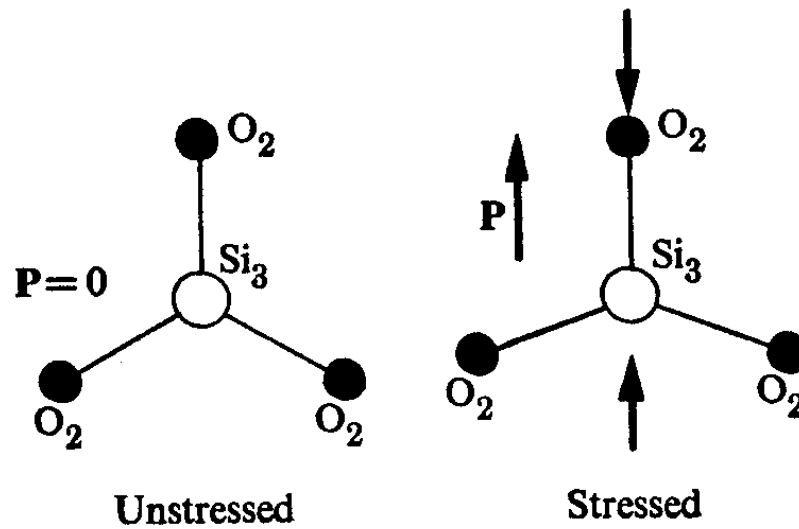


FIGURE 11.4-6 Using schematic illustrations of piezoelectric transducers, we see that (a) the dimensions of the unit cells in a piezoelectric crystal are changed by an applied stress, thereby changing their electrical dipoles. The result is a measurable voltage change. This is the “piezoelectric effect.” (b) Conversely, an applied voltage changes the dipoles and, thereby, produces a measurable dimensional change. This is the “reverse piezoelectric effect.”

- Related to the distortion in the crystal lattice resulting in changes in polarization used in pressure sensors – an example being the quartz crystal



Example 5.1: Compare the thermal energy E_{th} of a free electron at 300K with the value of the excitation energy (1.12 eV) for silicon.

Solution:

At 300K, $E_{th} = kT = 1.38 \times 10^{-23} \text{ J/K} \times 300 \text{ K} = 4.14 \times 10^{-21} \text{ J}$. $E_{excite} = 1.12 \text{ eV} = 1.12 \text{ eV} \times 1.6 \times 10^{-19} \text{ J/eV} = 1.79 \times 10^{-19} \text{ J}$. This is about 50 times larger than E_{th} and one can therefore expect only a small fraction of the electrons in silicon to be free electrons.

Example 5.2: Distinguish between the thermal velocity and the drift velocity, i.e., velocity due to an applied electric field, for electrons inside a solid.

Solution:

Thermal velocity is temperature dependent has no specific direction, while drift velocity for an electron is opposite to the direction of the applied field.

Quantitatively, thermal velocity is $\sqrt{2kT/m_e^*} \approx 1 \times 10^5 \text{ m/s}$, while the drift velocity is proportional to the electric field (and is zero when no current is flowing).

Example 5.3: Verify that $f(E) = 0.047$ when a) $E - E_F = + 3kT$, and b) $f(E) = 0.952$ when $E - E_F = - 3kT$.

Solution:

a) Based on the Fermi function, $f(E) = 1/[\exp((E - E_F)/(kT)) + 1] = 1/[\exp(3) + 1] = 0.047$.

b) Similarly, $f(E) = 1/[\exp(-3) + 1] = 0.952$.

Example 5.4: A hypothetical solid has 1×10^{26} atoms and each atom has 6 electrons. a) Suggest what would be the minimum number of energy states needed to accommodate all the electrons within a single energy band, and b) if the lowest energy state in the band is 0 eV and the energy states are separated by $0.5 \times 10^{-26} \text{ eV}$, compute the energy of the most energetic electron.

Solution:

- a) There are 6×10^{26} electrons. Since each energy state can accommodate only 2 electrons, there must be at least 3×10^{26} states.
- b) If these energy states are separated by 0.5×10^{-26} eV, the energy of the most energetic electron is $3 \times 10^{26} \times 0.5 \times 10^{-26}$ eV = 1.5 eV.

Example 5.5: A copper atom has 29 electrons. Compute the fraction of (free) electrons in copper if its conductivity is 1.7×10^7 S/m. Assume that the electron mobility is 1.26×10^{-3} m²/V.s and the atomic density of copper is 2×10^{29} /m³ (metals have electrons only).

Solution:

The (free) electron density in copper is $\sigma/(q\mu_n) = 1.7 \times 10^7$ S/m / (1.6×10^{-19} C $\times$ 0.00126 m²/V.s) = 0.84×10^{29} /m³. The total number of electrons present in copper is $2 \times 10^{29} \times 29 = 5.8 \times 10^{30}$. The fraction of (free) electrons = 0.84×10^{29} /m³ / (5.8×10^{30} /m³) = 0.014.

Example 5.6: A wire made of an aluminum alloy is 1 mm in diameter and 1 m in length. a) If there is a current of 10 mA flowing and the voltage across the wire is 0.432 mV, determine its conductivity. b) If the electron mobility is 3×10^{-3} m²/V.s, what is the electron density? c) What is the drift velocity for the electrons?

Solution:

a) The resistance R is $V_a/I = 0.432 \times 10^{-3}$ V / (10×10^{-3} A) = 0.0432Ω . This gives a conductivity $\sigma = L/(A_{cs}R) = 1 \text{ m} / (3.14 \times (0.5 \times 10^{-3} \text{ m})^2 \times 0.0432 \Omega) = 29.5 \times 10^6$ S/m.

b) The electron density n is $\sigma/(q\mu_n) = 29.5 \times 10^6$ S/m / (1.6×10^{-19} C $\times$ 3×10^{-3} m²/V.s) = 6.14×10^{28} /m³.

c) The drift velocity v_{drift} is: $\mu_n E' = 3 \times 10^{-3}$ m²/V.s $\times$ 0.432×10^{-3} V / 1 m = 1.3×10^{-6} m/s.

Example 5.7: From Table II.6 (p.14), estimate the resistivity of copper if there is 0.1 wt % of chromium present (use Table II-6 for the value of σ_1 for copper).

Solution:

Since $\sigma = \sigma_1 / (1 + \beta\kappa)$. For impure copper, $\rho = 1/\sigma = 1/(5.8 \times 10^{-11} \text{ S/m}) \times (1 + 0.1 \times 2.47) = 2.19 \times 10^{-10} \text{ }\Omega\cdot\text{m}$.

Example 5.8: Determine the Hall coefficient R_H if the current passing through a solid is 1 mA, the width of the solid is 0.005 m, the cross-sectional area is $1 \times 10^{-6} \text{ m}^2$, the Hall voltage is 2.5 mV, and the magnetic induction is 0.1 T.

Solution:

Since $R_H = t_h V_H / (IB) = A_{cs} V_H / (IBd_w) = 1 \times 10^{-6} \text{ m}^2 \times 2.5 \times 10^{-3} \text{ V} / (1 \times 10^{-3} \text{ A} \times 0.1 \text{ T} \times 5 \times 10^{-3} \text{ m}) = 5 \times 10^{-3} \text{ m}^3/\text{C}$.

Example 5.9: A chromel-alumel thermocouple is used to monitor the temperature of a hot furnace. If the voltage output from the thermocouple is 10 mV and the cold junction is at 0 °C, calculate the furnace temperature.

Solution:

The temperature of the furnace is $T_h = \Delta V_{AA} / (K_{SA} - K_{SB}) = 10 \text{ mV} / 0.040 \text{ mV}/^\circ\text{C} = 250^\circ\text{C}$ (see [Table 2.7](#) for the value of $K_{SA} - K_{SB}$).

Example 5.10: An intrinsic semiconductor has a conductivity of 250 S/m at 20°C and 1100 S/m at 100°C. What is the value of the energy gap?

Solution:

Since $\ln(\sigma_1/\sigma_2) = E_g/(2kT_2) - E_g/(2kT_1)$. Solving for the energy gap gives: $E_g = 2k \ln(\sigma_1/\sigma_2) / (1/T_1 - 1/T_2) = 2 \times 86.2 \times 10^{-6} \text{ eV/K} \times \ln(1100 \text{ S/m}/250 \text{ S/m}) / (1/293 \text{ K} - 1/373 \text{ K}) = 0.349 \text{ eV}$.

Example 5.11: Calculate the intrinsic conductivity of GaAs at 50°C.

Solution:

For GaAs, the intrinsic conductivity $\sigma(\text{intrinsic}) = n_i q (\mu_n + \mu_p) = 1.8 \times 10^{12} / \text{m}^3 \times 1.6 \times 10^{-19} \text{ C} \times (0.85 \text{ m}^2/\text{V.s} + 0.04 \text{ m}^2/\text{V.s}) = 2.56 \times 10^{-7} \text{ S/m}$ (see Table 2.8 for the parameters).

Since $\sigma = \sigma_0' \exp(-E_g/(2kT))$, $\sigma(50^\circ\text{C}) = 2.56 \times 10^{-7} \text{ S/m} \times \exp(1.42 \times (1/300\text{K} - 1/323\text{K})/(2 \times 8.62 \times 10^{-5} \text{ eV/K})) = 1.8 \times 10^{-6} \text{ S/m}$.

Example 5.12: In a n-type semiconductor, the Fermi level is 0.1 eV above its intrinsic value ($= E_g/2$). If the energy gap is 1.11 eV, what is the probability that an electron is found in the conduction band at 25°C .

Solution:

The Fermi level is $E_F = E_c - E_F(\text{intrinsic}) + 0.1 \text{ eV} = E_c - (1.11/2) + .1 \text{ eV} = E_c - 0.455 \text{ eV}$. The probability of finding an electron in the conduction band is $f(E_c) = 1/(\exp(E_c - E_F)/(kT) + 1) = 1/(\exp(0.455 \text{ eV}/(86.2 \times 10^{-6} \text{ eV/K} \times 298\text{K})) + 1) = 2.2 \times 10^{-8}$.

Example 5.13: A n-type semiconductor has a donor density $N_D = 1 \times 10^{22} / \text{m}^3$. At what temperature will the intrinsic carrier density equal to N_D ? We are given that $N_C = 2.46 \times 10^{25} / \text{m}^3$, $N_V = 1 \times 10^{25} / \text{m}^3$ and $E_g = 1.1 \text{ eV}$ (these parameters are assumed to be temperature independent).

Solution:

The intrinsic carrier density $n_i = (N_C N_V)^{0.5} \exp(-E_g/(2kT))$. This leads to $T = E_g/(2k \ln((N_C N_V)^{0.5} / N_D)) = 1.1 \text{ eV} \times 1.6 \times 10^{-19} \text{ J/eV} / (2 \times 1.38 \times 10^{-23} \text{ J/K} \times \ln((2.46 \times 10^{25} / \text{m}^3 \times 1 \times 10^{25} / \text{m}^3)^{0.5} / (1 \times 10^{22} / \text{m}^3))) = 867\text{K}$.

Example 4.14: List the electron and hole densities of silicon at 300K if it is a) intrinsic; b) doped with $1 \times 10^{20} / \text{m}^3$ of donors (N_D); c) doped with $1 \times 10^{20} / \text{m}^3$ of acceptors (N_A); and d) doped with $1 \times 10^{20} / \text{m}^3$ of donors (N_D) and $0.5 \times 10^{20} / \text{m}^3$ of acceptors (N_A). Note that: $n_i = 1.45 \times 10^{16} / \text{m}^3$.

Solution:

For silicon:	N_D	N_A	n	p	n_i
	0	0	n_i	n_i	n_i

10^{20}	0	10^{20}	$n_i^2/10^{20}$	n_i
0	10^{20}	$n_i^2/10^{20}$	10^{20}	n_i
1.0×10^{20}	0.5×10^{20}	0.5×10^{20}	$n_i^2/0.5 \times 10^{20}$	n_i

Example 5.15: A parallel plate capacitor has the following properties: $A_{cs} = 2 \times 10^{-3} \text{ m}^2$, $d = 0.01 \text{ m}$, $V_a = 100 \text{ V}$ and $\epsilon_r = 6$. a) Find the capacitance. b) If there are $1 \times 10^{15} \text{ dipoles/m}^3$ in the insulator, what is the polarizability? c) Determine the amount of surface charge on the capacitor plates. d) Repeat part c) in the absence of the insulator. e) What is the amount of polarization charge?

Solution:

- a) The capacitance $C = \epsilon_i A_{cs}/d = 6 \times 8.86 \times 10^{-12} \text{ F/m} \times 2 \times 10^{-3} \text{ m}^2 / (1 \times 10^{-2} \text{ m}) = 1.06 \times 10^{-11} \text{ F}$.
- b) The relative permittivity $\epsilon_r = 6 = 1 + \alpha' N_{\text{dipole}} / \epsilon_0$. Therefore, the polarizability $\alpha' = (6 - 1) \times 8.85 \times 10^{-12} \text{ F/m} / (1 \times 10^{15} / \text{m}^3) = 4.43 \times 10^{-26} \text{ F.m}^2$.
- c) The stored charge $QA_{cs} = CV_a = 1.06 \times 10^{-11} \text{ F} \times 100 \text{ V} = 1.06 \times 10^{-9} \text{ C}$.
- d) The stored charge without the insulator $Q_o A_{cs} = C_o V = CV_a / \epsilon_r = 1.06 \times 10^{-9} \text{ C} / 6 = 0.176 \times 10^{-9} \text{ C}$.
- e) The polarization charge $\Delta QA_{cs} = (Q - Q_o) A_{cs} = (1.06 - 0.176) \times 10^{-9} \text{ C} = 0.88 \times 10^{-9} \text{ C}$.

Example 5.16: Compute the polarizability of a water molecule if its dipole moment $p_o = 1 \times 10^{-29} \text{ C.m}$, $E' = 1 \times 10^5 \text{ V/m}$, and $T = 300\text{K}$. For ionic molecules, $\alpha' = p_o^2 / (3kT)$.

Solution:

For a water molecule, the polarizability $\alpha' = p_o^2 / (3kT) = (1 \times 10^{-29} \text{ C.m})^2 / (3 \times 1.38 \times 10^{-23} \text{ J/K} \times 300\text{K}) = 8.05 \times 10^{-39} \text{ F.m}^2$.

Example 5.17: Defining the dipole moment as the product of the dipole charge times separation, compute the dipole moment for the tetragonal BaTiO_3 unit cell shown in Fig.11-4-3 (p.43). Assume that the Ba^{2+} ions are stationary.

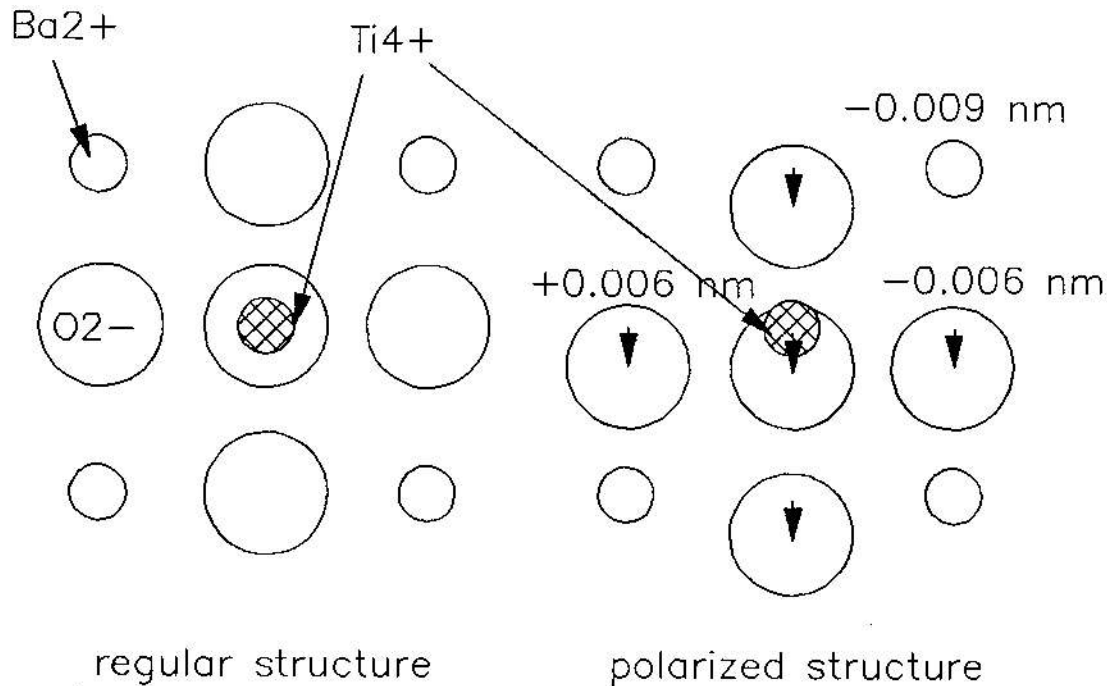


Fig.11-45: Polarization of BaTiO_3 crystal

Solution:

For the given unit cell structure, there is the equivalence of 3 O^{2-} ions, 1 Ti^{4+} ion and 1 Ba^{2+} ion. In the figure, the Ti^{4+} ion is shifted upward and the O^{2-} ions are shifted downward (note that the equatorial O^{2-} ions are shifted by a smaller amount when compared with the corner O^{2-} ions).

The dipole moment related to the shift in the Ti^{4+} ion is $p_{\text{Ti}^{4+}} = 4 \times 1.6 \times 10^{-19} \text{ C} \times 0.006 \times 10^{-9} \text{ m} = 3.84 \times 10^{-30} \text{ C.m}$. The dipole moment related to the shift in the O^{2-} ion is $p_{\text{O}^{2-}} = 2 \times 1.6 \times 10^{-19} \text{ C} \times (2 \times 0.006 \times 10^{-9} \text{ m} + 0.009 \times 10^{-9} \text{ m}) = 6.72 \times 10^{-30} \text{ C.m}$.

Since the Ba^{2+} ions are considered to be stationary, the total dipole moment is $p_{\text{Ti}^{4+}} + p_{\text{O}^{2-}} = (3.84 + 6.72) \times 10^{-30} \text{ C.m} = 1.06 \times 10^{-29} \text{ C.m}$.

Example 5.18: The equivalent circuit for a lossy capacitor is shown in Fig.E2.1, calculate the loss factor. Assume $V_a = V_o \sin(\omega t)$.

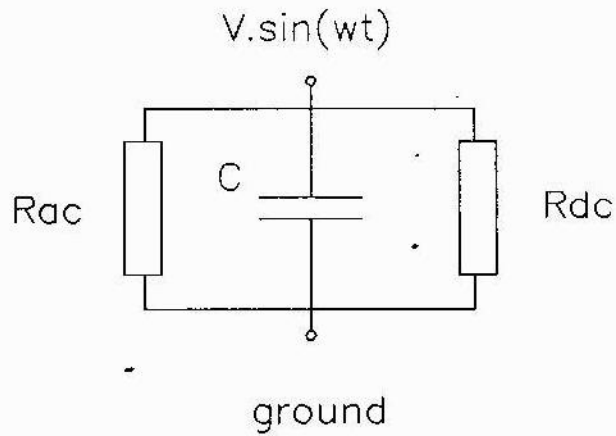


Fig.E2.1

Solution:

The energy stored per cycle in the capacitor is $E_{\text{stored}} = \int_{\text{cycle}} V_a i \, dt = \int_{\text{cycle}} V_o^2 \omega C \sin(\omega t) \cos(\omega t) \, dt = -V_o^2 C (\cos(2\omega t) - 1)/4$. Its peak value $= V_o^2 C/2$. Energy loss due to the resistor is given by $E'' = V_o^2/R_{ac} \times \int_{\text{cycle}} \sin(\omega t) \sin(\omega t) \, dt = V_o^2 \pi/(\omega R_{ac})$. The loss factor $\tan(\delta) = V_o^2 \pi/((\omega R_{ac}) \times (2 \pi V_o^2 C/2)) = 1/(\omega C R_{ac})$.