\[
e^{j\theta} = \cos \theta + j \sin \theta \\
\cos \theta = \frac{1}{2} \left( e^{j\theta} + e^{-j\theta} \right) \\
\sin \theta = \frac{1}{2j} \left( e^{j\theta} - e^{-j\theta} \right)
\]

\[
x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j(2\pi/T)kt} \\
\mathcal{F} x(\omega) = X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \, dt \\
\mathcal{F}^{-1} X(\omega) = \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} \, d\omega \\
S(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \omega - k\omega_b \\
H(\omega) = \frac{1}{\omega_b} \text{rect} \left( \frac{\omega}{\omega_b} \right)
\]

### Fourier Transform Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Time Domain</th>
<th>Frequency Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linearity</td>
<td>(a_1 x_1(t) + a_2 x_2(t))</td>
<td>(a_1 X_1(\omega) + a_2 X_2(\omega))</td>
</tr>
<tr>
<td>Time-Domain Shifting</td>
<td>(x(t - t_0))</td>
<td>(e^{-j\omega t_0} X(\omega))</td>
</tr>
<tr>
<td>Frequency-Domain Shifting</td>
<td>(e^{j\omega t_0} x(t))</td>
<td>(X(\omega - \omega_0))</td>
</tr>
<tr>
<td>Time/Frequency-Domain Scaling</td>
<td>(x(at))</td>
<td>(\frac{1}{</td>
</tr>
<tr>
<td>Conjugation</td>
<td>(x^*(t))</td>
<td>(X^*(-\omega))</td>
</tr>
<tr>
<td>Duality</td>
<td>(X(t))</td>
<td>(2\pi \delta(\omega))</td>
</tr>
<tr>
<td>Time-Domain Convolution</td>
<td>(x_1 \ast x_2(t))</td>
<td>(X_1(\omega) X_2(\omega))</td>
</tr>
<tr>
<td>Frequency-Domain Convolution</td>
<td>(x_1(t) x_2(t))</td>
<td>(\frac{1}{T} \delta(\omega - \omega_0))</td>
</tr>
<tr>
<td>Time-Domain Differentiation</td>
<td>(\frac{d}{dt} x(t))</td>
<td>(j\omega X(\omega))</td>
</tr>
<tr>
<td>Frequency-Domain Differentiation</td>
<td>(t x(t))</td>
<td>(\frac{j}{\omega_0} X(\omega))</td>
</tr>
<tr>
<td>Time-Domain Integration</td>
<td>(\int_{-\infty}^{\infty} x(t) d\tau)</td>
<td>(\frac{1}{\pi^2} X(\omega) + \pi X(0) \delta(\omega))</td>
</tr>
<tr>
<td>Parseval’s Relation</td>
<td>(\int_{-\infty}^{\infty}</td>
<td>x(t)</td>
</tr>
</tbody>
</table>

### Fourier Transform Pairs

<table>
<thead>
<tr>
<th>Pair</th>
<th>(x(t))</th>
<th>(X(\omega))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\delta(t))</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>(u(t))</td>
<td>(\pi \delta(\omega) + \frac{1}{j\omega})</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>(2\pi \delta(\omega))</td>
</tr>
<tr>
<td>4</td>
<td>(\text{sgn}(t))</td>
<td>(\frac{1}{j\omega})</td>
</tr>
<tr>
<td>5</td>
<td>(e^{j\omega t})</td>
<td>(2\pi \delta(\omega - \omega_0))</td>
</tr>
<tr>
<td>6</td>
<td>(\cos(\omega t))</td>
<td>(\pi [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)])</td>
</tr>
<tr>
<td>7</td>
<td>(\sin(\omega t))</td>
<td>(\frac{\pi}{j} [\delta(\omega - \omega_0) - \delta(\omega + \omega_0)])</td>
</tr>
<tr>
<td>8</td>
<td>(\text{rect}\left(\frac{t}{T}\right))</td>
<td>(</td>
</tr>
<tr>
<td>9</td>
<td>(\text{sinc}(Bt))</td>
<td>(\frac{\pi}{B} \text{rect} \left( \frac{B\omega}{2\pi} \right))</td>
</tr>
<tr>
<td>10</td>
<td>(e^{-at} u(t)), (\text{Re}{a} &gt; 0)</td>
<td>(\frac{1}{at + j\omega})</td>
</tr>
<tr>
<td>11</td>
<td>(e^{at} e^{-at} u(t)), (\text{Re}{a} &gt; 0)</td>
<td>(\frac{1}{(a+1)t + j\omega})</td>
</tr>
<tr>
<td>12</td>
<td>(e^{-at} \cos(\omega t) u(t)), (\text{Re}{a} &gt; 0)</td>
<td>(\frac{a+1}{(a+1)t + j\omega})</td>
</tr>
<tr>
<td>13</td>
<td>(e^{-at} \sin(\omega t) u(t)), (\text{Re}{a} &gt; 0)</td>
<td>(\frac{(a+1)^2 + \omega^2}{(a+1)^2 + \omega^2})</td>
</tr>
<tr>
<td>14</td>
<td>(e^{at} u(-t)), (\text{Re}{a} &gt; 0)</td>
<td>(\frac{1}{-at + j\omega})</td>
</tr>
</tbody>
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