

Useful Formulae and Other Information

$$\begin{aligned}
 x(t) &= \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} & \mathcal{F}x(\omega) = X(\omega) &= \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt & X(\omega) &= \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0) \\
 c_k &= \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt & \mathcal{F}^{-1}X(t) = x(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega & X(\omega) &= \sum_{k=-\infty}^{\infty} \omega_0 X_T(k\omega_0) \delta(\omega - k\omega_0) \\
 & & & & a_k &= \frac{1}{T} X_T(k\omega_0)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{L}x(s) = X(s) &= \int_{-\infty}^{\infty} x(t) e^{-st} dt & \mathcal{L}_u x(s) = X(s) &= \int_{0^-}^{\infty} x(t) e^{-st} dt \\
 \mathcal{L}^{-1}X(t) = x(t) &= \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} X(s) e^{st} ds
 \end{aligned}$$

$$\begin{aligned}
 e^{j\theta} &= \cos \theta + j \sin \theta \\
 \cos \theta &= \frac{1}{2} \left[e^{j\theta} + e^{-j\theta} \right] \\
 \sin \theta &= \frac{1}{2j} \left[e^{j\theta} - e^{-j\theta} \right]
 \end{aligned}$$

x	$\cos x$	$\sin x$
0	1	0
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0	1
$\frac{3\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
π	-1	0
$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
$\frac{3\pi}{2}$	0	-1
$\frac{7\pi}{4}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$

$$\begin{aligned}
 A_k &= (v - p_k) F(v) \big|_{v=p_k} \\
 A_{kl} &= \frac{1}{(q_k - l)!} \left[\frac{d^{q_k-l}}{dv^{q_k-l}} [(v - p_k)^{q_k} F(v)] \right] \bigg|_{v=p_k} \\
 ax^2 + bx + c &= 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
 \end{aligned}$$

Fourier Series Properties

Property	Time Domain	Fourier Domain
Linearity	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time-Domain Shifting	$x(t - t_0)$	$e^{-jk\omega_0 t_0} a_k$
Time Reversal	$x(-t)$	a_{-k}

Fourier Transform Properties

Property	Time Domain	Frequency Domain
Linearity	$a_1 x_1(t) + a_2 x_2(t)$	$a_1 X_1(\omega) + a_2 X_2(\omega)$
Time-Domain Shifting	$x(t - t_0)$	$e^{-j\omega t_0} X(\omega)$
Frequency-Domain Shifting	$e^{j\omega_0 t} x(t)$	$X(\omega - \omega_0)$
Time/Frequency-Domain Scaling	$x(at)$	$\frac{1}{ a } X\left(\frac{\omega}{a}\right)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Time-Domain Convolution	$x_1 * x_2(t)$	$X_1(\omega) X_2(\omega)$
Frequency-Domain Convolution	$x_1(t) x_2(t)$	$\frac{1}{2\pi} X_1 * X_2(\omega)$
Time-Domain Differentiation	$\frac{d}{dt} x(t)$	$j\omega X(\omega)$
Frequency-Domain Differentiation	$tx(t)$	$j \frac{d}{d\omega} X(\omega)$
Time-Domain Integration	$\int_{-\infty}^t x(\tau) d\tau$	$\frac{1}{j\omega} X(\omega) + \pi X(0) \delta(\omega)$
Parseval's Relation	$\int_{-\infty}^{\infty} x(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) ^2 d\omega$	

Fourier Transform Pairs

Pair	$x(t)$	$X(\omega)$
1	$\delta(t)$	1
2	$u(t)$	$\pi \delta(\omega) + \frac{1}{j\omega}$
3	1	$2\pi \delta(\omega)$
4	$\text{sgn}(t)$	$\frac{2}{j\omega}$
5	$e^{j\omega_0 t}$	$2\pi \delta(\omega - \omega_0)$
6	$\cos(\omega_0 t)$	$\pi [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
7	$\sin(\omega_0 t)$	$\frac{\pi}{j} [\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
8	$\text{rect}(t/T)$	$ T \text{sinc}(T\omega/2)$
9	$\text{sinc}(Bt)$	$\frac{\pi}{ B } \text{rect}\left(\frac{\omega}{2B}\right)$
10	$e^{-at} u(t), \text{Re}\{a\} > 0$	$\frac{1}{a + j\omega}$
11	$t^{n-1} e^{-at} u(t), \text{Re}\{a\} > 0$	$\frac{(n-1)!}{(a + j\omega)^n}$

Bilateral Laplace Transform Properties

Property	Time Domain	Laplace Domain	ROC
Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(s) + a_2X_2(s)$	At least $R_1 \cap R_2$
Time-Domain Shifting	$x(t - t_0)$	$e^{-st_0}X(s)$	R
Laplace-Domain Shifting	$e^{s_0t}x(t)$	$X(s - s_0)$	$R + \text{Re}\{s_0\}$
Time/Laplace-Domain Scaling	$x(at)$	$\frac{1}{ a }X\left(\frac{s}{a}\right)$	aR
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Time-Domain Convolution	$x_1 * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
Time-Domain Differentiation	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
Laplace-Domain Differentiation	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Time-Domain Integration	$\int_{-\infty}^t x(\tau)d\tau$	$\frac{1}{s}X(s)$	At least $R \cap \{\text{Re}\{s\} > 0\}$

Property

Initial Value Theorem	$x(0^+) = \lim_{s \rightarrow \infty} sX(s)$
Final Value Theorem	$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s)$

Unilateral Laplace Transform Properties

Property	Time Domain	Laplace Domain
Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(s) + a_2X_2(s)$
Laplace-Domain Shifting	$e^{s_0t}x(t)$	$X(s - s_0)$
Time/Laplace-Domain Scaling	$x(at), a > 0$	$\frac{1}{a}X\left(\frac{s}{a}\right)$
Conjugation	$x^*(t)$	$X^*(s^*)$
Time-Domain Convolution	$x_1 * x_2(t), x_1 \text{ and } x_2 \text{ are causal}$	$X_1(s)X_2(s)$
Time-Domain Differentiation	$\frac{d}{dt}x(t)$	$sX(s) - x(0^-)$
Laplace-Domain Differentiation	$-tx(t)$	$\frac{d}{ds}X(s)$
Time-Domain Integration	$\int_0^t x(\tau)d\tau$	$\frac{1}{s}X(s)$

Property

Initial Value Theorem	$x(0^+) = \lim_{s \rightarrow \infty} sX(s)$
Final Value Theorem	$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s)$

Bilateral Laplace Transform Pairs

Pair	$x(t)$	$X(s)$	ROC
1	$\delta(t)$	1	All s
2	$u(t)$	$\frac{1}{s}$	$\text{Re}\{s\} > 0$
3	$-u(-t)$	$\frac{1}{s}$	$\text{Re}\{s\} < 0$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$	$\text{Re}\{s\} > 0$
5	$-t^n u(-t)$	$\frac{n!}{s^{n+1}}$	$\text{Re}\{s\} < 0$
6	$e^{-at}u(t)$	$\frac{1}{s+a}$	$\text{Re}\{s\} > -a$
7	$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\text{Re}\{s\} < -a$
8	$t^n e^{-at}u(t)$	$\frac{n!}{(s+a)^{n+1}}$	$\text{Re}\{s\} > -a$
9	$-t^n e^{-at}u(-t)$	$\frac{n!}{(s+a)^{n+1}}$	$\text{Re}\{s\} < -a$
10	$\cos(\omega_0 t)u(t)$	$\frac{s}{s^2 + \omega_0^2}$	$\text{Re}\{s\} > 0$
11	$\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{s^2 + \omega_0^2}$	$\text{Re}\{s\} > 0$
12	$e^{-at} \cos(\omega_0 t)u(t)$	$\frac{s+a}{(s+a)^2 + \omega_0^2}$	$\text{Re}\{s\} > -a$
13	$e^{-at} \sin(\omega_0 t)u(t)$	$\frac{\omega_0}{(s+a)^2 + \omega_0^2}$	$\text{Re}\{s\} > -a$

Unilateral Laplace Transform Pairs

Pair	$x(t)$	$X(s)$
1	$\delta(t)$	1
2	1	$\frac{1}{s}$
3	t^n	$\frac{n!}{s^{n+1}}$
4	e^{-at}	$\frac{1}{s+a}$
5	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
6	$\cos(\omega_0 t)$	$\frac{s}{s^2 + \omega_0^2}$
7	$\sin(\omega_0 t)$	$\frac{\omega_0}{s^2 + \omega_0^2}$
8	$e^{-at} \cos(\omega_0 t)$	$\frac{s+a}{(s+a)^2 + \omega_0^2}$
9	$e^{-at} \sin(\omega_0 t)$	$\frac{\omega_0}{(s+a)^2 + \omega_0^2}$