

**Answer (g).**

We are asked to find the Fourier transform  $Y$  of

$$y(t) = \left[ t e^{-j5t} x(t) \right]^*.$$

In what follows, we use the **prime symbol to denote the derivative** (i.e.,  $f'$  denotes the derivative of  $f$ ). To begin, we have

$$\begin{aligned} y(t) &= \left[ t e^{-j5t} x(t) \right]^* \\ &= \left[ e^{-j5t} \underbrace{t x(t)}^{\text{red bracket}} \right]^*. \end{aligned}$$

Letting  $v_1(t) = t x(t)$ , we have

$$v_1(t) = t x(t) \quad \textcircled{1}$$

$$y(t) = \left[ e^{-j5t} v_1(t) \right]^*.$$

Letting  $v_2(t) = e^{-j5t} v_1(t)$ , we have

$$v_2(t) = e^{-j5t} v_1(t) \quad \textcircled{2}$$

$$y(t) = v_2^*(t). \quad \textcircled{3}$$

Thus, we have written  $y(t)$  as

$$\textcircled{3} \rightarrow y(t) = v_2^*(t)$$

where

$$\textcircled{1} \rightarrow v_1(t) = t x(t) \quad \text{and}$$

$$\textcircled{2} \rightarrow v_2(t) = e^{-j5t} v_1(t).$$

Taking the Fourier transforms of the preceding equations, we obtain

$$\begin{aligned} \textcircled{4} \quad V_1(\omega) &= jX'(\omega), & \leftarrow \text{FT of } \textcircled{1} \text{ using frequency-domain differentiation property} \\ \textcircled{5} \quad V_2(\omega) &= V_1(\omega + 5), \quad \text{and} & \leftarrow \text{FT of } \textcircled{2} \text{ using frequency-domain shifting property} \\ \textcircled{6} \quad Y(\omega) &= V_2^*(-\omega). & \leftarrow \text{FT of } \textcircled{3} \text{ using conjugation property} \end{aligned}$$

Combining the above equations, we have

$$\begin{aligned} Y(\omega) &= V_2^*(-\omega) & \leftarrow \textcircled{6} \\ &= [V_1(-\omega + 5)]^* & \leftarrow \text{substitute } \textcircled{5} \\ &= [jX'(-\omega + 5)]^* & \leftarrow \text{substitute } \textcircled{4} \\ &= -jX'^*(-\omega + 5). & \leftarrow (ab)^* = a^* b^* \end{aligned}$$