An Incremental/Decremental Delaunay Mesh-Generation Framework for Image Representation

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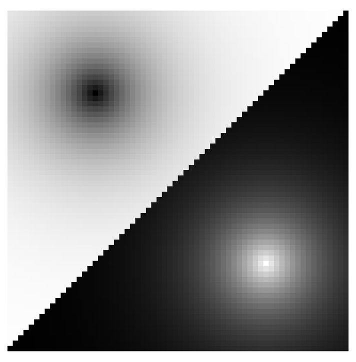
Motivation

- growing interest in image representations that are based on nonuniform sampling and also attempt to exploit geometric structure in images (e.g., image edges)
- triangle meshes well suited to nonuniform sampling as well as capturing geometric structure in images
- mesh representations of images useful in many diverse areas such as: enhancement, tomographic reconstruction, pattern recognition, computer vision, image/video coding

Conceptual Model of Image

- image modelled as function defined on continuous domain



Image

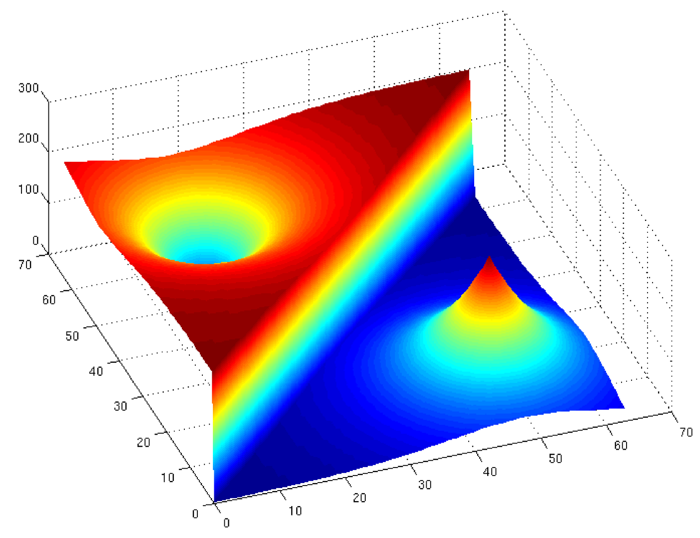


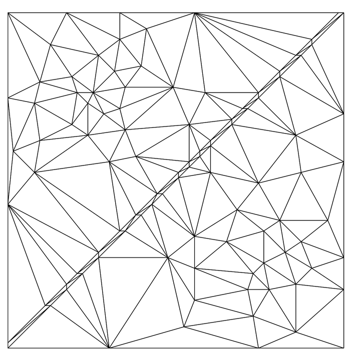
Image Modelled as Surface

Mesh Model

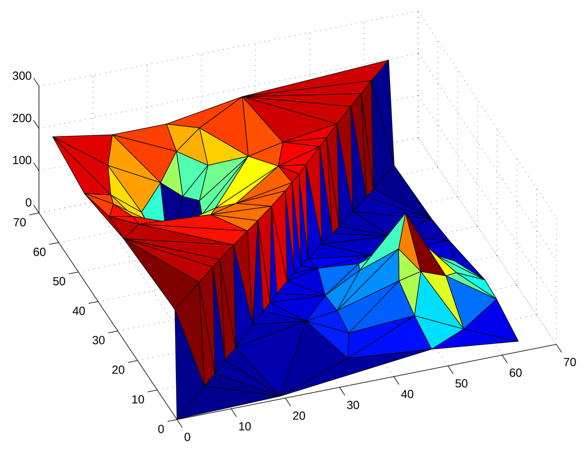
- mesh model of image ϕ defined on $\Lambda = \{0,1,\ldots,W-1\} \times \{0,1,\ldots,H-1\}$ (i.e., rectangular grid of width W and height H) completely characterized by:
  1. set $P = \{p_i\}_{i=1}^{|P|}$ of sample points
  2. set $Z = \{z_i\}_{i=1}^{|P|}$ of corresponding sample values (i.e., $z_i = \phi(p_i)$)
- P always chosen to include extreme convex hull points of Λ so triangulation of P covers entire image domain Λ
- mesh determined from model parameters as follows:
  1. construct triangulation of P
  2. for each face in triangulation with vertices (x_i, y_i), (x_j, y_j), and (x_k, y_k), and their respective sample values z_i, z_j, and z_k, form unique planar interpolant passing through points (x_i, y_i, z_i), (x_j, y_j, z_j), and (x_k, y_k, z_k)
  3. combine interpolants from faces to obtain continuous piecewise-planar interpolant that approximates ϕ over entire image domain Λ
- sampling density of mesh model defined as $|P| / |\Lambda|$

Mesh Approximation of Image (Sampling Density 2.5%)

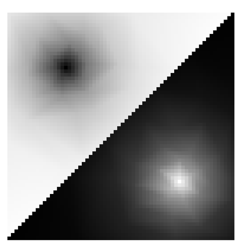
- reconstructed image obtained from mesh model by scan conversion



Triangulation of Image Domain



Resulting Triangle Mesh Approximation



Reconstructed Image

Mesh-Generation Framework

- proposed framework is iterative
- starts from initial mesh P_0; points added and deleted until target mesh size achieved and no further modifications desired
- in each iteration, point is either added or deleted using one of following operations:
  1. optimal-add operation: in face with highest squared error, point with highest candidate error is added to mesh
  2. optimal-delete operation: deletes point in mesh that will cause least increase in squared error

Bad-Point Replacement (BPR)

- propose postprocessing step to be applied after mesh generation called bad-point replacement (BPR)
- point in mesh said to be bad if its deletion from mesh would not cause approximation error to increase
- BPR scheme removes bad points from mesh, substituting new points in their place
- in more detail, consists of following steps:
  1. while point p that would be deleted by next optimal-delete operation is bad, perform optimal-delete operation, and mark p as permanently removed from mesh; if no points deleted in this step, stop
  2. perform n optimal-add operations, where n is number of points deleted in step 1
  3. go to step 1

IDDT Mesh-Generation Method

- IDDT mesh-generation method based on framework from above
- by using both optimal-add and optimal-delete operations, number of mesh points increased until target number N of points is reached
- finally, BPR scheme employed to remove any bad points
- in more detail, consists of following steps:
  1. let $n = N - |P|$ (where P is set of points currently in mesh); if $n \leqslant 0$, go to step 5
  2. perform n optimal-add operations
  3. perform $\lfloor n/2 \rfloor$ optimal-delete operations
  4. go to step 1
  5. apply BPR method to mesh

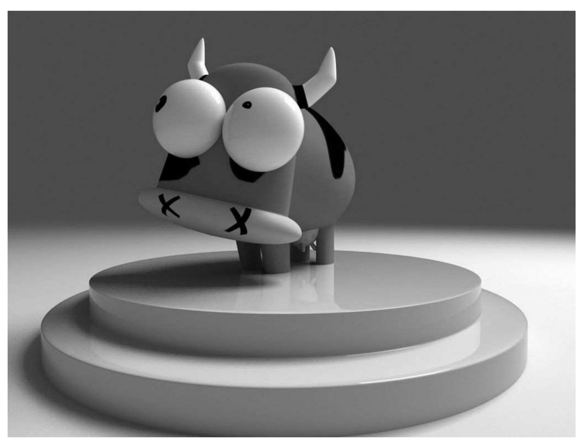
IDDT PSNR Performance Evaluation

- to evaluate PSNR performance of IDDT scheme, compare to error diffusion (ED) method of Yang et al. [3] and modified Garland-Heckbert (MGH) method [1] inspired by [2]

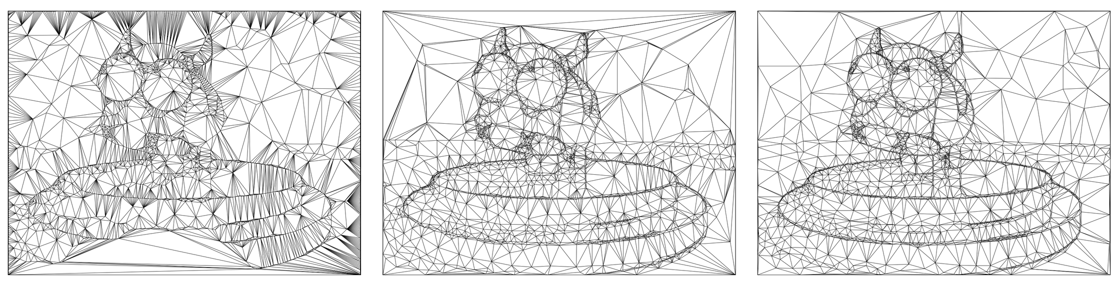
Image	Samp. Density (%)	PSNR (dB) ED	MGH	IDDT
bull	0.25	20.59	35.29	**37.56**
	0.50	25.89	38.76	**40.48**
	1.00	33.34	41.07	**42.46**
	2.00	37.56	43.07	**44.38**
peppers	0.50	16.03	24.68	**26.50**
	1.00	21.35	27.53	**29.15**
	2.00	26.09	29.85	**31.31**
	3.00	28.17	31.13	**32.45**

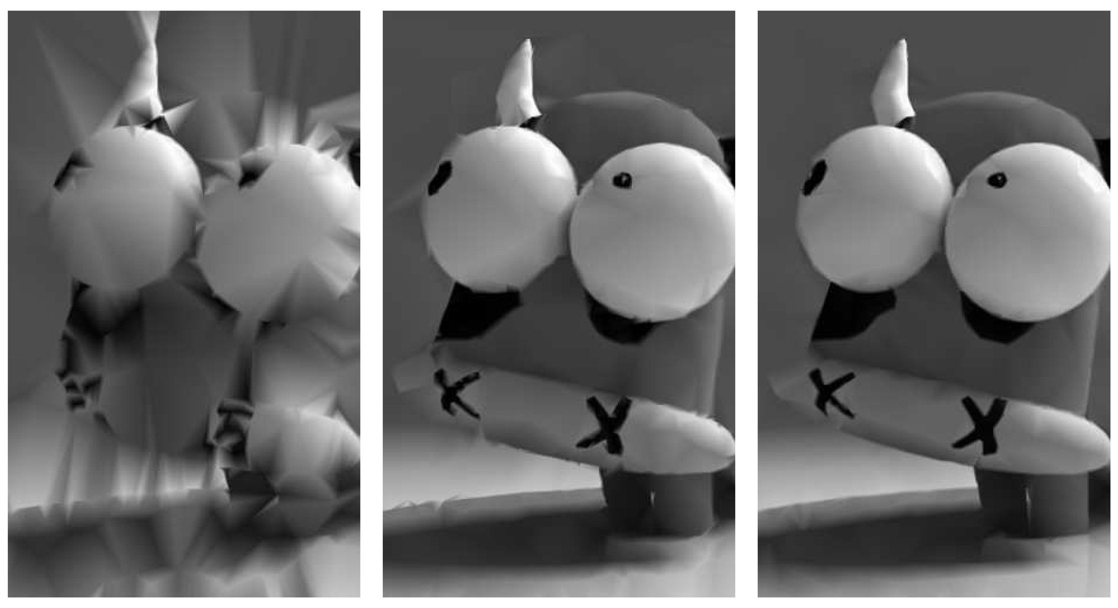
- IDDT method outperforms ED and MGH schemes by about 4.28 to 16.97 dB and 1.31 to 2.27 dB, respectively

IDDT Subjective Performance Evaluation: bull Image, Sampling Density 0.25%



Original (1024 × 768)





ED (20.59 dB) | MGH (35.29 dB) | IDDT (37.53 dB)

- clearly, IDDT method yields image reconstructions with better subjective quality than those obtained with ED and MGH schemes

BPR Performance Evaluation

Image	Samp. Density (%)	PSNR (dB) ED	ED with BPR	MGH	MGH with BPR
bull	0.25	20.59	**36.44**	35.29	**36.55**
	0.50	25.89	**40.15**	38.76	**40.13**
	1.00	33.34	**42.21**	41.07	**42.09**
	2.00	37.56	**44.06**	43.07	**43.97**
peppers	0.50	16.03	**25.59**	24.68	**25.66**
	1.00	21.35	**28.78**	27.53	**28.38**
	2.00	26.09	**31.09**	29.85	**30.76**
	3.00	28.17	**32.19**	31.13	**31.89**

- using BPR as postprocessing step with each of ED and MGH methods improves mesh quality by 4.02 to 15.85 dB and 0.76 to 1.37 dB, respectively

Conclusions

- proposed IDDT method for mesh generation yields superior meshes relative to other schemes, as demonstrated by experimental results
- BPR scheme, used in IDDT method, can also be used to optimize meshes produced by other mesh-generation methods

References

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[3] Y. Yang, M. N. Wernick, and J. G. Brankov.
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IEEE Trans. on Image Processing, 12(8):866–881, Aug. 2003.