An Optimization-Based Mesh-Generation Method for Image Representation

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2015 IEEE Pacific Rim Conference
August 2015

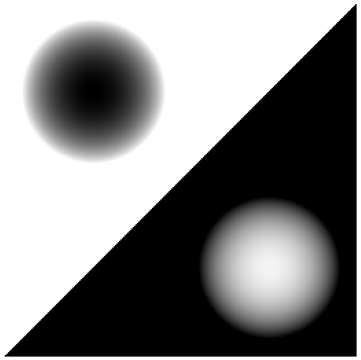
University
of Victoria

Outline

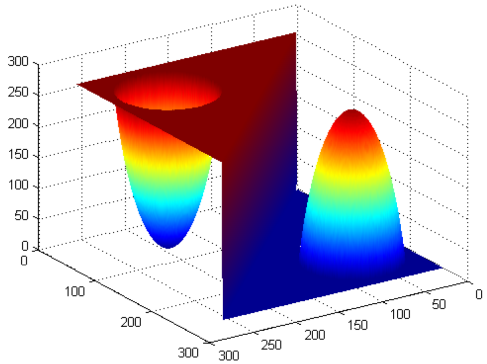
- Introduction to triangle-mesh models of images
- Background
- Proposed mesh-generation method
- Results
- Conclusions

Introduction

Triangle Mesh Models of Images



original image



surface model

Introduction

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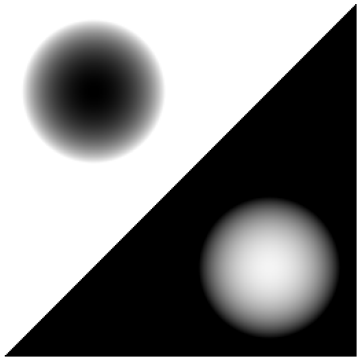

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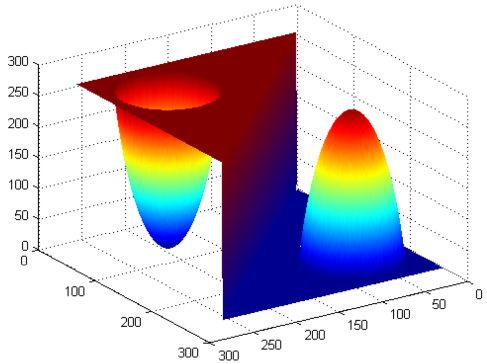
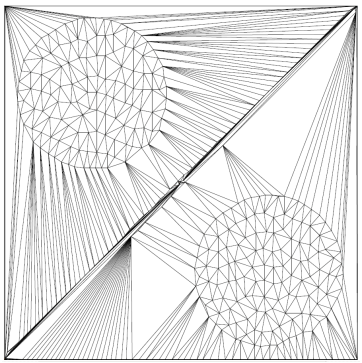
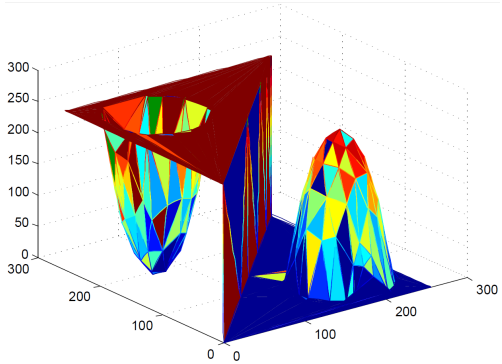


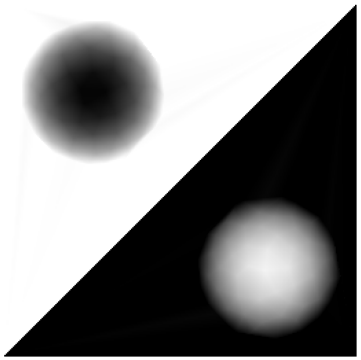
surface model



triangulation

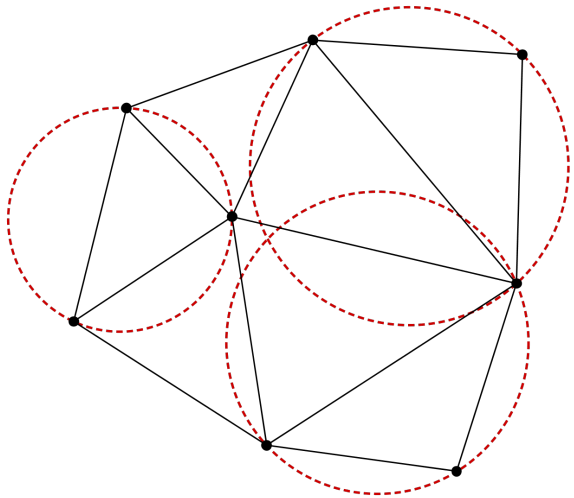


triangle-mesh model

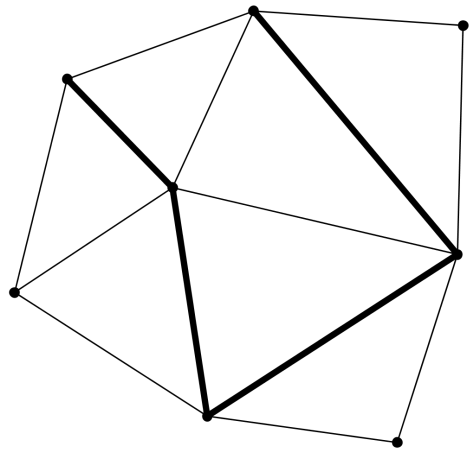


reconst. image

- Two classes of triangulation:
  - Delaunay triangulation (DT)
  - Constrained Delaunay triangulation (CDT)



DT



CDT

Mesh-Generation Problem

Find a mesh model of original image ϕ defined in Λ with N number of points that minimizes the measure ϵ of error between ϕ and reconstructed image $\hat{\phi}$

- Mean squared error: $\epsilon = \frac{1}{|\Lambda|} \sum_{(x,y)\in\Lambda} \left(\hat{\phi}(x,y) - \phi(x,y)\right)^2$
- Peak signal-to-noise ratio: $PSNR_{dB} = 20 \log_{10} \left(\frac{2^\rho - 1}{\sqrt{\epsilon}}\right)$
- ρ is the number of bits/pixel in the image
- Sampling density: $d = \frac{N}{|\Lambda|}$

Background

ERD Mesh Model

- Explicit representation of discontinuities (ERD)
- Originally proposed by Tu and Adams in 2013
- Based on constrained Delaunay triangulation (CDT)
- Image discontinuities allowed across constrained edges of triangulation
- Wedge-based discontinuity modeling

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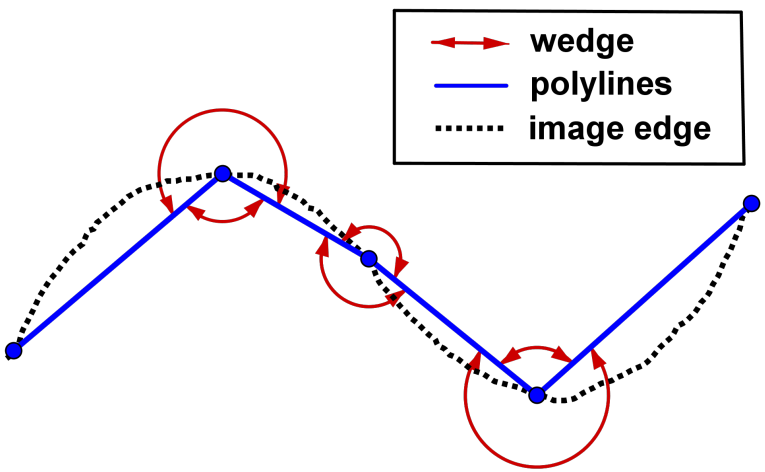

image edge representation

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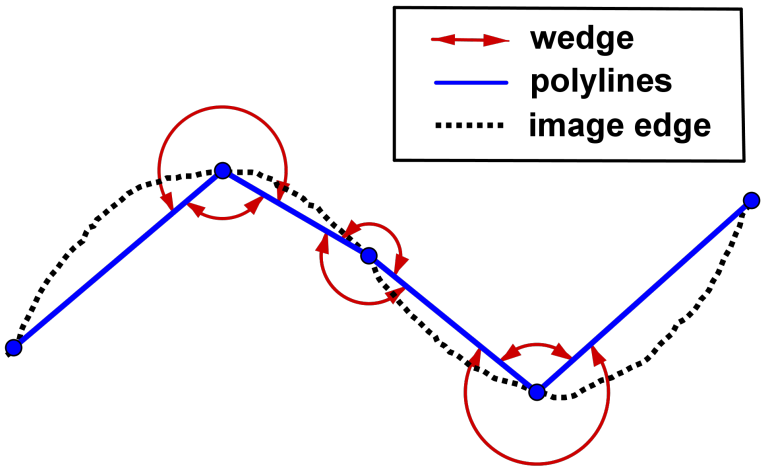
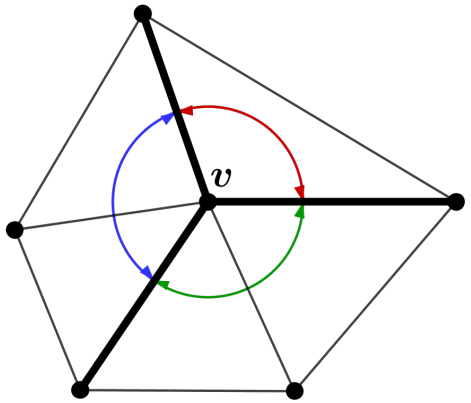
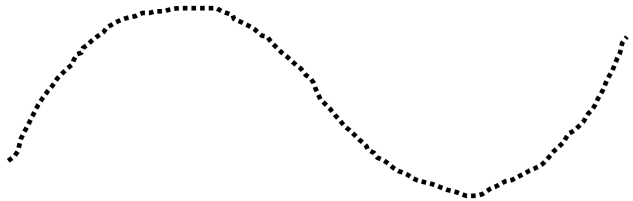


image edge representation



three wedges

1. *Initial triangulation.*
   - Detect image edges
   - Edges are approximated using polylines
   - Construct a constrained Delaunay triangulation



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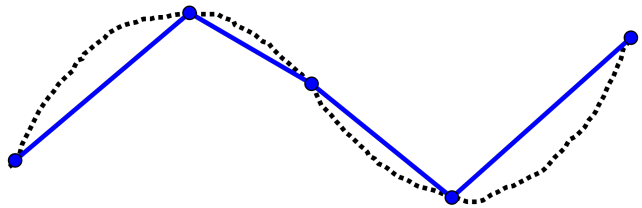

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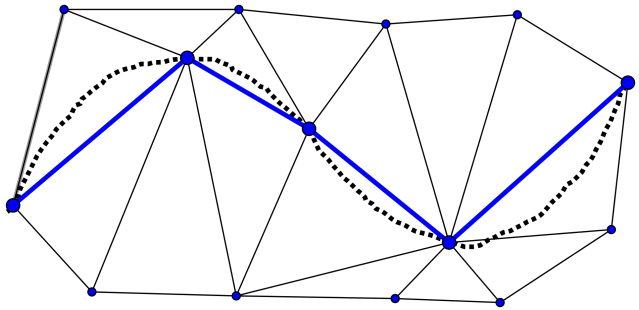

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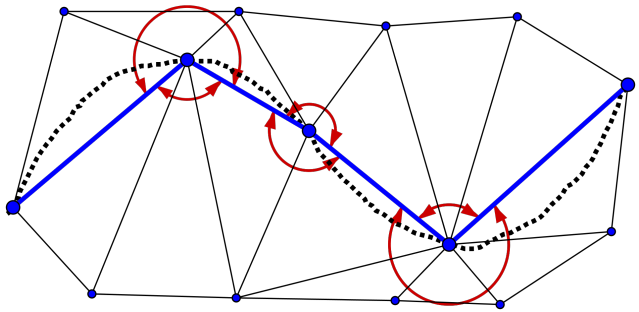

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3. *Point Selection.* Select a new sample point to add to the mesh by the error-diffusion technique
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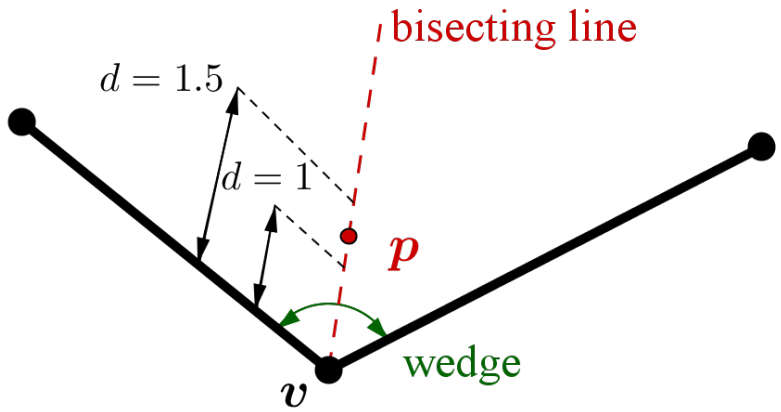
5. *Stopping criterion.* If the desired number of sample points is not achieved, go to step 3.

Background

Wedge-Value Calculation - Local Line Search

- The main focus of our work
- ERDED employs a local line search to calculate wedge values
- Line search is limited to $d \in [1, 1.5]$



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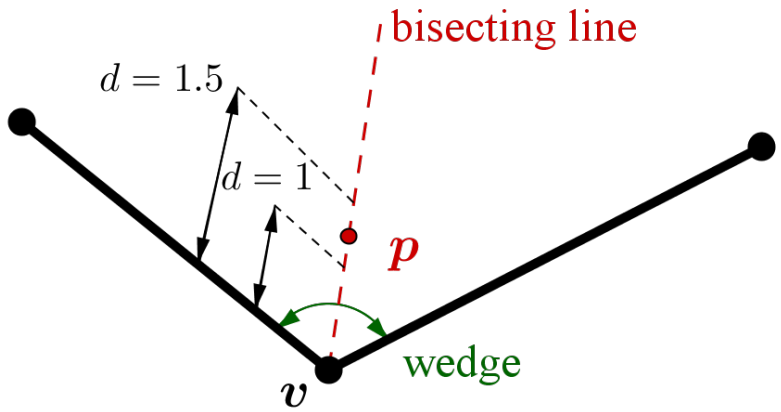
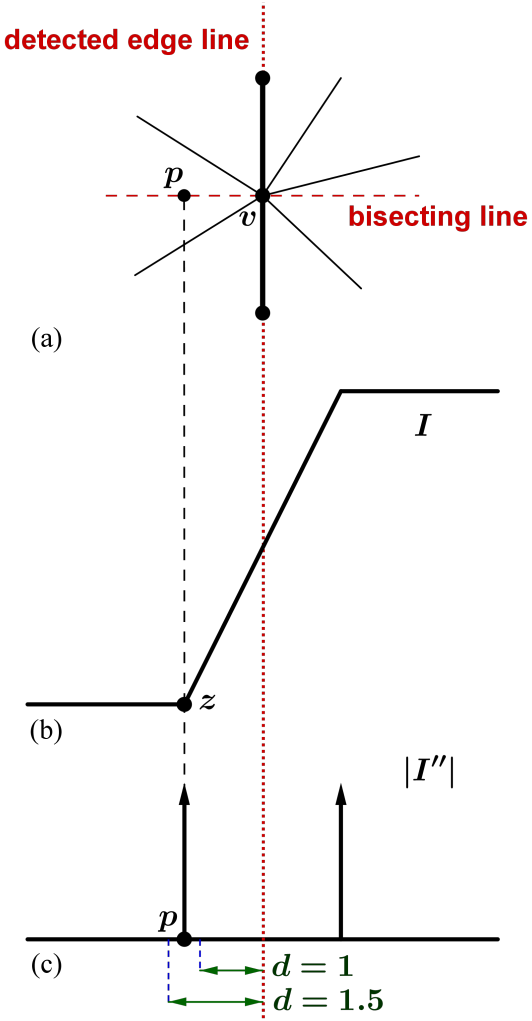


- p has the largest maximum magnitude of the second order directional derivative (MMSODD)
- z value at point p is selected as the wedge value associated with wedge

Proposed Mesh-Generation Method

Analysis of Local Line Search



sharp edge

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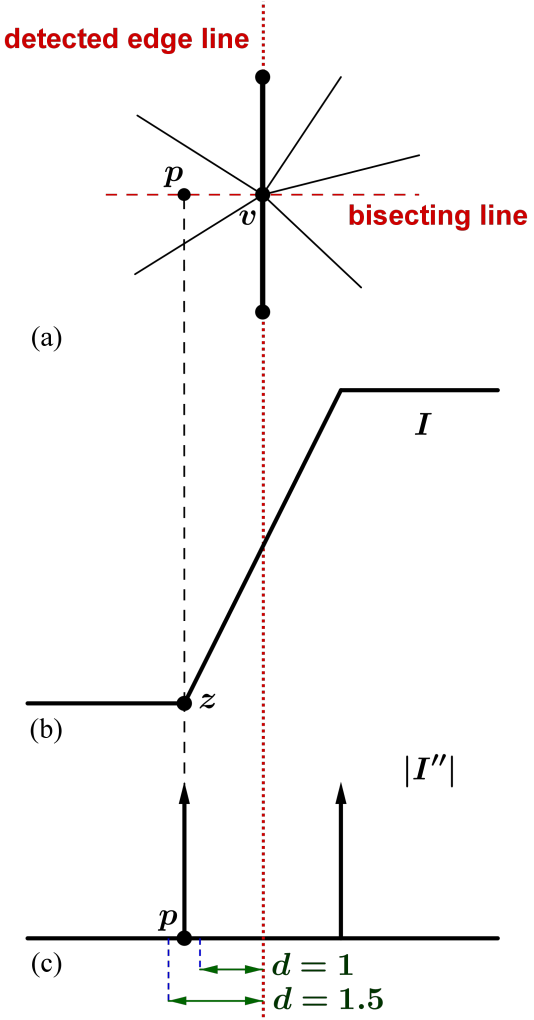
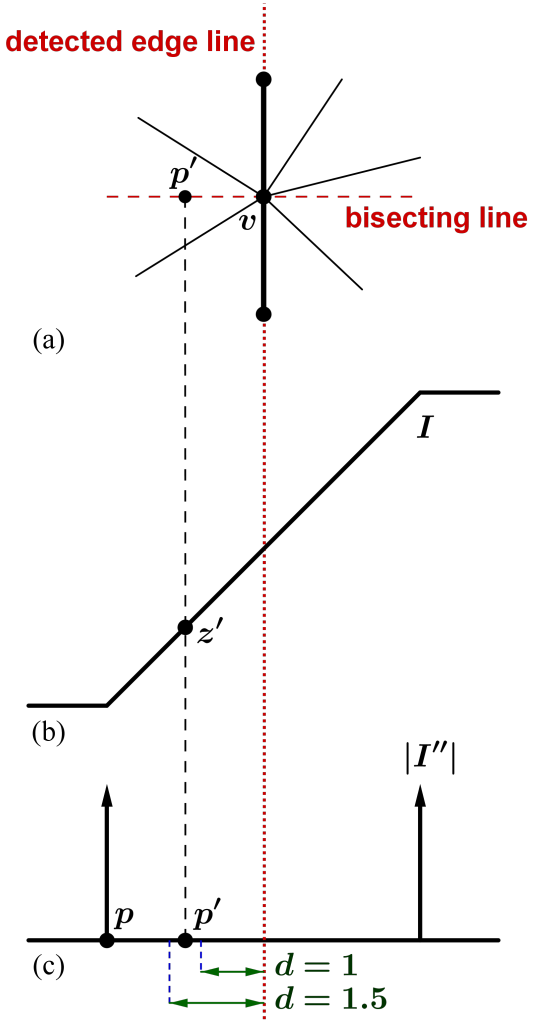


sharp edge

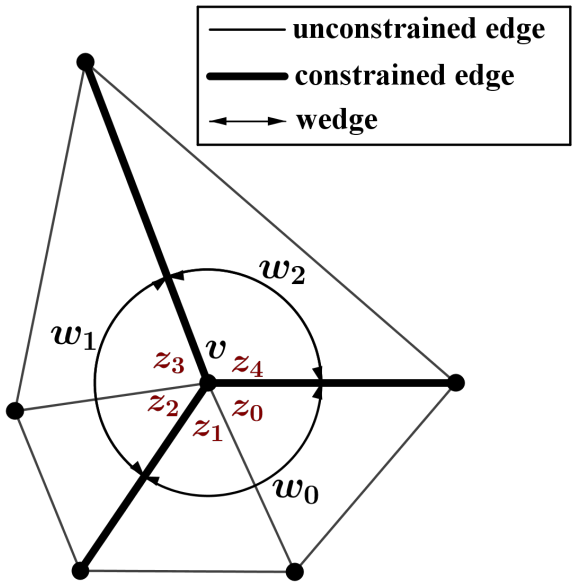


less-sharp edge

Proposed Mesh-Generation Method

Optimization-Based Algorithm

- Introducing corner z values:



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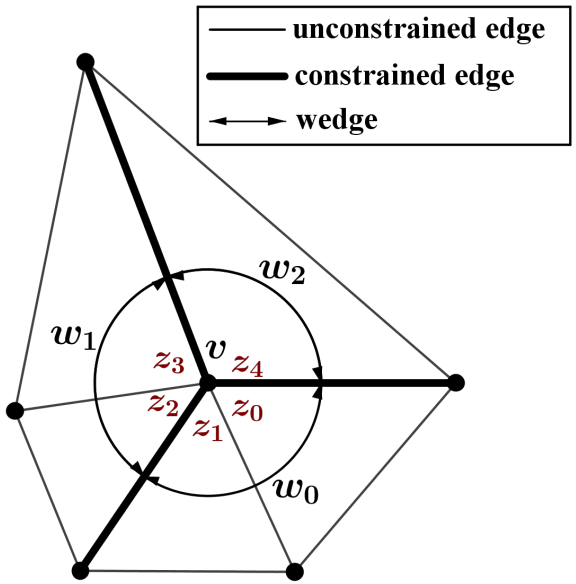

- Optimize corner z values to minimize the squared error inside each face $\rightarrow \underset{\{z_i\}}{\text{minimize}} \sum_{p \in \Omega} \left| \hat{\phi}(p) - \phi(p) \right|^2$
- Ω is the set of grid points in the face of interest

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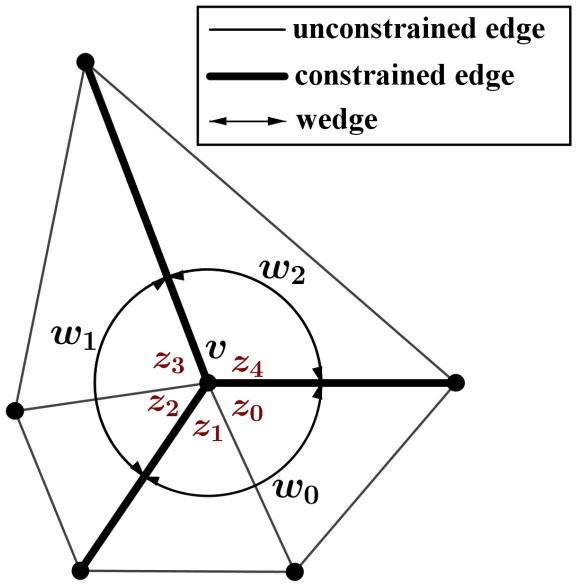

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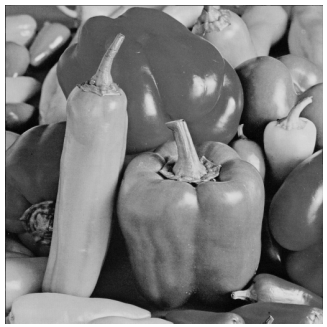
- Ω is the set of grid points in the face of interest
- $w_0 = \frac{z_0+z_1}{2}, w_1 = \frac{z_2+z_3}{2}, w_2 = z_4$

Results

Test Images

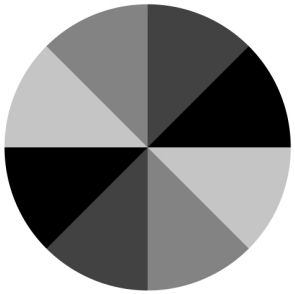
- Photographic, medical, and computer-generated imagery used



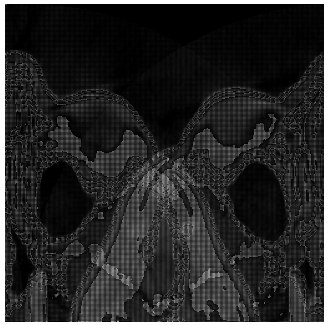
`peppers`

`lena`

`bull`



`wheel`



`ct`

Results

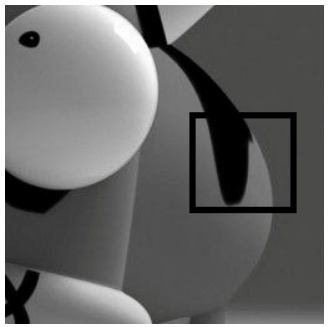
Proposed Method Vs. ERDED Method

- 18/20 of the test cases by a margin of up to 5.06 dB

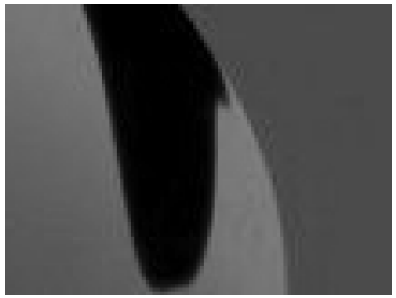
Image	Sampling Density (%)	PSNR(dB)	
		Proposed	ERDED
peppers	0.5	**22.49**	22.14
	1	**26.47**	25.97
	2	**29.26**	29.00
	3	**30.37**	30.17
lena	0.5	**20.81**	20.55
	1	**26.14**	25.81
	2	**29.38**	29.28
	3	31.30	**31.31**
ct	0.125	**18.72**	15.63
	0.25	**27.42**	25.97
	0.5	**31.62**	30.06
	1	**36.62**	36.51
bull	0.125	**25.22**	24.68
	0.25	**29.47**	28.86
	0.5	**35.35**	35.15
	1	**39.21**	39.02
wheel	0.0625	**35.75**	30.69
	0.125	**36.54**	34.55
	0.25	**37.24**	35.56
	0.5	37.38	**37.86**

Results

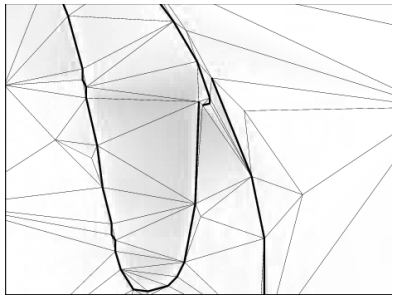
Subjective Comparison



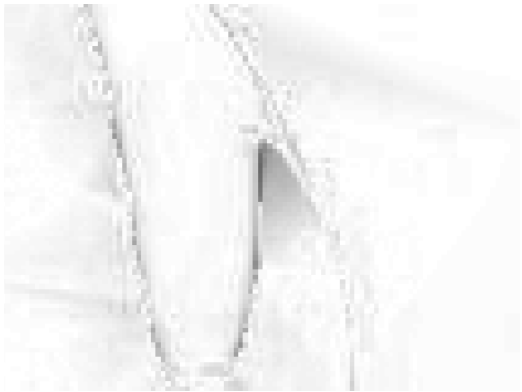
image



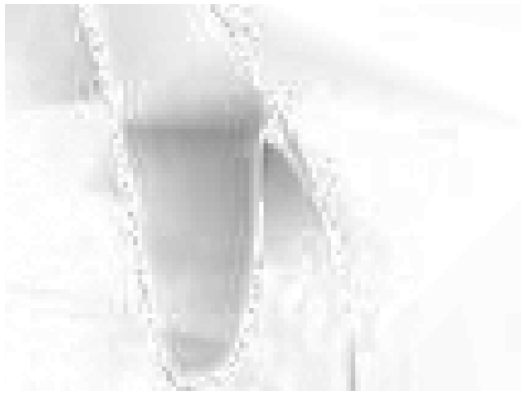
magnified area



triangulation



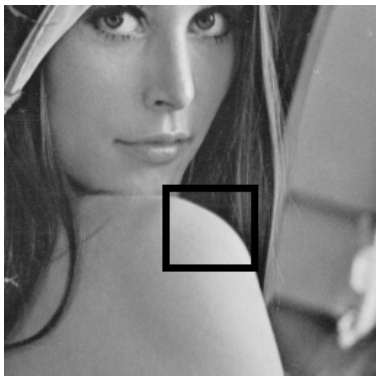
error image with proposed method



error image with ERDED

Results

Subjective Comparison Cont'd



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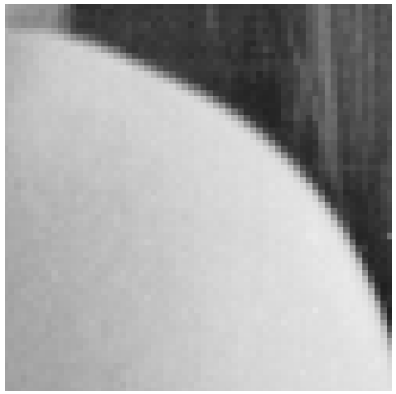

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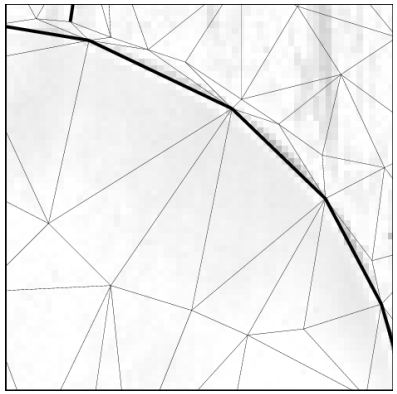

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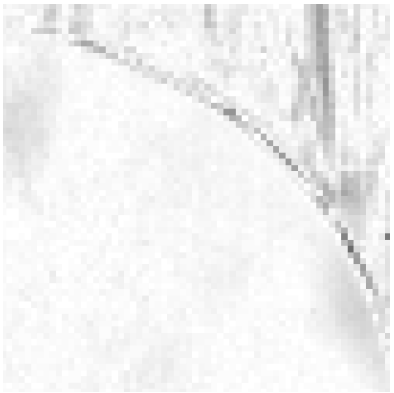

error image with proposed method

error image with ERDED

Conclusions

- Modified version of the ERDED mesh-generation method was proposed
- An optimization-based algorithm was employed which
  - Better exploits the image content
  - Reduces the approximation error in the reconstructed image
- The proposed method outperforms the ERDED method both in terms of PSNR and subjective quality
- The improved approximation quality comes at a relatively modest computational cost

THANK YOU