Effective Techniques for Generating Delaunay Mesh Models of Single- and Multi-Component Images

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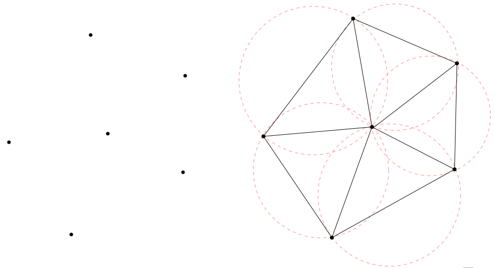
Overview

Triangulation

A triangulation of a set P of points in $\mathbb{R}^2$ is a set T of (non-degenerate) triangles satisfying the following conditions:

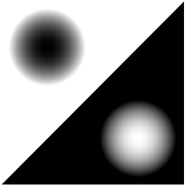
1. the set of all vertices of triangles in T is P;
2. the union of all triangles in T is the convex hull of P; and
3. the interiors of any two triangles in T are disjoint.

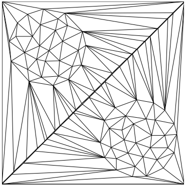
- Preferred-directions Delaunay Triangulation (PDDT) is employed in our work

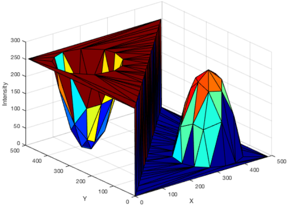


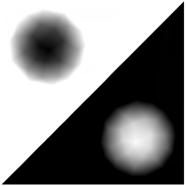
Triangle-Mesh Model of Image

- Triangle-mesh model:
  - a set P of sample points;
  - a (PDDT) triangulation T of P; and
  - a set $Z = \{z_i\}_{i=0}^{|P|-1}$ of function values, where $z_i = \phi(p_i)$.
- Mesh-generation:
  - target size (or sampling density)
  - minimize error
  - computational and memory complexity







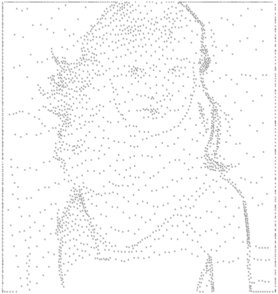


Floyd-Steinberg Error-Diffusion (FSED)

Given a density function d of an image of width W and height H, a threshold τ, and initial diffused-in errors $\tilde{e}$, FSED generates a binary-valued function b indicating position of selected points.

- classical FSED sets $\tilde{e}$ as zero
- to achieve desired size, can apply binary search to find an optimal τ
- issue: τ employed in our application is high, which can cause startup effect

(a) Density function

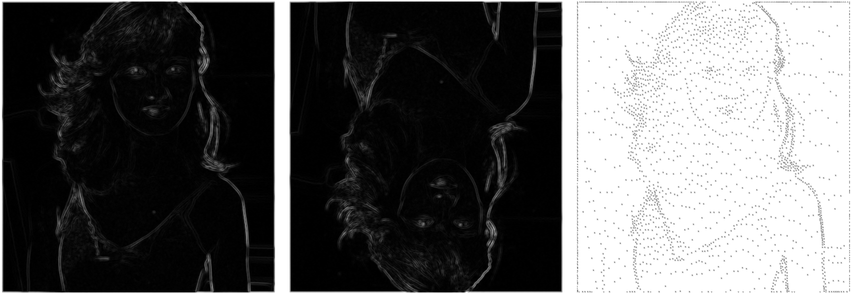


(b) Selected points

Propose Mirroring Method as Startup Policy

As a preprocessing step, obtain initial diffused-in errors $\tilde{e}$ as follows:

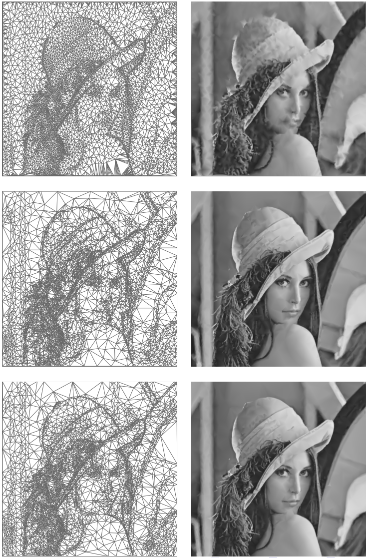
1. construct a mirrored version d_m of the density function d;
2. take d_m as the input density function, run FSED;
3. record and output the errors diffused to the last row.



(a) d (b) d_m (c) Selected points

Previously-Proposed Methods for Grayscale Images

- ED: select sample points in one shot using FSED
- GPR: select all sample points in the sampling grid as initial sample points, and then perform mesh-simplification
- GPRFSED: select a subset of sample points as initial sample points, and then perform mesh-simplification



Proposed Mesh-Generation Framework

Given an M-component ($M \geq 1$) image of width W and height H, and a desired number N of sample points, perform as follows (in order):

1. Initial sample-point selection. Select a set P of N_0 sample points, by applying the initial sample-point selection policy initSampSelPolicy, which employs FSED with the startup policy startupPolicy.
   - free parameters: N_0, initSampSelPolicy, startupPolicy
   - $N_0 \in [N..WH]$, startupPolicy $\in$ {classical, mirroring}
2. Initial mesh construction. Construct a PDDT T of P.
3. Mesh-simplification. While mesh size $>$ desired size N:
   - compute and update error increase over all components for deletion of each point
   - delete point that causes the least error increase

Initial Sample-Point Selection Policy

Significance Function σ:

- MMSODD
- magnitude of laplacian (MoL)

Aggregation Function F:

- $F_{\max}(x_1, x_2, \ldots, x_n) = \max\{x_1, x_2, \ldots, x_n\}$
- $F_{\text{avg}}(x_1, x_2, \ldots, x_n) = \frac{1}{n}\sum_{i=1}^{n} x_i$

Policy	Applicable Image	Free Parameter	Idea	initSampSelPolicy
PSA	grayscale ($M = 1$)	σ	normalize σ to get a density function d; invoke FSED	PSA(D) PSA(L)
PSB	RGB ($M = 3$)	σ	convert RGB image to grayscale image; compute and normalize σ to get d; invoke FSED	PSB(D) PSB(L)
PSC	RGB ($M = 3$)	N/A	obtain d by normalizing vector space operator; invoke FSED	PSC
PSD	any $M \geq 1$	σ_i, $i \in [0..M)$; F	aggregate M significance functions into one a; normalize a to get a density function d; invoke FSED	PSD(Max, D) PSD(Max, L) PSD(Avg, D) PSD(Avg, L)
PSE	any $M \geq 1$	σ_i, $i \in [0..M)$	normalize σ_i on each component to get d_i; perform FSED on each component and get M subsets of points; take the union of points	PSE(D) PSE(L)

- Relation: for $M = 1$, PSA, PSD, PSE are equivalent when having the same σ

Different Modes in the Proposed Framework

- by varying choice of N_0, we can operate in fundamentally different behaviors
- split into three modes of operations

Table: Different modes in the proposed framework

Modes	N_0	Mesh-simplification	Free parameters	Mesh quality	Complexity
ED-like	N	No	initSampSelPolicy, startupPolicy	low	low
GPRFSED-like	$(N..WH)$	Yes	N_0, initSampSelPolicy, startupPolicy	high	modest
GPR-like	WH	Yes	N/A	high	extremely high

Test Data

- 45 RGB images and 45 grayscale images
- taken from standard data sets, JPEG-2000, USC-SIPI, CIPR-Canon, and Kodak, and some computer-generated images.

ED-like Mode | RGB-Color Case | initSampSelPolicy

- fix parameter startupPolicy (e.g., classical)
- for each of test case, generate a mesh using each initSampSelPolicy
- compute approximation error and rank from 1 (best) to 9 (worst)

Samp. Density (%)	PSNR Average Rank (Standard Deviation)								
	PSB (D)	PSB (L)	PSC	PSD (Max, D)	PSD (Max, L)	PSD (Avg, D)	PSD (Avg, L)	PSE (D)	PSE (L)
0.5	4.00 (1.80)	5.82 (1.45)	**2.16** (2.01)	2.69 (1.28)	4.80 (1.63)	3.69 (1.60)	5.49 (1.90)	7.80 (1.17)	8.56 (1.09)
1.0	3.51 (1.67)	6.02 (1.36)	2.98 (2.29)	**2.60** (1.61)	4.93 (1.55)	2.96 (1.23)	5.42 (1.48)	7.93 (1.14)	8.64 (0.85)
2.0	3.49 (1.73)	5.87 (1.54)	4.24 (2.44)	**2.33** (1.48)	4.58 (1.67)	2.64 (1.23)	5.04 (1.28)	8.07 (0.71)	8.73 (0.49)
3.0	3.16 (1.40)	5.80 (1.45)	5.02 (2.47)	**2.20** (1.24)	4.44 (1.63)	2.42 (1.06)	5.16 (1.23)	7.93 (0.68)	8.87 (0.34)
4.0	3.24 (1.69)	5.98 (1.29)	5.84 (2.04)	**1.80** (0.91)	4.53 (1.42)	2.16 (0.76)	4.73 (0.90)	7.91 (0.78)	8.80 (0.40)
Overall	3.48 (1.69)	5.90 (1.42)	4.05 (2.63)	**2.32** (1.36)	4.66 (1.59)	2.77 (1.32)	5.17 (1.43)	7.93 (0.93)	8.72 (0.70)

- fix parameter initSampSelPolicy (e.g., PSD(Max, D))
- compute the win ratio and difference in PSNR by which the mirroring method beats the classical method

Samp. Density (%)	PSNR Change (dB)			Win Ratio (%)
	Minimum	Median	Maximum	
0.5	-2.29	0.76	3.87	77.8
1.0	-0.57	0.38	2.65	80.0
2.0	-0.53	0.19	3.68	68.9
3.0	-3.45	0.09	3.32	64.4
4.0	-0.90	0.05	1.58	66.7
Overall	-3.45	0.17	3.87	71.6

ED-like Mode | Grayscale Case |

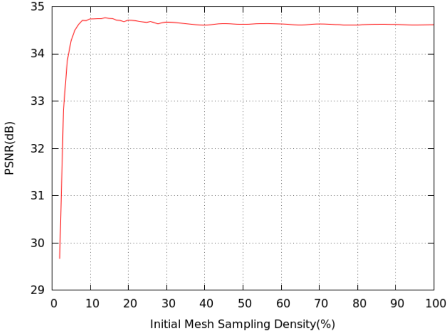
- fix parameter startupPolicy (e.g., classical)
- compute the win ratio and difference in PSNR by which MMSODD beats MoL in the PSA (or equivalently, PSD or PSE) policy

Samp. Density (%)	PSNR Change (dB)			Win Ratio (%)
	Minimum	Median	Maximum	
0.5	-0.53	0.47	2.13	82.2
1.0	-1.05	0.49	5.37	93.3
2.0	-0.20	0.36	1.27	91.1
3.0	0.02	0.38	1.94	100.0
4.0	-0.53	0.38	1.49	95.6
Overall	-1.05	0.42	5.37	92.4

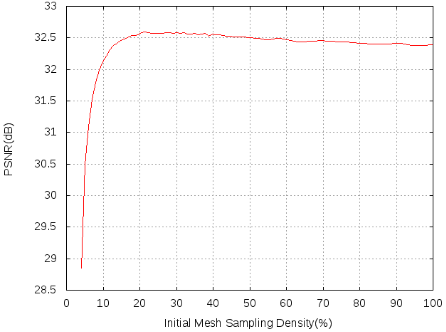
- fix parameter initSampSelPolicy (e.g., PSA(D))
- compute the win ratio and difference in PSNR by which the mirroring method beats the classical method

Samp. Density (%)	PSNR Change (dB)			Win Ratio (%)
	Minimum	Median	Maximum	
0.5	-1.63	0.61	2.40	84.4
1.0	-3.33	0.35	3.51	75.6
2.0	-2.72	0.10	4.72	55.6
3.0	-0.92	0.11	2.46	66.7
4.0	-1.81	0.03	3.08	62.2
Overall	-3.33	0.16	4.72	68.9

- study N_0 based on initial sampling density D_0, where $D_0 = \frac{N_0}{WH}$
- fix parameters initSampSelPolicy (i.e., PSD(Max, D)) and startupPolicy (i.e., mirroring)
- result: $D_0 = \gamma D$, or $N_0 = \gamma N$, where $\gamma \in [4, 5.5]$ and nominally chosen as 4



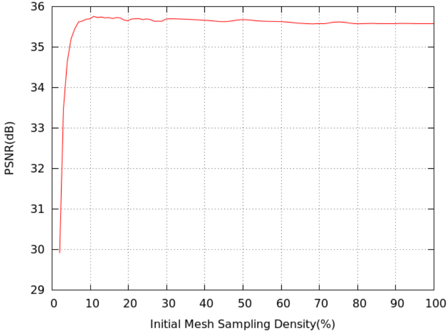
(a) color kodim23 | 2%



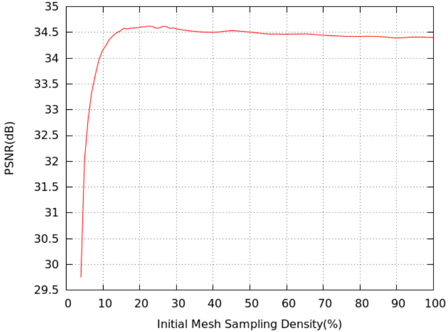
(b) color lena | 4%

GPRFSED-like Mode | Grayscale Case | N_0

- study N_0 based on initial sampling density D_0, where $D_0 = \frac{N_0}{WH}$
- fix parameters as initSampSelPolicy (e.g., PSA(D)) and startupPolicy (i.e., mirroring)
- result: $D_0 = \gamma D$, or $N_0 = \gamma N$, where $\gamma \in [4, 5.5]$ and nominally chosen as 4



(a) grayscale kodim23 | 2%



(b) grayscale lena | 4%

Proposed Methods

Two methods proposed: low complexity MED and higher complexity MGPRFS.

- initial mesh size N_0:
  - for MED: $N_0 = N$
  - for MGPRFS: $N_0 = \gamma N$, where $\gamma \in [4, 5.5]$ and nominally chosen as 4.
- initSampSelPolicy: PSD(Max, D)
- startupPolicy: mirroring

Comments:

- both of these methods can generate mesh models for any M-component images, where $M \geq 1$.
- MED and MGPRFS can be seen as extended and improved version of ED and GPRFSED methods, respectively.

Evaluation of Proposed Methods

Compare to:

- single-component mesh generators ED, GPRFSED, and GPR; and their color-capable version CED, CGPRFSED, and CGPR
- the method constructed from the GPR-like mode of our proposed framework, which we refer to as MGPR (for grayscale case, MGPR is equivalent to GPR).

Meaningful comparisons to be made:

- MED vs CED and ED
- MGPRFS vs CGPRFSED and GPRFSED
- MGPRFS vs CGPR, MGPR, and GPR

Evaluation of Proposed Methods | Mesh Quality

Average rank of methods considered for RGB-color images

Samp. Density (%)	PSNR Average Rank (Standard Deviation)					
	CED	MED	CGPRFSED $\gamma = 4$	MGPRFS $\gamma = 4$	CGPR	MGPR
0.5	5.88 (0.32)	5.12 (0.32)	3.81 (0.45)	2.24 (0.61)	2.67 (0.92)	**1.29** (0.55)
1.0	5.90 (0.29)	5.10 (0.29)	3.40 (0.66)	**1.57** (0.73)	3.36 (0.65)	1.67 (0.71)
2.0	5.90 (0.29)	5.10 (0.29)	3.12 (0.66)	**1.36** (0.53)	3.69 (0.51)	1.83 (0.69)
3.0	5.81 (0.39)	5.17 (0.43)	3.07 (0.63)	**1.33** (0.52)	3.76 (0.53)	1.86 (0.68)
4.0	5.81 (0.39)	5.19 (0.39)	3.02 (0.60)	**1.33** (0.56)	3.79 (0.51)	1.86 (0.60)
Overall	5.86 (0.34)	5.13 (0.35)	3.29 (0.67)	**1.57** (0.69)	3.45 (0.77)	1.70 (0.68)

- MED beats CED in 84.4% of all test cases by up to 7.08 dB
- MGPRFS beats CGPRFSED and CGPR respectively, in 97.8% and 89% of all test cases by up to 7.05 dB and 5.15 dB
- MGPRFS performs comparable to, or better than, the MGPR method at sampling densities of 1.0% and higher in 68.3% of test cases, by up to 0.29 dB.

Evaluation of Proposed Methods | Mesh Quality

Average rank of methods considered for grayscale images

Samp. Density (%)	PSNR Average Rank (Standard Deviation)				
	ED	MED	GPRFSED $\gamma = 4$	MGPRFS $\gamma = 4$	GPR/MGPR
0.5	4.83 (0.37)	4.17 (0.37)	2.79 (0.41)	2.12 (0.50)	**1.10** (0.37)
1.0	4.76 (0.43)	4.24 (0.43)	2.38 (0.72)	**1.71** (0.76)	1.90 (0.81)
2.0	4.57 (0.49)	4.43 (0.49)	**1.76** (0.68)	1.90 (0.78)	2.33 (0.86)
3.0	4.64 (0.48)	4.36 (0.48)	2.10 (0.72)	**1.52** (0.66)	2.38 (0.82)
4.0	4.62 (0.49)	4.38 (0.49)	**1.71** (0.59)	1.79 (0.74)	2.50 (0.85)
Overall	4.69 (0.46)	4.31 (0.46)	2.15 (0.75)	**1.81** (0.72)	2.04 (0.92)

- MED beats ED in 68.9% of all test cases by up to 4.72 dB
- MGPRFS beats GPRFSED at lower sampling densities (i.e., 0.5% and 1%) in 74.4% of the test cases by up to 2.88 dB, performs comparable to GPRFSED at higher sampling densities
- performs comparable to, or better than, the GPR/MGPR method at sampling densities of 1.0% and higher in 69.4% of test cases, by up to 0.42 dB.

PSNR results for some representative images

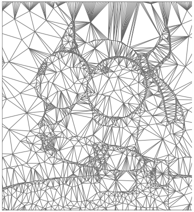
Image	Samp. Density (%)	PSNR (dB)					
		CED	MED	CGPRFSED $\gamma = 4$	MGPRFS $\gamma = 4$	CGPR	MGPR
lena (color)	0.5	17.48	19.18	25.63	26.04	26.09	**26.15**
	1.0	21.31	22.12	28.02	**28.38**	28.09	28.28
	2.0	25.54	25.77	30.33	**30.48**	30.13	30.44
	3.0	27.42	27.73	31.44	**31.68**	31.29	31.58
	4.0	28.82	28.83	32.13	**32.49**	32.04	32.39
pens (color)	0.5	13.96	15.78	22.49	24.05	23.60	**24.31**
	1.0	17.24	19.27	25.95	**26.77**	26.40	26.76
	2.0	21.98	23.48	29.08	**29.43**	29.12	29.42
	3.0	25.05	25.91	30.59	**31.16**	30.77	31.15
	4.0	27.05	27.92	31.97	**32.45**	32.01	32.44

Image	Samp. Density (%)	PSNR (dB)				
		ED	MED	GPRFSED $\gamma = 4$	MGPRFS $\gamma = 4$	GPR/MGPR
lena (grayscale)	0.5	17.17	17.96	26.22	26.49	**26.55**
	1.0	21.13	21.83	29.00	**29.11**	29.10
	2.0	25.83	26.14	31.86	**31.92**	31.80
	3.0	28.06	28.43	33.50	**33.51**	33.33
	4.0	29.59	29.76	34.54	**34.59**	34.40

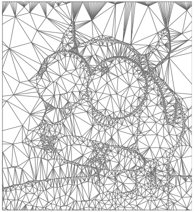
Evaluation of Proposed Methods | Mesh Quality

Subjective Quality Example:

Original



(a) CED

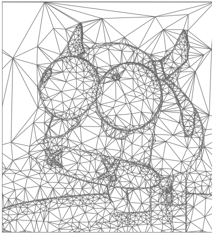


(b) MED

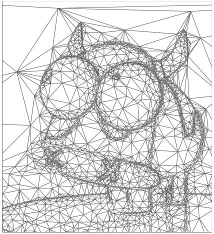
(c) CED

(d) MED

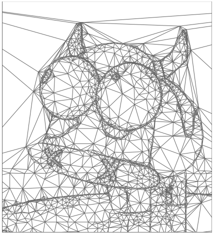
Evaluation of Proposed Methods | Mesh Quality



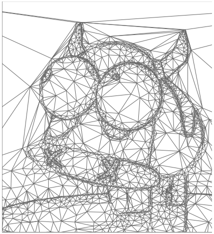
(a) CGPRFSED



(b) MGPRFS



(c) CGPR



(d) MGPR

(e) CGPRFSED

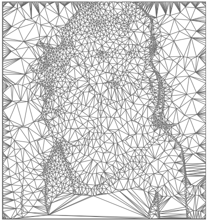
(f) MGPRFS

(g) CGPR

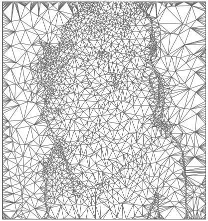
(h) MGPR

Evaluation of Proposed Methods | Mesh Quality

Original



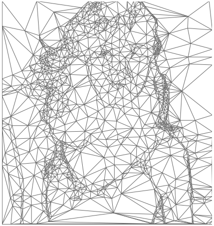
(a) ED



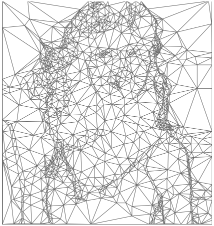
(b) MED

(c) ED

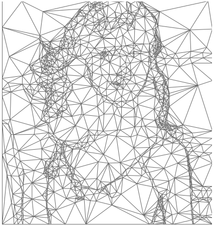
(d) MED



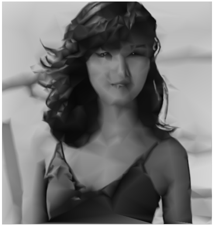
(a) GPRFSED



(b) MGPRFS



(c) GPR/MGPR



(d) GPRFSED

(e) MGPRFS

(f) GPR/MGPR

Evaluation of Proposed Methods | Computational Complexity

RGB-color case.

Image	Samp. Density (%)	Time (s)					
		CED	MED	CGPRFSED $\gamma = 4$	MGPRFS $\gamma = 4$	CGPR	MGPR
lena (color)	0.5	0.16	0.35	0.68	1.72	29.97	41.03
	1.0	0.18	0.46	1.03	2.09	29.62	40.46
	2.0	0.23	0.58	2.31	2.79	29.33	38.66
	3.0	0.30	0.73	3.15	3.49	29.19	37.32
	4.0	0.38	0.80	4.32	5.04	29.00	37.18

grayscale case.

Image	Samp. Density (%)	Time (s)				
		ED	MED	GPRFSED $\gamma = 4$	MGPRFS $\gamma = 4$	GPR/MGPR
lena (grayscale)	0.5	0.13	0.22	0.63	0.77	28.19
	1.0	0.15	0.31	1.00	1.19	27.60
	2.0	0.20	0.36	1.77	1.91	27.40
	3.0	0.27	0.44	2.74	2.76	27.00
	4.0	0.33	0.54	3.49	3.53	26.70

Table: Comparison of the maximum mesh size for the various methods

Method	Maximum Mesh Size	Relative Maximum Mesh Size		
		General	D = 0.5%	D = 4%
CED	DWH	1	1	1
ED	DWH	1	1	1
CGPRFSED, $\gamma = 4$	$4DWH$	4	4	4
GPRFSED, $\gamma = 4$	$4DWH$	4	4	4
CGPR	WH	$1/D$	200	25
GPR	WH	$1/D$	200	25
MGPR	WH	$1/D$	200	25
MED	DWH	1	1	1
MGPRFS, $\gamma = 4$	$4DWH$	4	4	4

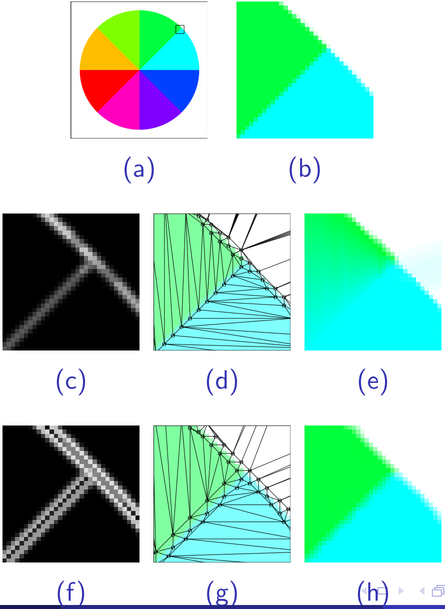
- MGPRFS requires $\frac{25}{4} \approx 6.2$ to $\frac{200}{4} = 50$ times less memory than CGPR, MGPR and GPR

Conclusions

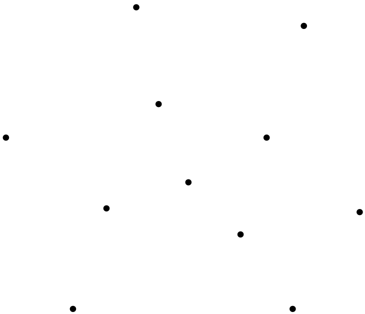
- Proposed methods outperforms methods with similar and higher complexity
  - MED outperforms ED and CED in mesh quality, with similar computational and memory costs
  - MGPRFS outperforms GPRFSED and CGPRFSED in mesh quality, with similar in computational and memory costs
  - MGPRFS yields meshes with quality comparable, or better than, GPR, CGPR, and MGPR in mesh quality, while requiring substantially less computational and memory costs
- tradeoff between mesh quality and computational cost: useful in a wide range of applications
- effective initial sample point selection policy and FSED startup policy

Thank you

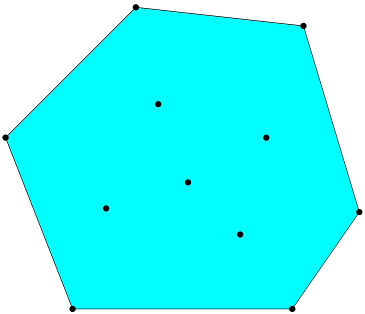
Single Response vs Double Response to Image Edges



Convex Hull

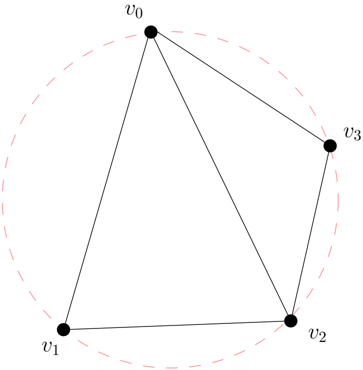


(a) A set P of points

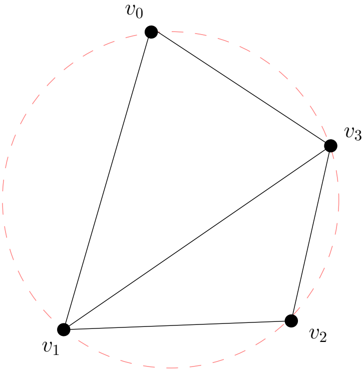


(b) convex hull of P

A set of points can have different triangulations

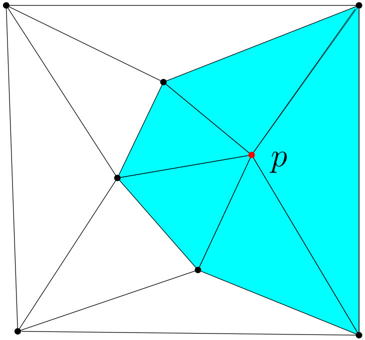


(a) Triangultion 1

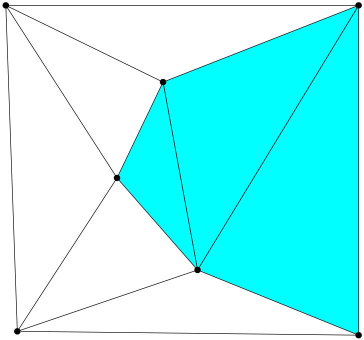


(b) Triangulation 2

Effect of Point Removal is Local in Delaunay Triangulation



(a) A Delaunay Triangulation with p to be removed



(b) The updated triangulation after removing p from T

MSE and PSNR

$$\text{MSE} = \frac{1}{M|\Lambda|} \sum_{i=0}^{M-1} \sum_{p \in \Lambda} [\hat{\phi}_i(p) - \phi_i(p)]^2,$$

$$\text{PSNR} = 20 \log_{10} \left(\frac{2^{\rho} - 1}{\sqrt{\text{MSE}}} \right)$$

MMSODD and MoL (applicable to single-component of images)

Maximum-magnitude second-order directional derivative (MMSODD):

$$d(x, y) = \max\Big\{|\alpha(x, y) + \beta(x, y)|, |\alpha(x, y) - \beta(x, y)|\Big\},$$

where

$$\alpha(x, y) = \frac{1}{2}\left[\frac{\partial^2}{\partial x^2}f(x, y) + \frac{\partial^2}{\partial y^2}f(x, y)\right]$$

and

$$\beta(x, y) = \sqrt{\frac{1}{4}\left[\frac{\partial^2}{\partial x^2}f(x, y) - \frac{\partial^2}{\partial y^2}f(x, y)\right]^2 + \left[\frac{\partial^2}{\partial x \partial y}f(x, y)\right]^2}.$$

Magnitude of laplacian (MoL):

$$L(x, y) = \frac{\partial^2}{\partial x^2}f(x, y) + \frac{\partial^2}{\partial y^2}f(x, y).$$

MMSODD and MoL for a grayscale image

(a) MMSODD

(b) MoL

Vector Gradient Operator (RGB Images)

$$\boldsymbol{u} = \frac{\partial R}{\partial x}\boldsymbol{r} + \frac{\partial G}{\partial x}\boldsymbol{g} + \frac{\partial B}{\partial x}\boldsymbol{b} \quad \text{and}$$
$$\boldsymbol{v} = \frac{\partial R}{\partial y}\boldsymbol{r} + \frac{\partial G}{\partial y}\boldsymbol{g} + \frac{\partial B}{\partial y}\boldsymbol{b},$$

based on which, the vector-space directional derivatives are computed as

$$g_{xx} = \boldsymbol{u} \times \boldsymbol{u} = \left|\frac{\partial R}{\partial x}\right|^2 + \left|\frac{\partial G}{\partial x}\right|^2 + \left|\frac{\partial B}{\partial x}\right|^2,$$
$$g_{yy} = \boldsymbol{v} \times \boldsymbol{v} = \left|\frac{\partial R}{\partial y}\right|^2 + \left|\frac{\partial G}{\partial y}\right|^2 + \left|\frac{\partial B}{\partial y}\right|^2, \text{ and}$$
$$g_{xy} = \boldsymbol{u} \times \boldsymbol{v} = \left|\frac{\partial R}{\partial x}\frac{\partial R}{\partial y}\right| + \left|\frac{\partial G}{\partial x}\frac{\partial G}{\partial y}\right| + \left|\frac{\partial B}{\partial x}\frac{\partial B}{\partial y}\right|.$$

$$\begin{aligned}\theta &= \frac{1}{2}\arctan\frac{2g_{xy}}{g_{xx} - g_{yy}} \quad \text{and} \\ F(\theta) &= \sqrt{\frac{1}{2}\left[(g_{xx} + g_{yy}) + cos2\theta(g_{xx} - g_{yy}) + 2g_{xy}sin2\theta\right]}.\end{aligned} \tag{1}$$

Vector Gradient Operator for a RGB Image

(a) An RGB image

(b) Vector gradient operator

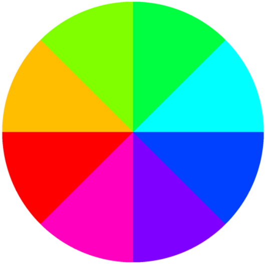
RGB to Grayscale conversion

RGB to grayscale conversion:

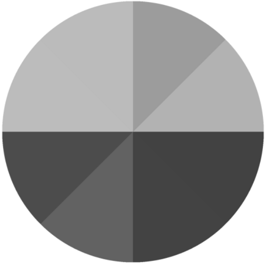
$$\tilde{f}(x,y) = w_r R(x,y) + w_g G(x,y) + w_b B(x,y)$$

$$w_r = 0.299,\ w_g = 0.587,\ w_b = 0.144$$

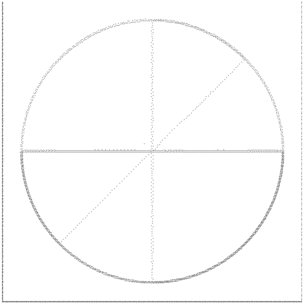
RGB to Grayscale conversion



(a) RGB image



(b) converted grayscale image



(c) selected points

(255,191,0) and (128,255,0) both result in gray value 188.

PSNR results for some representative RGB-color images

Image	Samp. Density (%)	PSNR (dB)					
		CED	MED	CGPRFSED $\gamma = 4$	MGPRFS $\gamma = 4$	CGPR	MGPR
lena (color)	0.5	17.48	19.18	25.63	26.04	26.09	**26.15**
	1.0	21.31	22.12	28.02	**28.38**	28.09	28.28
	2.0	25.54	25.77	30.33	**30.48**	30.13	30.44
	3.0	27.42	27.73	31.44	**31.68**	31.29	31.58
	4.0	28.82	28.83	32.13	**32.49**	32.04	32.39
pens (color)	0.5	13.96	15.78	22.49	24.05	23.60	**24.31**
	1.0	17.24	19.27	25.95	**26.77**	26.40	26.76
	2.0	21.98	23.48	29.08	**29.43**	29.12	29.42
	3.0	25.05	25.91	30.59	**31.16**	30.77	31.15
	4.0	27.05	27.92	31.97	**32.45**	32.01	32.44
bluegirl (color)	0.5	19.73	21.17	27.10	29.37	**29.68**	29.67
	1.0	22.49	25.30	31.99	32.67	32.54	**32.74**
	2.0	25.29	29.40	34.97	35.38	34.98	**35.42**
	3.0	29.49	31.90	36.39	**36.85**	36.33	36.82
	4.0	32.67	33.40	37.29	**37.86**	37.23	37.75

PSNR results for some representative grayscale images

Image	Samp. Density (%)	PSNR (dB)				
		ED	MED	GPRFSED $\gamma = 4$	MGPRFS $\gamma = 4$	GPR/MGPR
lena (grayscale)	0.5	17.17	17.96	26.22	26.49	**26.55**
	1.0	21.13	21.83	29.00	**29.11**	29.10
	2.0	25.83	26.14	31.86	**31.92**	31.80
	3.0	28.06	28.43	33.50	**33.51**	33.33
	4.0	29.59	29.76	34.54	**34.59**	34.40
pens (grayscale)	0.5	14.37	15.65	24.10	24.79	**25.36**
	10.	17.83	19.24	27.43	**27.79**	27.77
	2.0	22.87	23.96	**30.77**	30.73	30.68
	3.0	26.19	26.93	**32.69**	32.64	32.62
	4.0	28.10	28.79	**34.21**	34.20	34.16
bluegirl (grayscale)	0.5	19.99	20.76	27.72	30.59	**30.97**
	1.0	22.72	26.24	33.43	34.18	**34.31**
	2.0	25.42	30.14	37.40	**37.51**	37.49
	3.0	30.10	32.55	39.21	**39.31**	39.21
	4.0	33.58	34.34	40.52	**40.55**	40.47

Evaluation of Proposed Methods | Computational Complexity

RGB-color case

Image	Samp. Density (%)	Time (s)					
		CED	MED	CGPRFSED $\gamma = 4$	MGPRFS $\gamma = 4$	CGPR	MGPR
lena (color)	0.5	0.16	0.35	0.68	1.72	29.97	41.03
	1.0	0.18	0.46	1.03	2.09	29.62	40.46
	2.0	0.23	0.58	2.31	2.79	29.33	38.66
	3.0	0.30	0.73	3.15	3.49	29.19	37.32
	4.0	0.38	0.80	4.32	5.04	29.00	37.18
bluegirl (color)	0.5	0.14	0.40	0.62	1.66	26.90	36.65
	1.0	0.16	0.45	0.94	2.28	26.78	42.76
	2.0	0.25	0.62	1.79	2.91	26.40	38.46
	3.0	0.26	0.66	2.96	3.89	25.82	35.59
	4.0	0.31	0.72	3.37	5.45	25.29	35.05
cartoon_bull (color)	0.50	0.50	1.38	2.49	4.23	125.86	150.07
	1.0	0.60	1.54	4.20	6.06	114.16	146.56
	2.0	0.80	1.97	6.64	9.12	104.15	140.98
	3.0	1.02	2.40	10.28	11.90	100.82	133.59
	4.0	1.29	2.84	15.64	17.38	97.68	125.38

grayscale case

Image	Samp. Density (%)	Time (s)				
		ED	MED	GPRFSED $\gamma = 4$	MGPRFS $\gamma = 4$	GPR/MGPR
lena (grayscale)	0.5	0.13	0.22	0.63	0.77	28.19
	1.0	0.15	0.31	1.00	1.19	27.60
	2.0	0.20	0.36	1.77	1.91	27.40
	3.0	0.27	0.44	2.74	2.76	27.00
	4.0	0.33	0.54	3.49	3.53	26.70
bluegirl (grayscale)	0.5	0.10	0.23	0.56	0.71	25.42
	1.0	0.12	0.27	0.92	1.06	25.35
	2.0	0.18	0.32	1.68	1.74	24.79
	3.0	0.24	0.39	2.50	2.48	24.55
	4.0	0.29	0.47	3.22	3.30	24.61
cartoon _bull (grayscale)	0.5	0.43	0.86	2.35	2.52	101.10
	1.0	0.51	0.56	3.70	4.20	97.06
	2.0	0.72	0.74	6.43	6.35	95.89
	3.0	0.92	1.62	9.02	9.12	95.23
	4.0	1.18	1.84	11.87	12.00	93.79