Image Representation Using Triangle Meshes with Explicit Discontinuities

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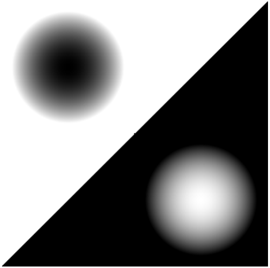
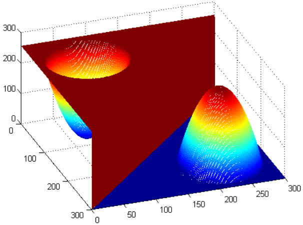
August 29, 2011

Introduction to Mesh Modelling (1)

An image can be modeled as a function defined on continuous domain. For example, the image in Figure (a) is modeled as a surface illustrated in Figure (b).



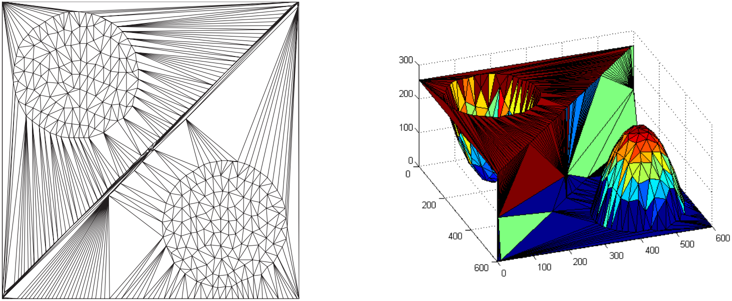
(a)



(b)

Figure: Image modeled as a function defined on continuous domain. (a) The original image, and (b) image modeled as surface.

Example of Mesh Modelling



(a) (b)

Figure: Mesh approximation of Image (sampling density 0.5%). (a) The triangulation of the original image, and (b) resulting triangle mesh.

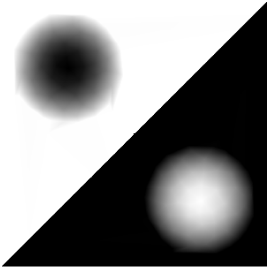


Figure: Reconstructed Image at sampling density 0.5%.

Mesh modelling of an image is an approach for approximating the image function ϕ, which involves partitioning the image domain into a collection of non-overlapping mesh elements (e.g., triangles).

A Mesh Model Explicitly Represent Discontinuities

Our mesh model for an image ϕ, which is based on constrained Delaunay triangulation [1], is completely characterized by

1. a set $P = \{p_i\}$ of sample points, where $p_i = (x_i, y_i) \in \frac{1}{2}\mathbb{Z}^2 \cap [0, W-1] \times [0, H-1]$;
2. a set E of constrained edges (i.e., a set of pairs of sample points from P); and
3. for each sample point p_i, one or more wedge values (where the term ”wedge value” will be defined precisely later).

Wedge

Definition

a **wedge** is a set of consecutive faces in a loop around a vertex *v* that are not separated by any constrained edge.

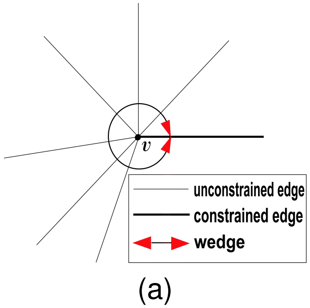


Figure: The relationship between vertices, constrained edges, and wedges. The (a) single wedge, and (b) multiple-wedge cases.

Wedge Value

Definition

The **wedge value** z of the wedge w belonging to vertex v specifies the limit of $\hat{\phi}(p)$ as p approaches v from points inside the wedge w.

Now, we specify precisely how the function $\hat{\phi}$ is defined at each point $p \in \Gamma$. There are two cases to consider: 1) p is not on a constrained edge; 2) p is on a constrained edge.

Definitions of Edge Point and Nonedge Point

Definition

a vertex with exactly one wedge is called a **nonedge point**.

Definition

a vertex with more than one wedge is called an **edge point**.

The set P_e of edge points allow for a good image approximation around image edges, while the set P_n of nonedge points permit a good approximation away from image edges.

General Mesh-Generation Steps

Our proposed method involves the following three steps:

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1. Select P_e and E.
2. Choose P_n, and let $P = P_e \cup P_n$.
3. Select the wedge values for each vertex in P.

Edge Detection

We first need to locate and represent the image edges [4].

Accuracy

Detect edge on half-pixel resolution. This is done by applying the edge detector to a higher resolution version of the image produced by linear interpolation

Consistency

Using modified Canny edge detector described in [2].

Polyline Generation

Each group of edge pixels in the edge map that are 8-connected are joined together to form a polyline.

The polyline is split at each intersection point.

In this manner, the final set of polylines obtained are guaranteed not to have any self-intersections.

Polyline Simplification

The Douglas-Peucker algorithm [3] is employed. It, in effect, removes points from a polyline such that the resulting simplified polyline approximates the original within a specified tolerance.

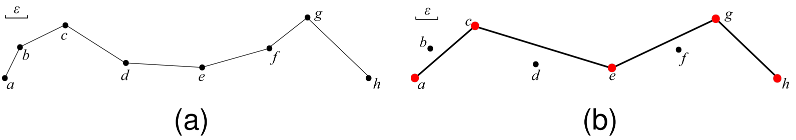


Figure: Polyline (a) before and (b) after simplified by Douglas-Peucker algorithm.

Process of Creating Constrained Edges

The process of producing simplified polylines from an image is illustrated in the figure below.

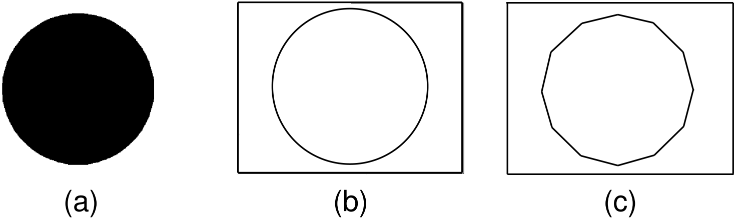


Figure: Process of producing simplified polylines. (a) Original image, (b) edge map, and (c) simplified polylines.

Selection of P_n using ED scheme

We employ the ED method of Yang et al. [5] to sample the set P_n of nonedge points.

Avoid sampling points near edges.

1. choose a small value of contrast parameter γ that controls the sensitivity of sample-point selection to local image structure (such as edges).
2. set the value of density function at each point near edges to 0.

Adjusting the threshold ρ of error diffusion until the desired number of points are obtained

Wedge Value Selection

We now construct the constrained Delaunay triangulation of P with the edge constraints E. For each vertex v of image ϕ,

1. v has exactly one wedge.
   $z_v = \phi(v)$.
2. v has more than one wedge.
   $z_v = \phi(v')$ and $v' = v + d$, where d is a displacement of length 1.5 away from v along the line that bisects the wedge.

Choice of Parameters γ, τ and ε

After numerous experiment, we choose $\gamma = 0.5$, and select ε, τ shown in the table below.

Table: Recommended choice of ε and τ (which depends on sampling density)

Samp. density (%)	ε	τ
$[0, 0.7)$	2	90
$[0.7, 1.5)$	2	70
$[1.5, 2.5)$	1	50
$[2.5, 5)$	1	40

Startup Effect

the ED method can sometimes place an abnormally small number of points in the first few rows of an image. This effect is evident in the top part of the triangulation shown later.

To solve this problem, we simply force our algorithm to sample a small number of points uniformly at the first row of the image.

Discussion

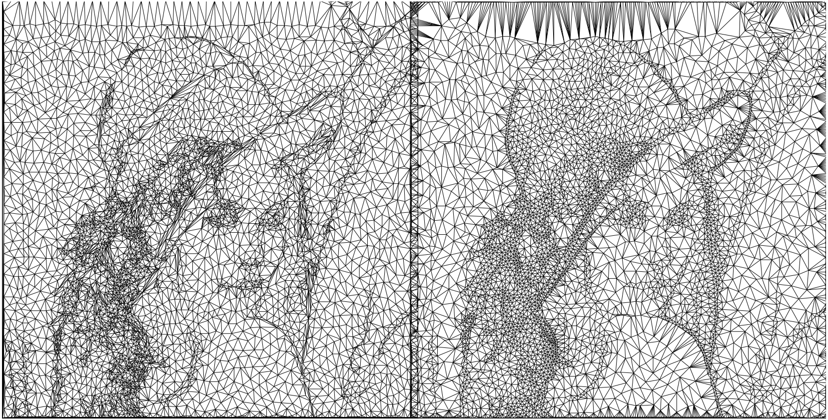
Comparison of mesh quality obtained with TA and TA-Random methods in terms of PSNR

Image	Samp. density (%)	PSNR (dB)	
		TA	TA-Random
lena	1.0	**25.72**	24.35
	2.0	**29.04**	27.39
	3.0	**30.16**	28.66
	4.0	**30.38**	29.48
peppers	1.0	24.02	**24.38**
	2.0	**28.08**	27.43
	3.0	**29.51**	28.66
	4.0	**30.07**	29.35

Comparison of image approximations for *lena* image

(a) (b)

Figure: Image approximations obtained with the (a) TA (29.04 dB) and (b) ED (26.25 dB) methods for the *lena* image at a sampling density of 2%.



(a) (b)

Figure: Triangulations obtained with the (a) TA (29.04 dB) and (b) ED (26.25 dB) methods and for the *lena* image at a sampling density of 2%.

Comparison of image approximations for *wheel* image

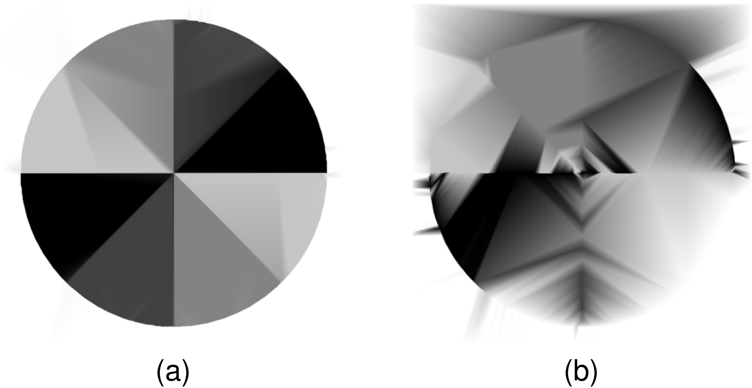


Figure: Image approximations obtained with the (a) TA (31.10 dB) and (b) ED (12.29 dB) methods for the *wheel* image at a sampling density of 0.25%.

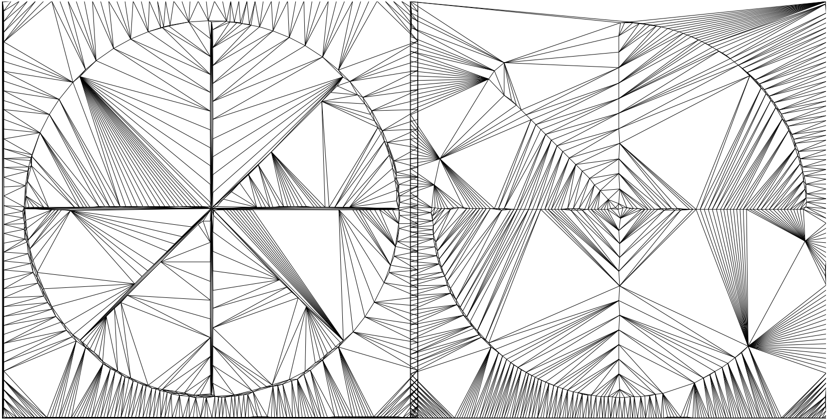


Figure: Triangulations obtained with the (a) TA (31.10 dB) and (b) ED (12.29 dB) methods and for the *wheel* image at a sampling density of 0.25%.

Comparison of mesh quality obtained with TA and ED methods

Image	Samp. density (%)	PSNR (dB)	
		TA	ED
lena	1.0	**25.72**	21.67
	2.0	**29.04**	26.25
	3.0	**30.16**	28.50
	4.0	**30.38**	29.67
peppers	1.0	**24.02**	21.69
	2.0	**28.08**	26.63
	3.0	**29.51**	28.79
	4.0	**30.07**	29.82

Comparison of mesh quality obtained with TA and ED methods (Cont'd)

Image	Samp. density (%)	PSNR (dB)	
		TA	ED
wheel	0.1	**25.27**	9.16
	0.25	**31.10**	12.29
	0.5	**34.19**	14.95
	1.0	**35.60**	22.36
bull	0.1	**17.06**	13.96
	0.25	**30.23**	17.60
	0.5	**34.33**	27.57
	1.0	**36.97**	34.00
glasses	1.0	**24.82**	20.67
	2.0	**27.98**	25.52
	3.0	**29.21**	27.98
	4.0	**29.63**	29.31

Conclusions

In this paper,

a mesh model that can explicitly represents discontinuities is introduced,

a mesh-generation method to construct the mesh model to represent the images is proposed, and

our proposed method is demonstrated to yield better quality mesh representations in terms of PSNR and subjective quality than the effective ED scheme.

References

[1] L. P. Chew. *Constrained Delaunay triangulations*, Algorithmica, 4:97-108, 1989.

[2] L. Ding and A. Goshtasby. *On the canny edge detector.* Pattern Recognition, 34(3):721-725, March 2001.

[3] D. Douglas and T. Peucker. *Algorithms for the reduction of the number of points required to represent a digitized line or its caricature.* Cartographica: The International Journal for Geographic Information and Geovisualization, 10(2):112-122, October 1973.

References

[4] M. A. Garcia, B. Vintimilla, and A. Sappa. *Approximation and processing of intensity images with discontinuity preserving adaptive triangular meshes.* Sixth European Conference on Computer Vision, LNCS Vol. 1842, Springer Verlag, Dublin, Ireland, July 2000.

[5] Weizhong Liu and Kunio Kondo. *A fast approach for accurate content-adaptive mesh generation.* IEEE Trans. on Image Processing, 12(8):866-881, August 2003.