

Example 3.23. Determine whether the system \mathcal{H} is invertible, where

$$\mathcal{H}x(t) = x(t - t_0)$$

and t_0 is a real constant.

Solution. Let $y = \mathcal{H}x$. By substituting $t + t_0$ for t in $y(t) = x(t - t_0)$, we obtain

$$\begin{aligned} y(t + t_0) &= x(t + t_0 - t_0) \\ &= x(t). \end{aligned}$$

Thus, we have shown that

$$x(t) = y(t + t_0).$$

This, however, is simply the **equation of the inverse system** \mathcal{H}^{-1} . In particular, we have that

$$x(t) = \mathcal{H}^{-1}y(t)$$

where

$$\mathcal{H}^{-1}y(t) = y(t + t_0).$$

Thus, we have found \mathcal{H}^{-1} . Therefore, the system \mathcal{H} is **invertible**. ■