

**Example 3.16** (Ideal integrator). Determine whether the system  $\mathcal{H}$  is memoryless, where

$$\mathcal{H}x(t) = \int_{-\infty}^t x(\tau) d\tau.$$

*Solution.* Consider the calculation of  $\mathcal{H}x(t)$  at any arbitrary point  $t = t_0$ . We have

$$\mathcal{H}x(t_0) = \int_{-\infty}^{t_0} x(\tau) d\tau.$$

Thus,  $\mathcal{H}x(t_0)$  depends on  $x(t)$  for  $-\infty < t \leq t_0$ . So,  $\mathcal{H}x(t_0)$  is dependent on  $x(t)$  for some  $t \neq t_0$  (e.g.,  $t_0 - 1$ ). Therefore, the system has **memory** (i.e., is not memoryless). ■

