

## A NEW TECHNIQUE FOR DOUBLE TUNING BIRDCAGE RESONATORS

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### Introduction

Double-tuned birdcage resonators have recently been designed using a variety of approaches [1]-[4]. Joseph and Lu showed that a nonuniform distribution of capacitances around the birdcage can also be used when the difference between the two frequencies is small [5]. In this last technique, the structure can not be operated in quadratures since the mode degeneracy is eliminated. In this work, we propose to show that a judicious, yet systematic, distribution of the capacitances in the columns of a low-pass birdcage resonator leads to double and multiple tuning while preserving the mode degeneracy as well as the sinusoidal current distribution. Given the two desired frequencies, explicit formulas for the capacitances are presented.

### Theory

To double-tune a birdcage resonator to two given frequencies  $\omega_1$  and  $\omega_2$ , we are going to make use of the phonon analogy from the theory of elastic waves in solids. It is well known that a one-dimensional lattice with two unequal masses per unit cell has two branches, the optical and the acoustic [6]. The mode  $j = 1$  corresponds therefore to two distinct frequencies. This result depends solely on the periodicity of the structure, only the values of the resonant frequencies are determined by the physical constants of the system. Following this analogy, we extend the period of the birdcage to twice that of the single-tuned structure. A possible way of achieving this is to allow the capacitances in the columns to be distributed such that  $C_1=C_3=\dots=C_{odd}$  and  $C_2=C_4=\dots=C_{even}$ . Since the period of the structure is now doubled, the mesh currents in two consecutive meshes are denoted by two different symbols  $X_n$  and  $Y_n$ . If only the mutual inductances between adjacent columns are included,  $X_n$  and  $Y_n$  are determined from Kirchoff's law, namely

$$\begin{aligned} & [(2L + 2l - 2M)\omega^2 - (\frac{1}{C_o} + \frac{1}{C_e})]X_n \\ & - M\omega^2[X_{n-1} + X_{n+1}] - [(L - 2M)\omega^2 - \frac{1}{C_e}]Y_n \quad 1.a \\ & - [(L - 2M)\omega^2 - \frac{1}{C_o}]Y_{n-1} = 0 \end{aligned}$$

and

$$\begin{aligned} & [(2L + 2l - 2M)\omega^2 - (\frac{1}{C_o} + \frac{1}{C_e})]Y_n \\ & - M\omega^2[Y_{n-1} + Y_{n+1}] - [(L - 2M)\omega^2 - \frac{1}{C_e}]X_n \quad 1.b \\ & - [(L - 2M)\omega^2 - \frac{1}{C_o}]X_{n+1} = 0 \end{aligned}$$

where  $L$  and  $l$  are the self-inductances of a column and a ring segment, respectively and  $M$  is the mutual inductance between two adjacent columns. Assuming running solutions, the resonant frequencies of mode  $J$  are found from setting the resulting determinant to zero. If the two desired angular frequencies are  $\omega_1$  and  $\omega_2$  for mode  $J = 1$ , which is required for field homogeneity, the corresponding values of the capacitances for an eight-column resonator are

$$C_{odd,even} = \frac{2}{S_r \pm \sqrt{S_r^2 - 4P_r}} \quad 2$$

where

$$S_r = \frac{a\omega_1^2 + \omega_2^2}{2L + 2l}, \quad 3.a$$

$$P_r = \frac{a}{2}(\omega_1^2\omega_2^2) \quad 3.b$$

and

$$a = (2L + 2l - 2M)^2 - 2(L - 2M)^2. \quad 3.c$$

When all the mutual inductances are included, equations (1) and (3) are modified accordingly. Multiple-tuning of the birdcage to  $p$  frequencies can similarly be achieved by increasing the period to include  $p$  periods of the single-tuned resonator in analogy with the phonon problem.

### Results

An eight-column birdcage resonator of diameter  $D=15.44$  cm and height  $h=19.02$  cm was constructed from copper foil of width 1.27 cm to test the present theory. The self and mutual inductances were calculated from expressions given in [7]. The measured values of the two capacitances necessary for double-tuning to the proton and carbon frequencies, 64 MHz and 16 MHz, respectively, are 522 pF and 38 pF. When all the mutual inductances are neglected, equation (1) gives 462.93 pF and 30.53 pF. If only the mutual inductances between adjacent columns are included, the calculated values are 453.6 pF and 39.87 pF. When the mutual inductances between adjacent ring segments are added, these values are 511.8 pF and 39.59 pF. Finally, when all the mutual inductances are included, the calculated values are 522.05 pF and 39.42 pF. For this case, the measured and calculated values agree to within less than 1% for the large capacitance and to within 4% for the smaller one. The mutual inductance between the columns affects more the small capacitance whereas that between the adjacent ring segments has a dramatic effect on the larger one. This is due to the phase differences between the currents in adjacent meshes for the two modes (acoustic and optical) [6].

### Conclusions

A new method, based on the phonon problem analogy, to double-tune birdcage resonators was presented. By increasing the period of the structure to  $p$  periods of the single-tuned birdcage, it is possible to multiple-tune the resonator to  $p$  different frequencies all with sinusoidal current distribution ( $J = 1$ ). Predicted and measured values of the two capacitances necessary to double-tune an eight-column resonator to the proton and carbon frequencies agree to within 1% and 4% respectively.

### References

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