IMPEDANCE GREEN'S FUNCTIONS IN THE SPECTRAL DOMAIN FOR LAYERED ANISOTROPIC MEDIA

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ABSTRACT

This paper presents new formulations of the impedance Green's functions in the spectral domain for general anisotropic media. The main advantages are: first, decoupling of the relationship between electric and magnetic field as opposed to dealing with coupled equations obtained when using other methods, and secondly, obtaining closed form expressions of the different TE and TM wave propagation constants. This is essential for modelling substrates involving tensor components and for the rigorous analysis of (M)MIC structures on multiple layered anisotropic substrates. The theory is demonstrated at the example of microstrip lines on ferrite-dielectric substrates with different directions of magnetic bias. The numerical results are found to be in good agreement with previously published data.

I. INTRODUCTION

Several investigations have been published on the incorporation of either uniaxial or biaxial anisotropic media, e.g. [1]-[3], in the theoretical analysis of (M)MIC structures. Most of the methods that have been used up to now, however, are very complex. In the conventional case involving scalar material constants, the formulation process of the well-known spectral domain immittance approach (SDIA) results in a simple solution for multilayered structures by decoupling the TE-wave and TM-wave components [4]. In the case of an anisotropic substrate, however, the key problem is to find the wave immittances and related transverse propagation constants of both TE and TM waves for general anisotropy or magnetization in x, y or z directions. Attempts to solve this problem by using SDIA [5], however, involve the concept of common transverse propagation constants for the TE and TM components, which is only valid if certain relationships between the tensor components are satisfied.

Therefore, this paper presents a theoretical treatment to apply the SDIA concept of TE and TM waves to a structure with anisotropic medium. In this formulation, not only decoupled relations between electric and magnetic fields can be obtained, but also formulations for the different TE and TM wave propagation constants are derived. Without losing generality, we restrict the demonstration of the procedure to microstrip structures on ferrite-dielectric substrates. Magnetic bias in x and y as well as in z direction are investigated, and the results are compared with those available in literature.

II. FORMULATION

The microstrip line with magnetized ferrite substrate is used to illustrate the procedure. The geometry of the structure and its coordinate system with the z axis as the chosen direction of wave propagation are shown in Fig. 1a.

If the substrate is magnetized in y direction, the permeability tensor can be expressed as [6]

$$<\vec{\mu}>=\mu_o\begin{pmatrix}\mu & 0 & jk\\ 0 & \mu_y & 0\\ -jk & 0 & \mu\end{pmatrix} \tag{1}$$

where μ, κ and μ_y are complex quantities to account for the losses in the magnetic material. By using the concept of SDIA, the six-component electromagnetic field considered in the coordinate system (x,y,z) can be decomposed into TM-to-y and

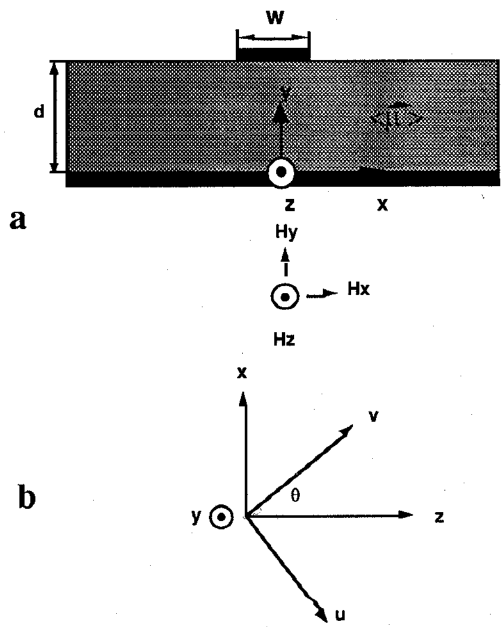

Fig. 1 Microstrip on an anisotropic substrate; a) cross-section and coordinate system, b) coordinate transformation.

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TE-to-y waves in the (u,v,y) system (c.f. Fig.1b). In this case the Fourier transform of the field components in x direction are defined as:

$$\tilde{\phi}(\alpha) = \int_{-\infty}^{\infty} \phi(x)\, e^{j\alpha x} dx \tag{2}$$

The y-direction propagation constant γ_m for TM waves can be obtained from

$$\frac{\partial^2 \tilde{E}_y}{\partial y^2} - \gamma_m^2 \tilde{E}_y = 0 \tag{3}$$

with

$$\begin{aligned} \gamma_m^2 &= -\kappa_o^2 \varepsilon_r \mu_\Delta + \alpha^2 + \beta^2 \\ \kappa_o^2 &= \omega_o^2 \varepsilon_o \mu_o \qquad \mu_\Delta = (\mu^2 - k^2)/\mu \end{aligned} \tag{4}$$

and α, β are the propagation constants in x, z directions, respectively.

The propagation constant γ_e in y direction for TE waves can be found from

$$\frac{\partial^2 \tilde{H}_y}{\partial y^2} - \gamma_e^2 \tilde{H}_y = 0 \tag{5}$$

with

$$\gamma_e^2 = \frac{\mu}{\mu_y}(-\kappa_o^2 \varepsilon_r \mu_\Delta + \alpha^2 + \beta^2) \tag{6}$$

In the SDIA, the coordinate system (x,y,z) of the structure is transformed into a new coordinate system (u,v,y) with relations referring to Fig.1b:

$$\begin{aligned} u &= z\sin\theta - x\cos\theta \\ v &= z\cos\theta - x\sin\theta \end{aligned} \tag{7}$$

When applying Maxwell's equations in the transformed coordinate system, the immittances Y_{TM} and Y_{TE} corresponding to the TM and TE components are given by

$$Y_{TM} = \frac{j\omega\varepsilon}{\gamma_m} \qquad Y_{TE} = \frac{\gamma_e}{j\omega\mu_o\mu_\Delta} \tag{8}$$

Note that the propagation constants of the TM waves differ from those of the TE waves and cannot be assumed identical as in [5].

After obtaining the wave immittances of the TM and TE components, the elements of the impedance Green's functions for the structure shown in Fig. 1a can be expressed as

$$\begin{aligned} \tilde{Z}_{11} &= N_z^2 \tilde{Z}_e + N_x^2 \tilde{Z}_h \\ \tilde{Z}_{22} &= N_x^2 \tilde{Z}_e + N_z^2 \tilde{Z}_h \\ \tilde{Z}_{12} &= \tilde{Z}_{21} = (\tilde{Z}_e - \tilde{Z}_h) N_x N_z \end{aligned} \tag{9}$$

with

$$N_x = \alpha / \sqrt{\alpha^2 + \beta^2} \qquad N_z = \beta / \sqrt{\alpha^2 + \beta^2} \tag{10}$$

Here, $\tilde{Z}_e, \tilde{Z}_h$ are the driving-point impedance of the TM and TE modes, respectively, and are given by

$$\begin{aligned} \tilde{Z}_e &= \frac{1}{Y_{TM1} + Y_{TM}\coth\gamma_m d} \\ \tilde{Z}_h &= \frac{1}{Y_{TE1} + Y_{TE}\coth\gamma_e d} \end{aligned} \tag{11}$$

with

$$Y_{TM1} = \frac{j\omega\varepsilon_o}{\gamma_1} \qquad Y_{TE1} = \frac{\gamma_1}{j\omega\mu_o} \tag{12}$$

and

$$\gamma_1 = \sqrt{\alpha^2 + \beta^2 - \omega^2 \mu_o \varepsilon_o} \tag{13}$$

Following the above steps, the wave admittances of the TE and TM waves with magnetic bias in z direction [7] can then be obtained from Maxwell's equations.

$$Y_{TE} = \frac{(\beta^2 \mu_\Delta + \alpha^2 \mu_z)\gamma_e + \alpha\mu_z \kappa(\alpha^2 + \beta^2)/\mu}{j\omega\mu_o\mu_z\mu_\Delta(\alpha^2 + \beta^2)} \tag{14}$$

$$Y_{TM} = j\omega\varepsilon_o\varepsilon_r / \gamma_m \tag{15}$$

with

$$\begin{aligned} \gamma_{m,e} = &\left(-\frac{q_{m,e}}{2} + \sqrt{(\frac{q_{m,e}}{2})^2 + (\frac{p_{m,e}}{3})^3}\right)^{1/3} \\ &+ \left(-\frac{q_{m,e}}{2} - \sqrt{(\frac{q_{m,e}}{2})^2 + (\frac{p_{m,e}}{3})^3}\right)^{1/3} \end{aligned} \tag{16}$$

and

$$\begin{aligned} q_m &= \frac{2}{27}a^3 - \frac{ab}{3} + c \\ p_m &= b - \frac{a^2}{3} \\ q_e &= -\kappa_o^2 \mu_z \varepsilon_r \kappa\alpha / \mu \\ p_e &= (\kappa_o^2 \mu_x \varepsilon_r - \alpha^2 - \beta^2 \mu_x / \mu) \\ a &= -\kappa_o^2 \varepsilon_r \kappa / 2 \\ b &= \kappa_o^2 \mu_x \varepsilon_r - \alpha^2 - \beta^2 \\ c &= \kappa_o^2 \varepsilon_r \kappa(\alpha^2 - \mu_x \varepsilon_r)/\alpha \end{aligned} \tag{17}$$

If the biasing field is in the x direction [7], the positions of α, β are exchanged in (14, 15), and $q_{m,e}$ and $p_{m,e}$, are replaced by

$$\begin{aligned} q_m &= \kappa_o^2 \varepsilon_r \kappa\beta / 2 \qquad p_m = \kappa_o^2 \mu\varepsilon_r - \alpha^2 - \beta^2 \\ q_e &= \kappa_o^2 \varepsilon_r \kappa\beta / \mu \qquad p_e = \kappa_o^2 \varepsilon_r - \alpha^2 / \mu - \beta^2 \end{aligned} \tag{18}$$

in equation (16).

III. RESULTS

In order to verify the procedure, the results obtained with this theory are compared to the mode-matching results of [8]. Fig. 2 shows the square of the normalized propagation constant of a microstrip line on ferrite substrate where ε_r=16,

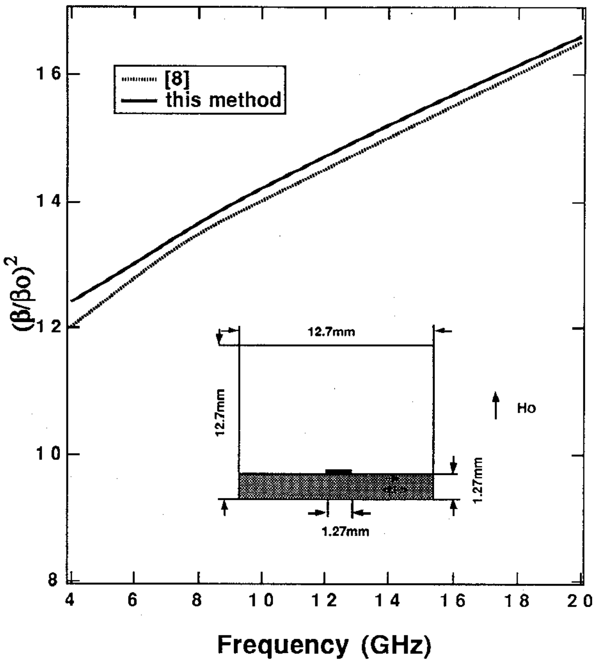

Fig. 2 Normalized propagation constant with biasing field H_0 in y direction; H_0=10KA/cm, M_s=1400A/cm, ε_r=16.

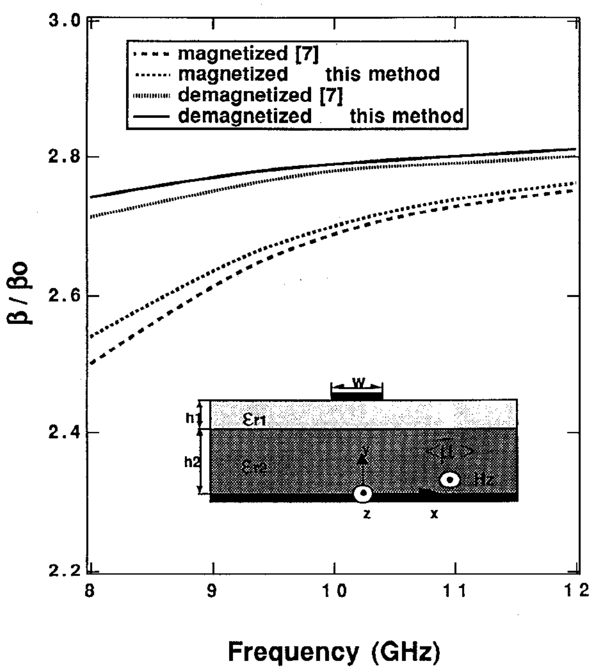

Fig. 3 Normalized propagation constant with biasing field H_0 in z direction; H_0=1740G, M_s=2300G, ε_{r2}=16.6, ε_{r1}=9.9, h_1=0.254mm, h_2=1.150mm.

and the applied dc magnetic field is in y direction. The magnitude of the biasing field H_0 and the saturation magnetization M_s are 10KA/cm and 1400A/cm, respectively. Good agreement is achieved for this case of a structure with reciprocal behavior.

In order to show the flexibility of this procedure, the algorithm is applied to analyze the characteristics of a dielectric-ferrite structure. The results are shown in Fig.3. The dielectric substrate has ε_r=9.9 and a thickness of 0.254mm. A Thomson/CSF ferrite is used with ε_r=16.6, H_0=1740G and M=2300G. The influence of demagnetized and partially magnetized levels in z-direction are investigated. Close agreement with the results given in [7] are obtained.

Fig. 4 shows a direct comparison between the method in [5] using identical transverse propagation constants and this approach that considers different values for γ_{TE} and γ_{TM}. Using the formulations given in [5], the agreement between the two methods is very close. This is due to the fact that in this case, the non-diagonal tensor elements of the ferrite are relatively small compared to the diagonal elements. Hence the influence of the non-diagonal quantities can be neglected which is exactly the assumption made in [5]. Although this is the common case for the ferrite structure, larger differences are expected if the influence of the non-diagonal elements increases as, e.g., in an anisotropic dielectric medium. The different tendencies encountered for different magnetization directions and structures (this theory's values are higher than [7,8] but lower that [5]) demonstrate the importance to consider the different transverse propagation constants in the impedance Green's functions for anisotropic substrates.

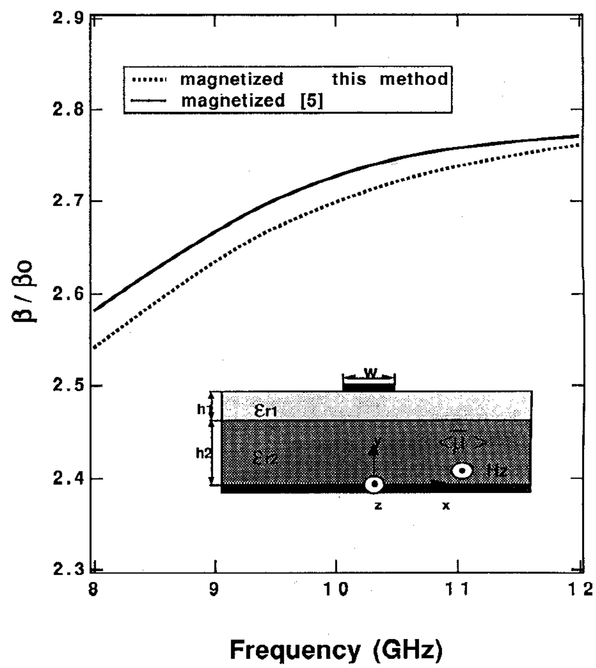

Fig. 4 Normalized propagation constant with biasing field H_0 in z direction (parameters refer to Fig. 3).

IV. CONCLUSIONS

In order to apply SDIA to the analysis of structures involving tensor-based media, a simple procedure is presented to rigorously derive the admittances of TM and TE waves for the anisotropic substrate. Since the impedance Green's functions can be formulated in closed forms, this method offers an attractive solution to problems with layered anisotropic media. By including the effects of different transverse TE- and TM-mode propagation constants in the spectral domain, the algorithm places no restrictions on the magnitudes of and relationships between tensor components. Hence any structure with any tensor forms for the anisotropic medium can be analyzed. Computed results agree well with those available in the literature.

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