Full-Wave Analysis of Planar Transmission Lines with Layered Lossy Anisotropic Media and High-T_C Superconductors

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ABSTRACT

A modified spectral domain immittance approach is presented which takes into account all practical parameters for the calculation of slow-wave and loss characteristics in modern MMIC and high-Tc superconductor microstrip transmission line structures. The method is capable of considering multilayered anisotropic substrates, conductor, dielectric and gyromagnetic losses, alternative directions for the magnetic bias, the finite-thickness of conventional or super-conducting strips, and lossy ground metallizations. At the example of ferrite-dielectric layered high-Tc superconducting microstrip lines, it is demonstrated that the line characteristics largely depend on both losses and the tensor parameters of anisotropic materials.

I. INTRODUCTION

Thin high-temperature superconducting film in (M)MIC transmission lines and circuits offer attractive solutions in applications such as microwave resonators, filters, delay lines and antenna systems [1-3]. Besides the low loss and low dispersion properties of superconducting line structures [4], the slow-wave effect is of particular importance [3, 5]. Ferromagnetic semiconductor (FMS) structures exhibit more desirable guided wave behaviour than conventional metal-insulator semiconductors (MIS) [6]. However, the influences of substrate and conductor losses and layered ferrite-dielectric materials on slow-wave and loss characteristics have not been investigated yet for superconductor applications.

Therefore, this paper focuses on the analysis of an alternative slow-wave transmission line structure based on high-Tc superconductors on ferrite-dielectric substrates (c.f. Fig. 1). A modified spectral domain immittance approach [7] incorporates the concept of complex resistive boundary conditions [5] and simplifies the analysis of multilayered structures with tensor properties. The procedure allows the finite conductor thickness, a lossy ground plane as well as complex material constants and different directions of the magnetic bias to be considered.

II. THEORY

Compared with the conventional spectral domain immittance approach [7], the key problem in the case of an anisotropic substrate is to find the wave immittances of transversely traveling TE and TM waves for the commonly used anisotropy in x or z direction (c.f. Fig. 1). For a substrate magnetized in z direction, the permeability tensor is given by

$$< \vec{\mu} > = \mu_o \begin{pmatrix} \mu & -j\kappa & 0 \\ j\kappa & \mu & 0 \\ 0 & 0 & \mu_z \end{pmatrix} \tag{1}$$

where μ, κ and μ_z are complex quantities. The tensor elements or the scalar permeability for

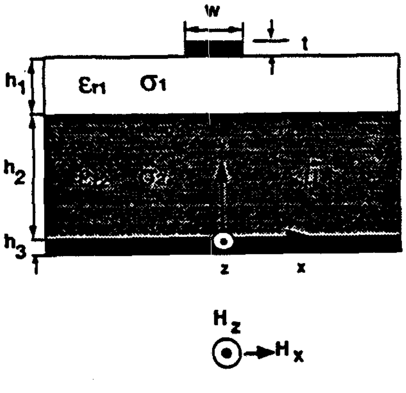

Fig. 1 Cross-section of a multilayered microstrip slow-wave transmission line.

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partially magnetized or demagnetized ferrites, respectively, are given by experimental expressions [8]. The immittances of transversely traveling TE and TM waves are derived as

$$Y_{TE} = \frac{(\beta^2\mu_\Delta + \alpha^2\mu_z)\gamma_y}{j\omega\mu_o\mu_z\mu_\Delta(\alpha^2+\beta^2)} + \frac{\alpha\mu_z\kappa(\alpha^2+\beta^2)/\mu}{j\omega\mu_o\mu_z\mu_\Delta(\alpha^2+\beta^2)} \qquad (2a)$$

$$Y_{TM} = j\omega\varepsilon_o\varepsilon_r / \gamma_y \qquad (2b)$$

with

$$\gamma_y{}^2 = \alpha^2 + \beta^2(1+\mu_z/\mu)/2 - k_o^2\varepsilon_r(\mu_z+\mu_\Delta)/2 \pm q/2 \qquad (3)$$

$$q^2 = k_o^2\varepsilon_r(\mu_\Delta - \mu_z) - \beta^2(1-\mu_z/\mu) + 2k_o^2\kappa^2\beta^2\varepsilon_r\mu_z/\mu^2 \qquad (4)$$

$$\mu_\Delta = (\mu^2 - \kappa^2)/\mu \qquad (5a)$$

$$k_o^2 = \omega^2\mu_o\varepsilon_o \qquad (5b)$$

and α, β, γ_y are the propagation constants in x, z and y directions, respectively.

The permeability tensor of a substrate magnetized in x direction can be obtained from [9]. The TE and TM wave immittances are then found by mutually replacing α and β in (2).The multilayered dyadic Green function is modified by considering a complex boundary condition [3] to incorporate the finite thickness of the conductor. If the thickness t of the strip of finite conductivity σ is greater than three or four penetration depths, the surface impedance is adequately represented by the real part of the wave impedance: $Z=(\omega\mu/2\sigma)^{1/2}$ [3]. If t is less than three penetration depths, the surface impedance is given by Z=1/(tσ) [5], where σ is real for conventional conductors and

$$\sigma = \sigma_n(T/T_c)^4 + \left(1-(T/T_c)^4\right)/\left(j\omega\mu\lambda_{eff}^2\right) \qquad (6)$$

for superconductors [3]. σ_n is often associated with the normal state conductivity at T_c; λ_{eff} is the effective field penetration depth.

Finally, the spectral-domain electric field at the conductor-dielectric interface is related to the current distribution by the modified dyadic Green's function elements

$$\tilde{E}_x = (\tilde{Z}_{xx} - Z)\tilde{J}_x + \tilde{Z}_{xz}\tilde{J}_z \qquad (7)$$

$$\tilde{E}_z = \tilde{Z}_{zx}\tilde{J}_x + (\tilde{Z}_{zz} - Z)\tilde{J}_z \qquad (8)$$

where $\tilde{Z}_{xx}$, $\tilde{Z}_{zx}$, $\tilde{Z}_{xz}$, $\tilde{Z}_{zz}$ are the dyadic Green's function elements for the multilayered structure in the spectral domain, $\tilde{E}_x, \tilde{E}_z$ and $\tilde{J}_x, \tilde{J}_z$ are the Fourier transforms of the electric field components and current distributions, and Z is the surface impedance of a metallic sheet of thickness t and conductivity σ. The current density is expanded in a set of Bessel functions which take the edge singularities into account [5]. The normalized phase constants β/β_0 and the attenuation constants are determined by the roots of the characteristic equation.

III. RESULTS

Fig. 2 shows the characteristics of a microstrip on ferrite-dielectric substrate. Close agreement with [8] is obtained in the lossless case and for an infinitely thin conductor. When using an extremely thin superconductor $YBa_2Cu_3O_7$ (YBCO: λ_{eff} =1500Å at T/Tc =0.5, σ_n=210000S/m), however, the propagation constant is reduced significantly.

The slow-wave and loss characteristics of an MMIC-applicable configuration [10] are shown in Fig. 3. The increase of the slow-wave factor and the reduction of the loss behaviour for the superconductor case compared with a copper conductor (σ_c=40000S/m) of identical thickness is owing to the fact that part of the electromagnetic field is drawn out of the substrate and into the superconductor.

The effect of the thickness of the superconductor on the backward-wave characteristics of a microstrip biased in x-direction is investigated in Fig. 4. This figure clearly demonstrates the increase of the

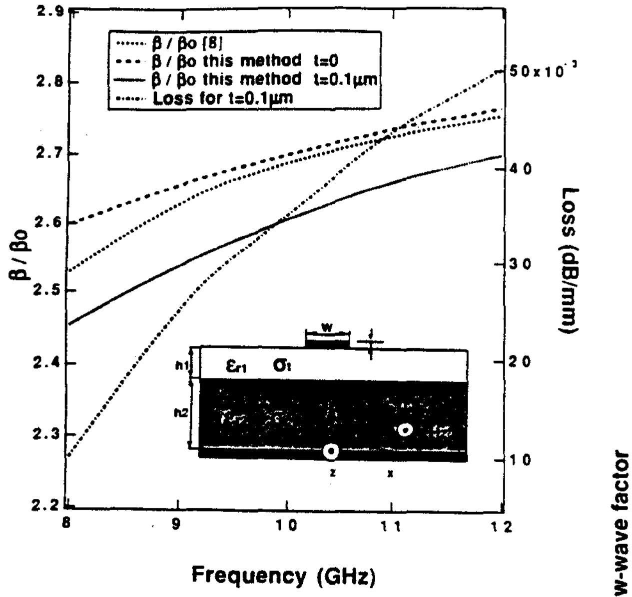

Fig. 2 Propagation characteristics of microstrip line with z-direction magnetized ferrite. h_1=1.15mm, h_2=0.254mm, w=0.9mm, ε_{r1}=16.6, ε_{r2}=9.9, M_s=2300G, H_z=1740G. Superconductor: T/T_c=0.5, λ_{eff}=1500Å, σ_n=210S/mm, t=0.1μm.

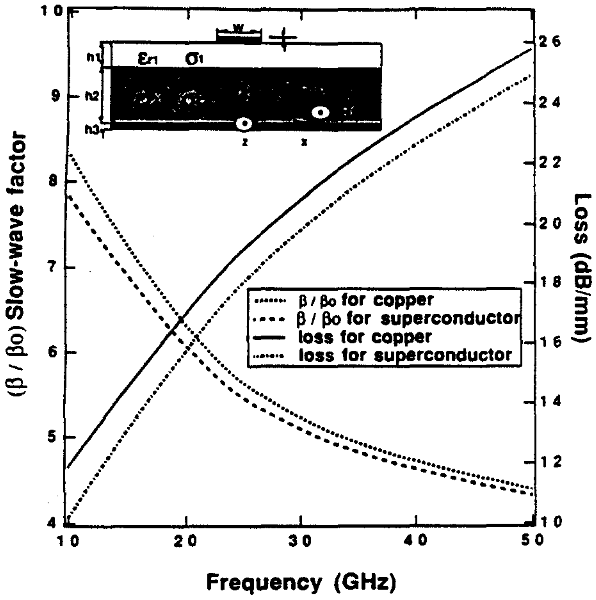

Fig. 3 Slow-wave factor and attenuation constant of multilayered microstrip line with magnetic bias in z direction. Dimensions: w=0.1mm, h_1=1μm, h_2=0.1mm, h_3=0.5μm, ε_{r1}=16.6, ε_{r2}=9.9, σ_c=40000S/m, σ_1=0.1S/mm, M_s=870G, Hz=2200Oers, ΔH=50Oers. Superconductor: T/T_c=0.5, λ_{eff}=1500Å, σ_n=210S/mm, t=0.1μm.

propagation constant caused by the kinetic inductance associated with the superconductor current. As the thickness of the superconducting layer decreases, the fractional amount of magnetic energy stored in the superconductor increases, hence increasing the slow-wave factor as well as the losses. A similar behaviour can be observed for the forward wave.

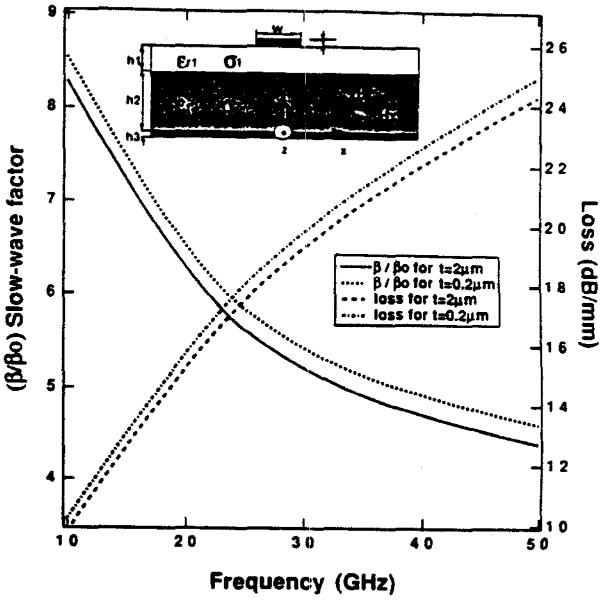

Fig.4 Slow-wave factor and attenuation constant of multilayered microstrip line with magnetic bias in x direction. Dimensions: w=0.1mm, h_1=1μm, h_2=0.1mm, h_3=0.5μm, ε_{r1}=16.6, ε_{r2}=9.9, σ_c=40000S/m, σ_2=0.1S/mm, σ_1=0.1S/m, M_s=870G, Hz=2200Oer, ΔH=50Oers. Superconductor: T/T_c=0.5, λ_{eff}=1500Å, σ_n=210S/mm.

Fig. 5 demonstrates the influence of the magnetization level on the slow-wave and loss characteristics with the thickness of the dielectric layer as parameter. While the slow-wave factor changes only slightly with magnetization and dielectric-substrate thickness, the influence of the latter on the loss behaviour is significant. With increasing thickness of the dielectric layer, a substantial part of the electromagnetic field propagates within the dielectric layer rather than in the ferrite of higher losses. Thus the losses of the structure are considerably reduced.

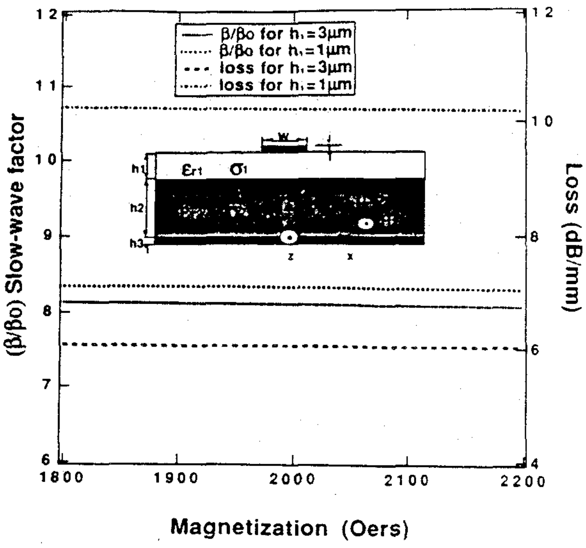

Fig.5 Slow-wave factor and attenuation constant of multilayered microstrip line with magnetic bias in z direction. Dimensions: $h_1=1\mu m$, $h_2=0.1mm$, $h_3=0.5\mu m$, $\varepsilon_{r1}=16.6$, $\varepsilon_{r2}=9.9$, $\sigma_c=40000S/m$, $\sigma_2=0.1S/mm$, $\sigma_1=0.1S/m$, $M_s=870G$, $\Delta H=50Oers$, frenquecy=10GHz. Superconductor: $T/T_c=0.5$, $\lambda_{eff}=1500Å$, $\sigma_n=210S/mm$, $t=0.3\mu m$.

IV. CONCLUSIONS

This paper introduces a general and efficient spectral domain immittance approach for the analysis of multilayered coplanar waveguide slow-wave structures with high-Tc superconductor. The method takes into account multilayered substrates, conductor, dielectric and gyromagnetic losses, alternative directions for the magnetic bias, the finite strip thickness and lossy ground metallizations. It is found that previously published zero-conductor-thickness approximations produce unreliable results. Slow-wave factors and losses decrease with increasing thickness of the superconductor since part of the electromagnetic energy propagates within the superconductor.

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