

# Spectral Domain Analysis of Coupled Microstrip Resonators With Tensor Substrates and High-Tc Superconductors

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## I. INTRODUCTION

Various types of interacting resonant structures are used in microwave integrated circuits. In the case of antenna, there is a tendency for coupled microstrip resonators to be used as antenna array elements[1]. One of the problems associated with designing such arrays is directly tied to mutual coupling between these elements. Mutual coupling must be considered when determining some of the array characteristics, because it can influence the value of some of its parameters, such as: the resonant frequency, losses and the radiation pattern. Several analytical methods have been used[2]-[3] to investigate the effect of mutual coupling. In most of these investigations, it was assumed that the substrate was isotropic and the patches were ideal conductors. Only a few investigate anisotropic substrate and high-Tc superconductor patches. Both of these enhancement to the classical patch antenna can improve its performance if proper design procedures are followed[4].

Therefore, this paper focuses on the analysis of coupled rectangular patch resonators with anisotropic substrates and high-Tc superconductor patches. The extended spectral-domain approach combined with the concept of imperfect conductors[5] is used. A decoupling procedure for the electric and magnetic field is developed which makes it possible to arrive at closed form impedance dyadic Green's functions in the spectral domain[5]. The resonant frequencies in the even and odd resonance modes of coupled resonator are evaluated from the numerical solution of the characteristic equation. The radiation patterns are also calculated.

## II. THEORY

The topology for a coupled patch antenna is shown in Fig. 1. The extended spectral domain immittance approach presented by [5] was used to include the conductivity and thickness of the ground metallization, the properties of conventional or high-Tc superconducting patches and the lossy anisotropic substrate. To describe the substrate we assume:

$$\langle \hat{\epsilon} \rangle = \epsilon_o \begin{pmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_x \end{pmatrix} \quad (1)$$

The current distributions on a coupled resonator can be obtained by[1]:

$$\tilde{J}_{zC,xC}(\alpha, \beta) = [\pm \eta e^{-j\alpha(s+w)/2} + e^{j\alpha(s+w)/2}] \tilde{J}_{x,z}(\alpha, \beta) \quad (2)$$

with  $\eta = +1$  (even mode),  $-1$  (odd mode), and the positive sign before the quantity  $\eta$  is taken for the x-directed current, while the negative sign is taken for the z-directed current. For the even mode,  $\tilde{J}_x(\alpha, \beta)$  and  $\tilde{J}_z(\alpha, \beta)$  are the same as the conventional expressions for current on a single patch. In order to obtain the radiation pattern of the structure, the complex resonant frequency of the coupled resonator should be solved by applying Galerkin's procedure. Once the complex resonant frequencies are obtained, far field radiation patterns for the resonator can be obtained from  $\tilde{E}_x$  and  $\tilde{E}_z$  since they are the Fourier transforms of the electric fields. Such an approach avoids the evaluation of Sommerfeld-type integrals when calculating the far fields. The expressions for far fields can be expressed as:

$$\begin{aligned} E_\theta(\phi, \theta) &\propto \sin\phi \tilde{E}_x(\alpha, \beta) + \cos\phi \tilde{E}_z(\alpha, \beta) \\ E_\phi(\phi, \theta) &\propto \cos\phi \cos\theta \tilde{E}_x(\alpha, \beta) - \cos\theta \sin\phi \tilde{E}_z(\alpha, \beta) \end{aligned} \quad (3)$$

using  $\alpha = \kappa \sin\phi \sin\theta$  and  $\beta = \kappa \cos\phi \sin\theta$   
 $\alpha, \beta$  are transformed into spherical coordinates.  $\kappa$  is the free space wavenumber  $\tilde{E}_x$  and  $\tilde{E}_z$  are given by:

$$\begin{aligned} \tilde{E}_x(\alpha, \beta, 0) &= (\tilde{Z}_{11}(\alpha, \beta) - Z_s) \tilde{J}_x(\alpha, \beta) + \tilde{Z}_{22}(\alpha, \beta) \tilde{J}_z(\alpha, \beta) \quad (4) \\ \tilde{E}_z(\alpha, \beta, 0) &= \tilde{Z}_{21}(\alpha, \beta) \tilde{J}_x(\alpha, \beta) + (\tilde{Z}_{22}(\alpha, \beta) - Z_s) \tilde{J}_z(\alpha, \beta) \quad (5) \end{aligned}$$

where  $\tilde{J}_x, \tilde{J}_z$  are the Fourier transform of current distributions of  $J_x$  and  $J_z$ . The E plane and H plane radiation patterns correspond to setting  $\phi=0$ , and  $\phi=\pi/2$ , respectively.

### III. NUMERICAL RESULTS

Fig. 2 shows the effect of spacing on the even and odd mode resonant frequencies of superconducting resonators on anisotropic substrate. Increasing the distance between the patches increases the even but decreases the odd mode resonant frequencies. This is due to the fact that the interaction between the two patches becomes weaker with increasing patch spacing. The same behavior can be observed in Fig. 3 for the Q-factor of the coupled resonators. As expected, higher losses, lower the Q factor for the even mode, since the field is mainly concentrated on the lossy substrate. Note that larger losses are obtained for the odd mode than for even mode.

The normalized far-field radiation patterns for even and odd modes in the E- and H-planes are investigated in Fig. 4 and Fig. 5, respectively. The resonant frequencies for even and odd modes are  $f_{er}=4.14$ GHz for the even mode and  $f_{or}=4.58$ GHz for the odd mode. A YBCO substrate with  $\epsilon_r=23$  for superconductor application was chosen. As expected and shown in Fig. 4, the radiation pattern for the even mode is quite similar to that of the single patch. The odd-mode patterns in Fig. 5 contain a null in broadside direc-

tion. This is due to the fact that the field radiated by the odd mode is out of phase and cancels in broadside direction.

#### IV. CONCLUSION

The extended spectral domain immittance approach was used to rigorously analyze the resonance frequencies, losses and radiation patterns of coupled rectangular patch antennas. This was done for both its odd and even modes. The demonstrations show that the interaction between two patches has a severe effect on the resonant frequencies and Q-factors of the structure. Therefore it is necessary to take mutual coupling into account in the applications of modern integrated circuit antenna designs.

#### REFERENCES

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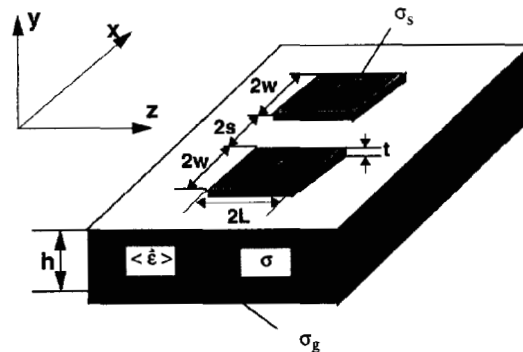


Figure 1 Illustration of coupled patch antenna

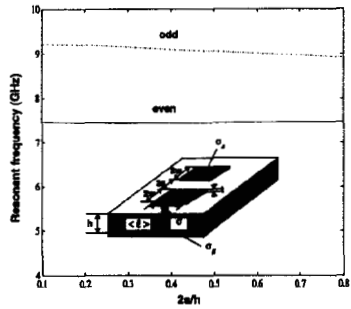


Figure 2 Resonant frequencies of even and odd modes versus ratio of  $2a/h$  for coupled patches. Parameters:  $b=0.5mm$ ,  $2l=6mm$ ,  $2w=1.5mm$ ,  $\epsilon_r/\epsilon_0=9.4/11.6$ ,  $\sigma=0.055/m$ , superconductor:  $t=0.5\mu m$ ,  $\sigma_s=2000/m$ ,  $T/T_c=7/92.5$ ,  $\lambda_{gp}=1500\mu m$ , ground plane:  $\sigma=600/m$ ,  $h=0.5mm$

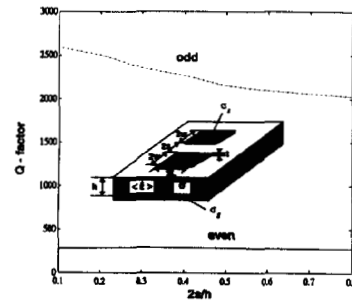


Figure 3 Q-factor of coupled patches of Fig. 2

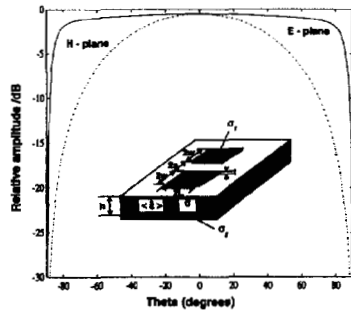


Figure 4 Radiation patterns of coupled patches for even mode excitation. Parameters:  $b=1mm$ ,  $2l=8mm$ ,  $2w=0.5mm$ ,  $2a=0.5mm$ ,  $\epsilon_r/\epsilon_0=23$ ,  $\sigma=0.015/m$ , superconductor:  $t=0.5\mu m$ ,  $\sigma_s=2000/m$ ,  $T/T_c=7/92.5$ ,  $\lambda_{gp}=1500\mu m$ , ground plane:  $\sigma=60 S/m$ ,  $h=0.5mm$ ,  $f_c=4.14GHz$

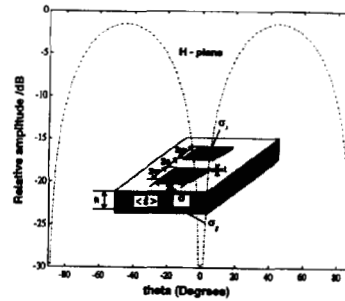


Figure 5 Radiation pattern of coupled patches for odd mode. Parameters are the same as Fig. 4 except  $f_c=4.58GHz$  and without E-plane pattern for odd mode.