MODIFIED ANALYSIS AND DESIGN OF WAVEGUIDE HORNS

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ABSTRACT

A modified analysis of standard waveguide horns is presented which accurately accounts for the aperture phase error. It is demonstrated that standard analysis techniques fail to produce the correct gain, particularly in the presence of low-to-medium-gain horns. Optimum dimensions are presented for conical and various pyramidal horns. It is found that, if a comparison is made on the basis of the actual horn length, standard and improved synthesis techniques are near-optimum. Recent claims of almost constantly higher gains, even for high-gain designs, cannot be substantiated.

I. INTRODUCTION

Waveguide horns in their classical pyramidal or conical form are well known and, therefore, are part of almost every standard text on antennas, e.g. [1] - [3]. While pyramidal horns are usually introduced via E-plane and H-plane sectoral horns, and design procedures are derived from the individual properties of these two components, guidelines with respect to the conical horn are mostly presented by reprinting the graphs of King [4]. In both cases, analysis procedures and, consequently, designs based on such analyses, are based on the assumption of a phase center, which is located within the feeding waveguide and which emits a spherical wave front traveling with free-space propagation constant through the horn section, thus causing a phase error distributed across the aperture of the antenna. The maximum phase error influences the performance of the horn and, therefore, affects the optimum horn dimensions for a specified gain.

Although this assumption, in the following refered to as standard analysis, constitutes an excellent approximation for high-gain or long horns, three fundamental problems are associated with it.

1. From the phase center to the throat of the horn, the standard analysis predicts already a small phase error, when in fact this phase error does not exist, because the wave is still propagating through a waveguide of constant cross-sectional boundary.
2. The standard analysis neglects the dispersion in the horn section and hence assumes that the wavelength is that of free space, when it actually is slightly longer.
3. In the aperture, the standard analysis assumes that electric and magnetic fields are related by the free-space impedance, when it still is the wave impedance of a waveguide of aperture cross-section dimensions that governs the ratio between electric and magnetic field components.

It is immediately obvious that the effects of all three points increase with decreasing horn dimensions, thus with decreasing gain. Moreover, the first two points clearly lead to an overestimation of the aperture phase error in the standard analysis and, therefore, might restrict the design engineer from obtaining an optimum compact horn for a specified gain. Consequently, it has been claimed recently [5] that, compared with standard [1] and improved [6] synthesis procedures and because of the first two items raised above, the gain of a pyramidal horn can be improved for a given length or, in other words, a shorter horn can be obtained for a given gain. However, the dimension of comparison in [5] is the phase-center-to-aperture distance h_1 and not the actual horn length h (c.f. Fig. 1). It follows from simple geometry that these two distances do not necessarily vary concurrently.

Therefore, this paper focuses on the relationship between gain and horn length with respect to the synthesis of conical and pyramidal horns. The modified analysis in accordance with the three points raised above is demonstrated and compared with the standard analysis technique. Optimum gain dimensions are presented and compared with well-known standard and improved synthesis procedures.

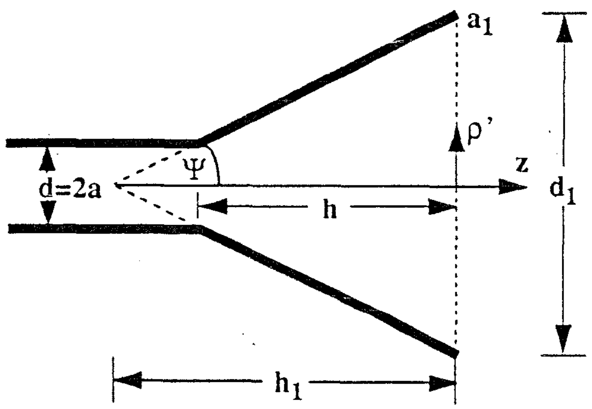

Figure 1: *Geometry of a conical waveguide horn.*

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II. THEORY

In this section, the modified analysis of the conical horn is presented. The pyramidal case follows well-known equations in [1] - [3] with the aperture phase error and integration for pattern calculations according to [5].

The phase error across the aperture is given by (Fig.1)

$$\phi_{ap}(\rho') = (\phi_{cwg} + \phi_{horn}) \left[\frac{1}{\cos\{\arctan(\rho'/h_1)\}} - 1 \right] \quad (1)$$

where

$$\phi_{cwg} = \frac{2\pi}{\lambda_{g11}} (h_1 - h) \quad (2)$$

represents the phase delay from the phase center to the throat of the horn and

$$\phi_{horn} = 2\pi \int_{-h}^{0} \sqrt{\frac{1}{\lambda^2} - \frac{1}{\left(\frac{2\pi}{p'_{11}}(a + z\tan\Psi)\right)^2}}\, dz \quad (3)$$

that of the conical horn section of length h. In (1) - (3), λ_{g11} is the wavelength of the fundamental TE_{11} mode in the feeding waveguide

$$\lambda_{g11} = \frac{\lambda}{\sqrt{k^2 - \left(\frac{p'_{11}}{a}\right)^2}} \quad (4)$$

$\lambda = 2\pi/k$ is the free-space wavelength, and p'_{11} is the first zero of the derivative of Bessel function $J_1(x)$. If the electric field is oriented with its maximum at $\phi = \pi/2$, then the far field components can be evaluated from

$$E_\theta \propto \sin\phi \left(1 + \frac{\beta_{11}}{k}\cos\theta\right)(I_{\theta 1} + I_{\theta 2}) \quad (5)$$

$$E_\phi \propto \cos\phi \left(\frac{\beta_{11}}{k} + \cos\theta\right)(I_{\phi 1} + I_{\phi 2}) \quad (6)$$

where the term

$$\frac{\beta_{11}}{k} = \sqrt{1 - \left(\frac{p'_{11}/a_1}{k}\right)^2} \quad (7)$$

accounts for the fact that the wave impedance of free space differs slightly from that of the TE_{11} mode in the aperture. Note that this factor decreases with decreasing frequency and with decreasing aperture dimensions, i.e., lower gain. For the conical horn discussed here, integrals $I_{\theta 1,2}$ and $I_{\phi 1,2}$ are given in [7]. The phase error calculation for the pyramidal horn is presented in [5] and need not be repeated here. With (5) and (6), the far field for $z>0$ is obtained, and the gain is evaluated by numerical integration of E_θ and E_ϕ [1].

III. RESULTS

Whereas for given aperture dimensions, the optimum horn length tends to infinity for optimum gain, conical and pyramidal horns can be gain-optimized for the aperture dimensions if the horn length is specified. Fig. 2 displays such aperture dimensions (solid lines) for a square horn. Note that since the feeding waveguide is usually operated below TE_{11} cutoff, a/λ is chosen as 0.65. For comparison, the dashed lines represent the solutions obtained by the standard analysis technique [1] - [3] which neglects items 1 - 3 discussed in the introduction. It is obvious that the standard analysis technique fails for low-gain application, e.g., below 20 dB.

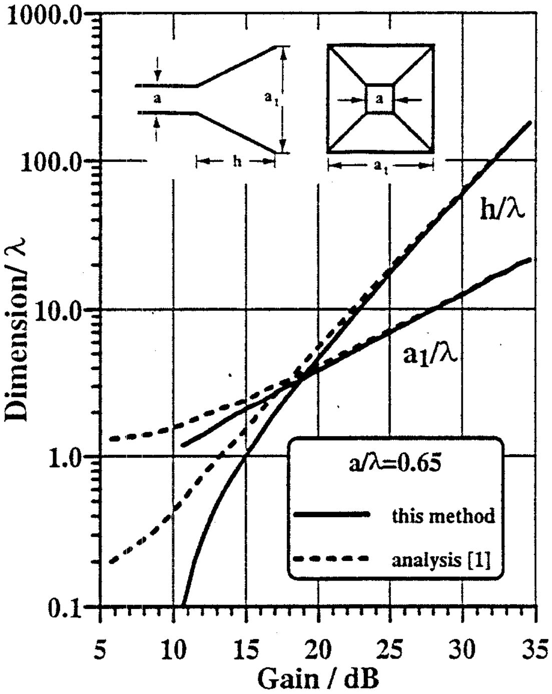

Figure 2: *Optimum dimensions of square horn (solid lines) and comparison with standard analysis in [1].*

Therefore, it is interesting to see how the synthesis procedure bases on the standard analysis performs. However, the standard synthesis technique in [1], [2] has already been investigated in [6], where the number of approximations in the design procedure has been reduced. A comparison between the optimum values and the synthesis of [6] for the square horn is depicted in Fig. 3. Note that in both cases, and all following ones, the method described in this paper is used for the gain analysis. Obviously, synthesis guidelines in [6] are very good. However, the optimum design yields slightly reduced aperture dimensions in the higher gain range and shorter horns below 12 dB.

For the conical horn, design guidelines were first given in [4] and then modified to fit a closed-form expression in [1]. The fundamental-mode range in the feeding circular waveguide is $0.586 < d/\lambda < 0.766$. Fig. 4 shows that the design guidelines for the conical horn are excellent and do not require any modifications.

So far, aperture dimensions were specified by one parameter only. The general pyramidal horn, however, requires

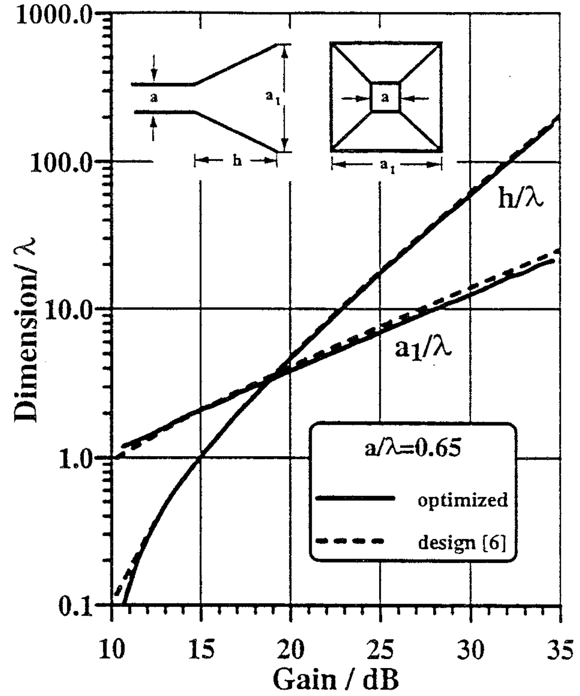

Figure 3: *Optimum dimensions of square horn (solid lines, same as in Fig. 2) and comparison with improved synthesis in [6].*

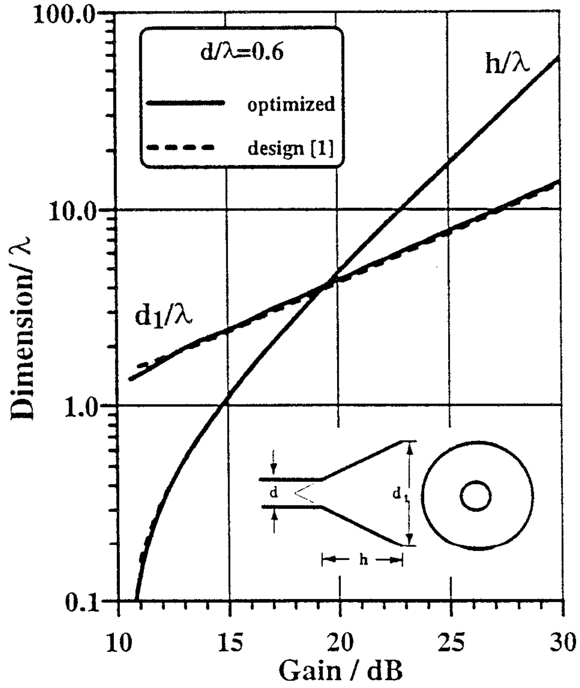

Figure 4: *Optimum dimensions of conical horn (solid lines) and comparison with synthesis in [1].*

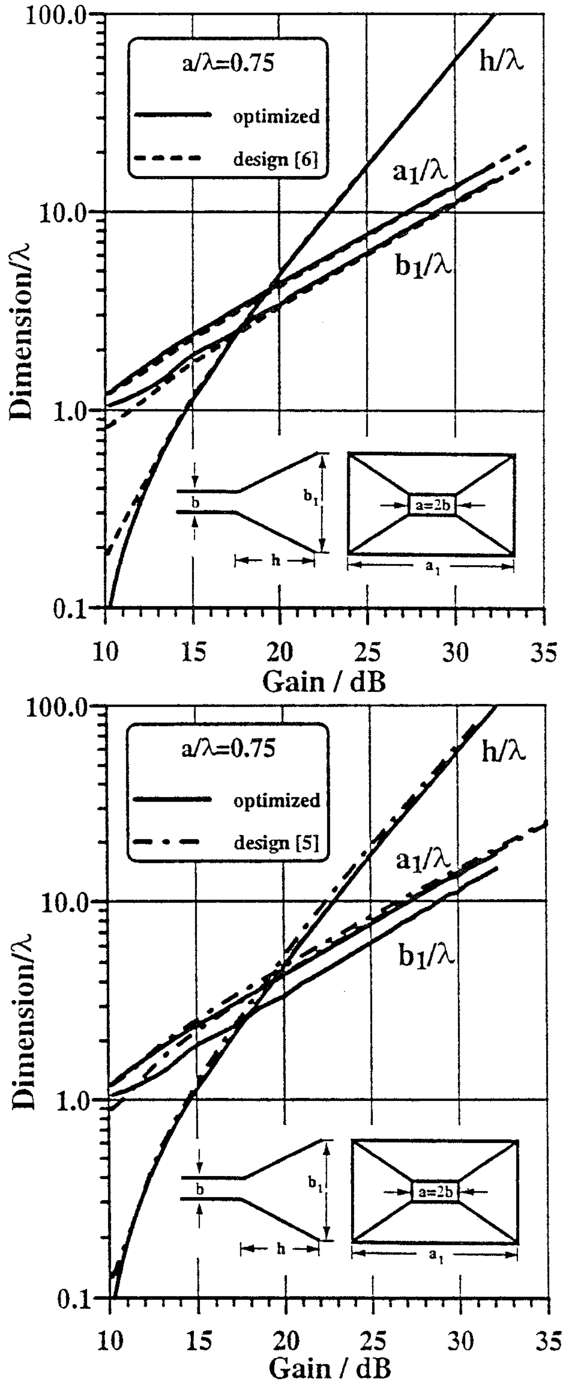

Figure 5: *Optimum dimensions of pyramidal horn with standard rectangular waveguide feed (solid lines, same in both figures) and comparison with syntheses in [6] (top) and [5] (bottom).*

two dimensions to be optimized. A simple gradient method is used here for this purpose. Results for the standard rectangular waveguide (a=2b) are depicted in Fig. 5. While the solid lines represent the optimum dimensions, dashed and dash-dotted lines show results from [6] and [5], respectively. Again, the design guidelines in [6] are very good except in the very-low-gain range (Fig. 5, top). However, the claim of [5], whose design procedure is supposed to provide higher gain or shorter horns than the synthesis procedure in [6], cannot be substantiated (compare dashed lines in Fig. 5, top, with dash-dotted lines in Fig. 5, bottom). It should be noted that this is also not the case, if the horn length h_1 is plotted instead of h (Fig. 1).

Reduced-height rectangular waveguides (a=4b) are frequently used in antenna feed systems. For such a feeding waveguide, Fig. 6 presents the optimum (solid lines) and synthesized [6] (dashed lines) dimensions. Again, the design guidelines of [6] are near optimum and only lead to longer horns below a gain of 12 dB.

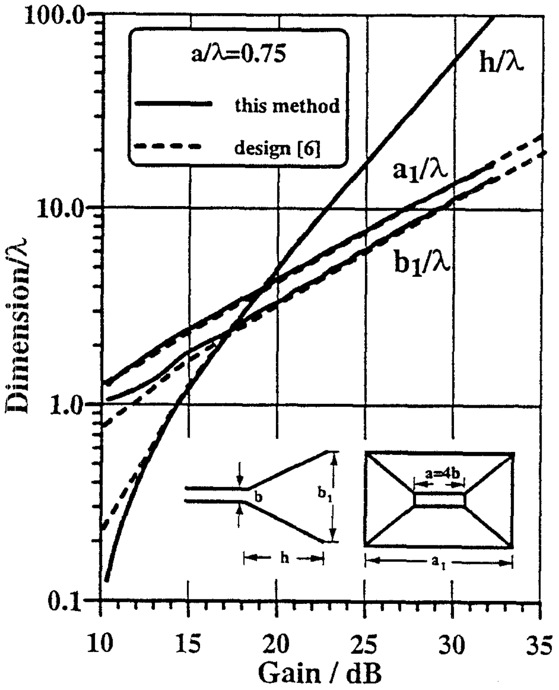

Figure 6: *Optimum dimensions of pyramidal horn with reduced-height rectangular waveguide feed (solid lines) and comparison with synthesis in [6].*

IV. CONCLUSIONS

A modified analysis of waveguide horns is presented, which solves for the gain and the accurate phase variation over the aperture by including the correct dispersion relations in the feeding guide, the actual horn section, and the aperture. It is demonstrated that this technique is superior to the analysis available in standard antenna texts. However, the design guidelines available in the literature are shown to be sufficient for medium-to-high-gain applications. Results obtained by a recently published synthesis, which claims an improvement in horn gain on the basis of a very similar analysis, cannot be confirmed.

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