

# Mode-matching CAD method for broad-wall couplers with square apertures of different size

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## Abstract

A rigorous field theory CAD method for rectangular waveguide broad wall aperture couplers is introduced which are coupled by square apertures of different size. Based on the corresponding mode-matching key-building blocks, i.e. rectangular waveguide T-junction and double-aperture step discontinuity, and their modal S-matrix combination, the design takes rigorously into account both the finite wall thickness and the higher order mode interaction between all discontinuities. The presented 5dB-coupler design example in the waveguide Ku-band with a double array of ten square apertures demonstrates the efficiency of the design method. The coupling variation is less than  $\pm 0.5$ dB (together with more than 40dB isolation and return loss) within 11.0 and 14.8 GHz. The theory is verified by excellent agreement with measurements.

## Introduction

Aperture coupling by holes in the broad wall of waveguides is a well-known technique to build a useful class of directional couplers for a great variety of applications [1] - [5]. Many refined equivalent-network synthesis methods are available for deriving the necessary coupling coefficients for desired directional coupler characteristics [1], [2], [4]. However, the design of the practical aperture geometries is still based on the approximate Bethe-Cohn theory [6], [7], together with thickness correction factors [2], [8], [9].

Although for small slots in the common waveguide wall improved calculations based on the variational [9], reaction [10], or moment method [11] are available, no rigorous analysis for large rectangular apertures in the broad wall of waveguides has been derived so far. The increasing demand for high-quality low-cost components and the requirement for shorter development cycles have prompted the need for accurate field theory based CAD methods which take

adequately into account the effects of large apertures, finite wall thickness, and the higher-order mode interaction at all discontinuities.

This paper presents a rigorous and efficient computer-aided design method for aperture coupled rectangular waveguide broad-wall couplers of the class shown in Fig. 1. The design method is based on the full wave mode-matching method which takes rigorously into account the finite wall thickness and the higher-order mode interaction of all discontinuities. In contrast to e.g. recent moment method solutions for narrow slots [11], the effect of large apertures is accurately included. Moreover, the suitable modal S-matrix formulation for multiport structures based on the asymmetric double-step discontinuity [13] or the asymmetric transition from rectangular to smaller circular waveguide [14] allows the rigorous formulation for double-aperture structures which have turned out to be more convenient for such types of couplers [2].

A typical design example, e.g. for beamforming network application purposes in the waveguide Ku-band (12 - 18 GHz), a 5dB-coupler with a double array of ten rectangular apertures, demonstrates the efficiency of the rigorous CAD method. The coupling variation is less than  $\pm 0.5$ dB together with more than 40dB isolation and return loss, respectively, between 11.0 and 14.8 GHz. For computer optimization, the evolution strategy method [12] is applied. The initial values are derived by utilizing the classical network theory synthesis methods [1], [2], [4], [8]. The presented theory is verified by excellent agreement with measurements.

## Theory

For the rigorous computer-aided design of the coupler structure to be investigated (Fig. 1), the full wave mode-matching method is applied. The required key-building block elements are shown in Fig. 2: The general T-junction (Fig. 2a) [12], and the transitions from rectangular waveguide (Fig. 2b) to two smaller rectangular waveguides (for modelling rectangular double-apertures), or (cf. Fig. 2c) to two smaller circular waveguides (for modelling circular double-apertures), respectively. The generalized modal S-matrix (or modal Y-matrix) combination of all building blocks together with empty waveguide sections of finite lengths [12] - [16], achieves the rigorous description of the waveguide multiple-aperture coupling element as well as of the complete coupler structure.

The full set of  $TE_{mn}$  and  $TM_{mn}$  modes in all sections is required for each key-building block in order to model adequately the composed general structure. For the waveguide subregion under consideration, the fields [12] - [14]

$$(1) \quad \begin{aligned} \vec{E}^\nu &= \frac{1}{j\omega\epsilon} \nabla \times \nabla \vec{A}_e^\nu + \nabla \times \vec{A}_h^\nu, \\ \vec{H}^\nu &= -\frac{1}{j\omega\mu} \nabla \times \nabla \vec{A}_h^\nu + \nabla \times \vec{A}_e^\nu, \end{aligned}$$

are derived from the z-components of the electric and magnetic vector potentials  $A_e$ ,  $A_h$ , respectively,

$$(2) \quad \begin{aligned} \vec{A}_{hz} &= \sum_{i=0}^{N_h} Q_{hi} T_{hi} (a_{hi} e^{-\gamma_{hi} z} + b_{hi} e^{+\gamma_{hi} z}), \\ \vec{A}_{ez} &= \sum_{i=1}^{N_e} Q_{ei} T_{ei} (a_{ei} e^{-\gamma_{ei} z} - b_{ei} e^{+\gamma_{ei} z}), \end{aligned}$$

where  $a_i$ ,  $b_i$  are the still unknown eigenmode amplitude coefficients of the forward (-) and backward (+) waves in z direction,  $\gamma_{h,e}$  are the propagation factors of the  $N_h$  and  $N_e$  considered  $TE_{pq}$  and  $TM_{pq}$  modes, respectively, where i stands for p,q.  $Q_{h,e}$  is the normalization factor [12] - [14], and  $T_{h,e}$  are the cross-section eigenfunctions for the rectangular waveguide [12], [13], or circular waveguide [14], respectively.

By matching the tangential field components at all interfaces at the individual step discontinuities, the wave amplitude coefficients of equation (2) can be related to each other after multiplication with the appropriate orthogonal functions. This yields the corresponding key building block three-port modal scattering matrix of the T-junction [12], and the two-port modal S-matrices of the transitions rectangular to asymmetrically located smaller rectangular [13] or circular [14] waveguide.

These general asymmetric single transitions contain already the full information for modelling the required double-apertures in Figs. 2b, 2c for the coupler structure to be investigated. For modelling multiport transitions in rectangular waveguides, in [15], the related coupling matrices of the single transition are combined by the modal admittance matrix formulation. It is also possible, however, to choose the modal S-matrix combination. The appropriate efficient algorithm given below avoids the inversion of a large matrix.

The relations for the modal scattering matrix elements ( $S_{LL}, S_{Lk}, \dots$ ) for the general waveguide (M + 1)-port, where the normalized coupling matrices of the corresponding two-ports L,k are already known, may be written analogously to the general n-furcation expression in [17]. If L designates the large rectangular waveguide input port (cf. Figs. 2b, 2c), k designates the smaller rectangular or circular output ports ( $k=I, II, III, \dots, M$ ) of the (M+1)-port, and the single normalized coupling matrices  $\underline{G}^{Lk}$  of the corresponding two-ports are already known, the following relations hold for the scattered wave amplitudes

$A^{L-}$ ,  $A^{k+}$  as a function of the incident wave amplitudes  $A^{L+}$ ,  $A^k$ ,  $A^u$

$$(3) \quad \begin{aligned} A^{L-} &= \underline{Y} \left( \sum_{k=I}^M \overbrace{(\underline{G}^{Lk} \underline{G}^{Lk'})}^{S_{LL}} - \underline{E} \right) A^{L+} + \sum_{k=I}^M \overbrace{Y \underline{G}^{Lk}}^{S_{Lk}} A^k, \\ A^{k+} &= \overbrace{(-\underline{G}^{Lk'} \underline{S}_{LL} + \underline{G}^{Lk'})}^{S_{Lk}} A^{L+} + \overbrace{(\underline{E} - \underline{G}^{Lk'} \underline{S}_{Lk})}^{S_{kk}} A^k \\ &\quad + \sum_u^M \overbrace{-\underline{G}^{Lk'} \underline{S}_{Lu}}^{S_{ku}} A^u, \end{aligned}$$

$$\underline{Y} = \left( \underline{E} + \sum_{k=I}^M \underline{G}^{Lk} \underline{G}^{Lk'} \right)^{-1},$$

$\underline{E}$  = unit matrix,  $k=I, II, III, \dots, M$ , and  $u = I, II, III, \dots, M$  ( $u \neq k$ ). Note that only one matrix inversion is required for deriving the relations in (3) and that the size of the matrix is only dependent from the number of modes involved, but independent from the number of ports.

For the calculation of the overall coupler, the four-port may be reduced conveniently to a two-port structure by placing an electric or magnetic wall, i.e. odd (o) or even (e) symmetry plane, respectively, along the longitudinal plane of geometrical symmetry. A computer program was written using the preceding relations and utilizing the evolution strategy method [12] - [14] for optimizing the geometrical parameters. The initial values for the field-theory optimization are calculated by the classical network-theory synthesis method [2], [8]. For the field theory CAD of the aperture couplers, sufficient asymptotic behavior has been obtained by consideration of 150  $TE_{mn}$  and 120  $TM_{mn}$  modes (including the  $TE_{m0}$  and  $TE_{0n}$  modes) in the order of increasing cut-off in all waveguide sections.

## Results

For comparison, Fig. 3 shows the calculated scattering parameter  $S_{13}$  of a slot aperture with finite thickness in the broad wall of rectangular WR-90 waveguides (22.860mm x 10.16mm) for which recent reference values are available [11]. Good agreement can be stated. Note that the moment method calculations of [11] are only valid for such narrow slots since a uniform field along the slotwidth has been assumed.

For a verification of the theory for large apertures, Fig. 4 shows the calculated and measured scattering parameters of a rectangular aperture with finite thickness in the broad wall of rectangular WR-62 waveguides (15.799mm x 7.899mm). Excellent agreement with the measurements can be stated.

A typical design example for beamforming network application purposes in the waveguide Ku-band (12 - 18 GHz) is chosen as the next example, Fig. 5. The excellent broadband characteristics of this 5dB-coupler with a double array of ten optimized rectangular apertures demonstrates the efficiency of the rigorous CAD method. The coupling variation is less than  $\pm 0.5$ dB together with more than 40dB isolation and return loss between 11.0 and 14.8 GHz. For computer optimization, the evolution strategy method [12] is applied. The initial values are derived by utilizing the classical network theory synthesis methods [1], [2], [4], [8]. Good agreement is obtained between theory and measurements. In Fig. 6 the phases of the corresponding scattering parameters are shown. As

can be stated, the transmission of the output ports of the coupler demonstrate a stable phase shift of  $90^\circ$ .

## Conclusion

A rigorous field theory CAD of rectangular waveguide broad wall aperture couplers is introduced which takes rigorously into account the finite wall thickness of the coupling holes, large apertures and the higher mode interaction between all discontinuities. The adequate combination of the rectangular waveguide T-junction and the double-step discontinuity key-building block modal S-matrices yields directly the overall scattering matrix of the coupler structure. For computer optimization, the evolution strategy method is applied, where the initial values are derived by utilizing the classical network theory synthesis method. Fast and reliable designs are achieved. The rigorous theory is verified by excellent agreement with measurements.

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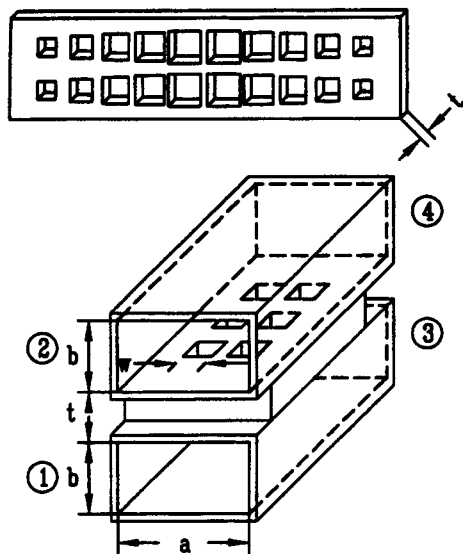


Figure 1: Broad-wall aperture coupler structure with a double array of square apertures of different side-lengths  $w_i$ .

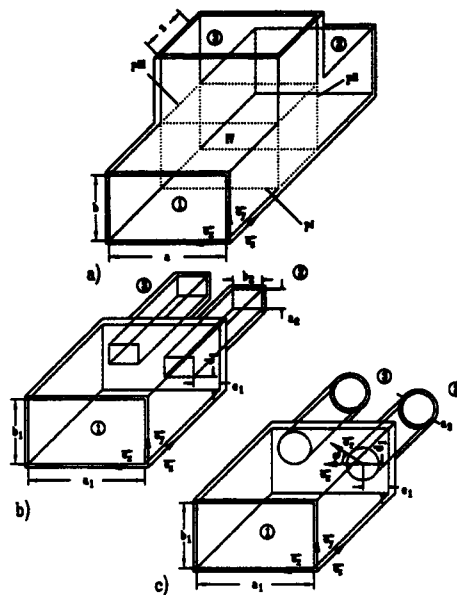


Figure 2: Mode-matching key-building block discontinuities for modelling the coupler structure of Fig. 1. (a) T-junction. (b) Rectangular waveguide transition to two rectangular waveguides of smaller size. (c) Rectangular waveguide transition to two double circular waveguides of smaller size.

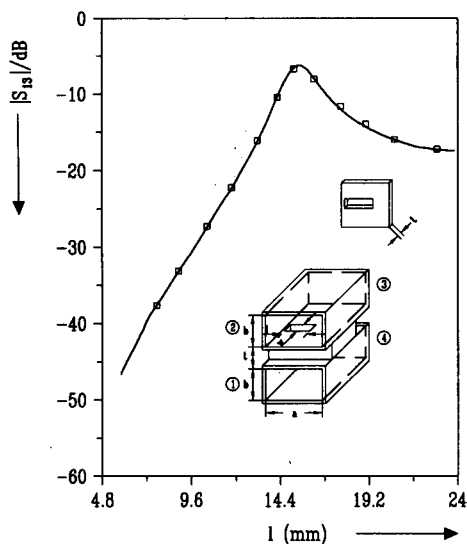


Figure 3: Calculated scattering parameter  $S_{13}$  of a slot aperture with finite thickness in the middle of the broad wall of rectangular WR-90 waveguides ( $22.860\text{mm} \times 10.16\text{mm}$ ) for which recent reference values are available [11] ( $\square \square \square$ ).  $f=9.375\text{GHz}$ ,  $w=1.5875\text{mm}$ ,  $t=1.58\text{mm}$ .

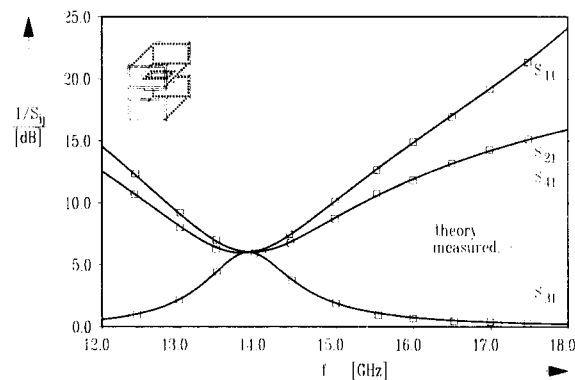


Figure 4: Calculated and measured scattering parameters of a large broad-wall aperture with finite thickness. Waveguide: Ku-band (12-18 GHz, R140, WR-62 waveguide housing:  $15.799\text{mm} \times 7.899\text{mm}$ ), aperture dimensions (mm):  $a_1 = 10.3$ ,  $b_1 = 3.0$ ,  $t = 2.5$ .

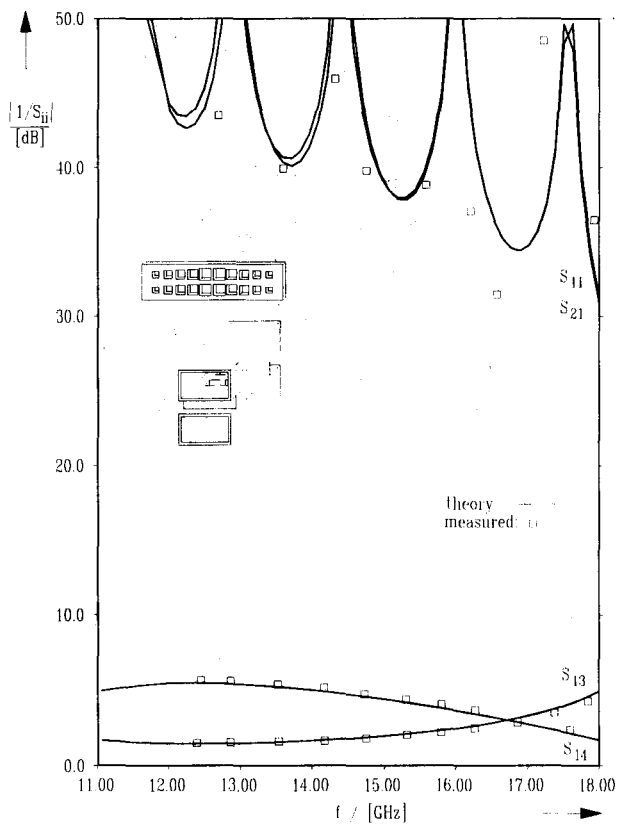


Figure 5: 5dB-coupler with an array of ten double rectangular apertures. Waveguide: Ku-band (12-18 GHz, R140, WR-62 waveguide housing: 15.799mm × 7.899mm) Calculated and measured scattering parameters.

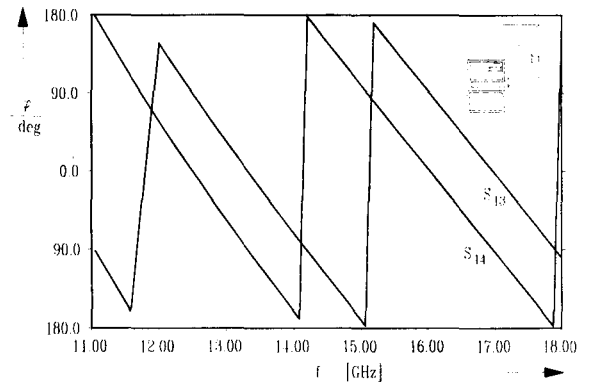


Figure 6: Calculated phases of the scattering parameters  $S_{13}$  and  $S_{14}$  of the 5dB-coupler of Fig. 5. parameters.