EFFICIENT EIGENVALUE ANALYSIS OF RECTANGULAR COAX FOR SLOT-FED PATCH ANTENNAS

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I. INTRODUCTION

Microstrip patch antennas have been investigated by several authors in the past, using different analysis techniques, e.g. [1]. With spectral methods, good results for the microstrip resonator can be achieved, but it is not practical to include the feed circuitry in the analysis. This might introduce a considerable error depending on the strength of the coupling between the feed and the patch. In order to account for its influence, the method of moments generally is applied. This, however, requires a considerable computational effort, especially for large matrices, since the elements in the computed matrix are frequency-dependent, and must be computed for each frequency point [2].

An alternative approach is to look at the patch as a very short length of rectangular coax line. On one side it is bound by free space, on the other, by a dielectric layer and an electric wall, the ground plane. A slot in the wall enables aperture-coupling from a microstrip line on the other side of the ground plane. This problem is solved by determining the coupling from the slot to the patch. The first, and most time-consuming step of the analysis is to find the mode spectrum of the coax line and the corresponding field distributions. Therefore, this will be the main topic of this paper.

Typically, modal field-matching techniques are applied to this kind of structures, and good results can be achieved [3]. However, there are several shortcomings. The fields along the interface between different subregions of the cross-section are expressed in terms of pure sine and cosine functions. Therefore, they usually do not satisfy the boundary condition for the larger region, nor the edge condition at a metal corner, i.e., the singularity of the fields. In order to achieve a satisfactory approximation of the field at the interface, thus to get reliable results, a large number of modes is usually required. This is particularly true in this case, where the electromagnetic field itself is to be computed. In addition, we encounter the problem of relative convergence which is especially important for large dimensional differences between subregions.

Therefore, an Integral-Equation Technique is applied in this paper which overcomes these deficiencies. The electric field along the interface is expressed in terms of suitable basis functions which already include the proper edge conditions [4]. Also, the tangential electric field along the metal wall is, a priori, identically zero for any number of basis functions whereas, in standard modal field-matching techniques, the vanishing tangential E-field is approximated by a superposition of many sine and/or cosine terms. The theory is verified by comparing our results with the standard mode-matching technique. It is shown that a few basis functions are sufficient, not only to calculate the cut-off frequencies, but also to accurately represent the fields in the cross-section, which, in standard mode-matching, requires a very large number of modes.

II. THEORY

The geometry of the analyzed structure is shown in Fig. 1. Magnetic walls are introduced to limit the space to a finite computational domain. Since they are placed far away from the conductor, their influence on the field is negligible. Field symmetries are assumed with respect to the planes x=0 and y=0, to further reduce the computational effort.

For TE modes, the tangential electric field at the interface plane, y=d, is expanded as

$$E_x = \sum_{n=1}^{N} c_n \frac{\sin((2n-1)\pi(a-x)/(2b))}{(1-((a-x)/b)^2)^{1/3}} \tag{1}$$

Here, c_n are the unknown expansion coefficients. Similarly, for TM modes, the tangential electric field is expanded as

$$E_z = \sum_{n=1}^{N} c_n \frac{\cos((2n-1)\pi(a-x)/(2b))}{(1-((a-x)/b)^2)^{1/3}} \tag{2}$$

Note that these expressions satisfy both, the boundary conditions of the electric field at the interface as well as the edge conditions. The axial fields inside the waveguide are written in terms of Fourier series for the two regions, e.g. [5], [6]. For TE- and TM modes, the fields E_x and H_x are known to be $E_x \sim \partial H_z/\partial y$ and $H_x \sim \partial E_z/\partial y$, respectively.

The next step is to write the Fourier coefficients in terms of the expansion coefficients, c_n. This is done by matching the Fourier sine or cosine integrals of the tangential electric fields in the two different regions, E_t^{I} and E_t^{II}. For the example of TE modes with magnetic-wall symmetry at x=0, this yields:

$$\int_0^b E_x^{\mathrm{I}}(x, y{=}\mathrm{d}) \cdot \sin\frac{(2m-1)\pi(a-x)}{2b}dx = \int_0^b E_x \cdot \sin\frac{(2m-1)\pi(a-x)}{2b}dx \tag{3}$$

$$\int_0^a E_x^{\mathrm{II}}(x, y{=}\mathrm{d}) \cdot \sin\frac{(2m-1)\pi(a-x)}{2a}dx = \int_0^b E_x \cdot \sin\frac{(2m-1)\pi(a-x)}{2a}dx \quad , \tag{4}$$

where E_x is as given in (1). These integrals can be solved analytically and lead to a sum of Bessel functions [7]. The results are frequency independent and, consequently, need to be computed only once which explains the fast execution of each iteration. Now, the Fourier coefficients can be expressed in terms of the expansion term coefficients c_n, and all field components depend only on the c_n's. Galerkin's method is used to match the transverse magnetic fields and to determine the cut-off wave numbers. Once these are known, the expansion coefficients, and thus all field components, are found from the solution of a linear equation.

The results for the TM modes and for the other symmetries are obtained in a similar manner.

III. RESULTS

The technique has been applied to the structure in Fig. 1. The result for the cut-off wave numbers are shown in Fig. 2. Identical results were obtained using the Integral-Equation and mode matching technique. The results from the Integral-Equation Technique, however, were generated using only two basis functions. In most cases, already one basis function gave accurate results for the wave number, leading to only one scalar to be minimized, while in mode-matching, the singular value of a 10x10 matrix was minimized.

Fig. 3 and Fig. 4 compare the convergence of the calculated tangential electric field E_x along the interface between the two regions using the different techniques. In Fig. 3, results from mode matching are shown, using 1, 5, 80 and 200 modes in region I. Note that for the given dimensions, $b/a=10$, the number of modes in region II is chosen 10 times higher, to ensure proper matching on the different sides of the interface, and to minimize the effect of relative convergence. In Fig. 4, the same structure is analyzed with the Integral-Equation Technique using 1, 3 and 5 basis functions. Both methods converge to the same result, however, the much faster convergence of the Integral-Equation Technique becomes apparent. Also, the non-physical ripple of the field in the vicinity of a singularity can be observed for the mode-matching technique. This is eliminated in the Integral-Equation Technique.

IV. CONCLUSIONS

An Integral-Equation Technique is presented to solve for the mode spectrum of a rectangular coax line with magnetic side walls. The method is based on directly expanding the tangential electric field at the interface plane in terms of basis functions. These basis functions are chosen so that they not only comply with the boundary conditions of the electric fields, but also satisfy the edge conditions at the metallic corners. Therefore, the solution depends only on one parameter, namely the number of basis functions, which eliminates the phenomenon of relative convergence. The results show that rapid convergence is achieved and that, using only few basis functions, sufficient accuracy is obtained, not only for the cut-off wave numbers, but also to properly represent the fields.

The method can be applied to many other structures with, in most cases, increased efficiency and reliability in comparison to standard modal field-matching techniques.

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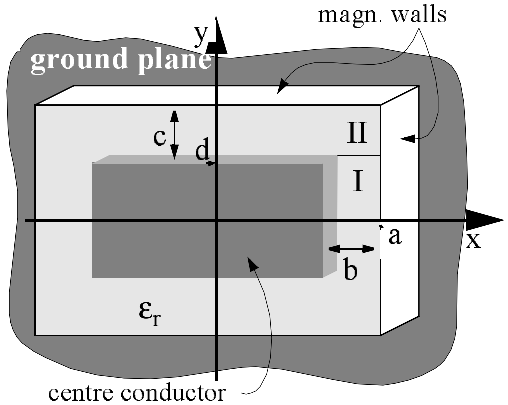

Fig. 1 Geometry of analyzed structure.

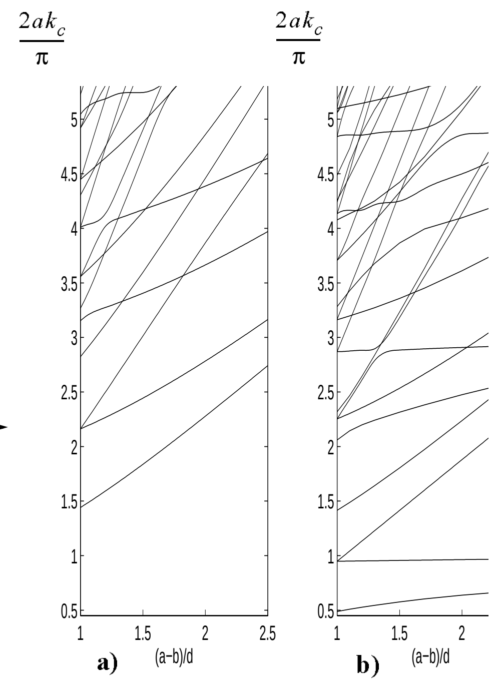

Fig. 2: Normalized cut-off frequencies as a function of the side ratio (a-b)/d for a coax waveguide with magnetic sidewalls and *a/b=(c+d)/c=10*.
a) TE modes, b) TM modes

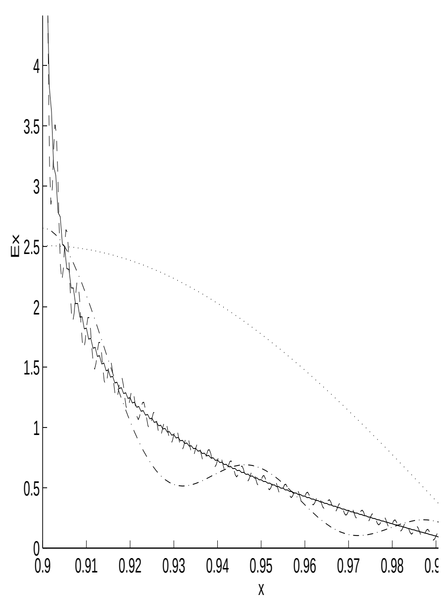

Fig. 3: Tangential electric field E_x along the interface *y=d* from mode-matching using 1 (dotted line), 5 (dash-dotted line), 80 (dashed line) and 200 modes (solid line).
Dimensions: *b/a*=0.1, *c/a*=0.1, *d/a*=0.9.

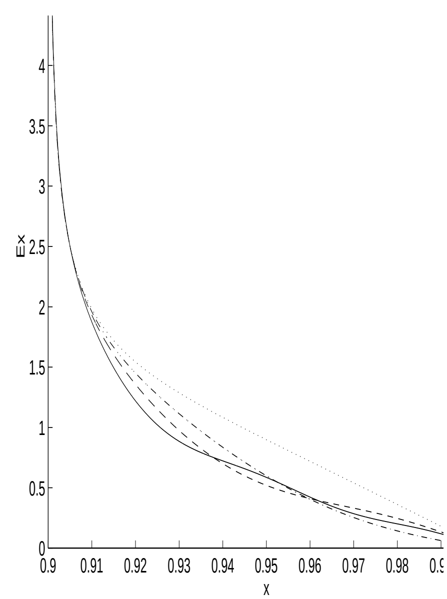

Fig. 4: Tangential electric field E_x along the interface *y=d* from the Integral Equation Technique using 1 (dotted line), 2 (dash-dotted line), 3 (dashed line) and 5 basis functions (solid line).
Dimensions: *b/a*=0.1, *c/a*=0.1, *d/a*=0.9.