SCATTERING OF TE_{11} MODE FROM TWO ASYMMETRIC RIDGES OF FINITE THICKNESS IN A CIRCULAR WAVEGUIDE

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Abstract

The scattering properties of the TE_{11} mode from two asymmetric ridges of finite thickness in a circular waveguide are determined using the Coupled-Integral-Equations Technique (CIET). Two vector coupled integral equations for the transverse electric field at the two discontinuities are derived and then solved by the moment method. The eigenmodes of the ridged section, in terms of which the kernels of the integral equations are expressed, are themselves determined using the CIET where basis functions with the proper edge conditions are used.

1 INTRODUCTION

Ridged structures are important components in modern microwave communication systems such as dual-mode filters and polarizers whose frequency response is primarily determined by the dimensions and positions of the ridges. Accurate prediction of the response functions of these devices is contingent upon an efficient and precise determination of the scattering properties of their ridged sections.

The first step in determining the scattering properties of ridged sections in waveguides consists in solving for the cutoff frequencies and eigenmodes of the corresponding infinitely long ridged structures. In this work, this first and important step is carried out through the Coupled-Integral-Equations Technique (CIET) where a set of coupled integral equations for the tangential electric field are solved by the moment method using basis functions which include the proper edge conditions at all the metallic wedges [1].

The second step consists in the analysis of the scattering of incident modes of the empty waveguide at the different discontinuities of the ridged sections. In this work, we establish sets of coupled vector integral equations for the transverse electric field at the different discontinuities. More specifically, we are concerned with the scattering of the fundamental modes of a circular waveguide, TE_{11}, with arbitrary polarization, by a finite section of an asymmetric double ridge structure. Instead of following the Mode-Matching Technique (MMT) and determining the scattering matrix by cascading the scattering matrices of the two discontinuities, we determine the scattering matrix of the section directly. This alternative approach allows us to concentrate directly on the dominant physics of the problem which takes place at the discontinuities and also take advantage of the fact that the coupling between the modes of the empty waveguide and those of the ridged waveguide are identical at both discontinuities. In addition, by solving the vector integral equations for the electric field at both discontinuities simultaneously, we eliminate the arbitrariness in determining what is commonly referred to as accessible modes between the two interacting discontinuities.

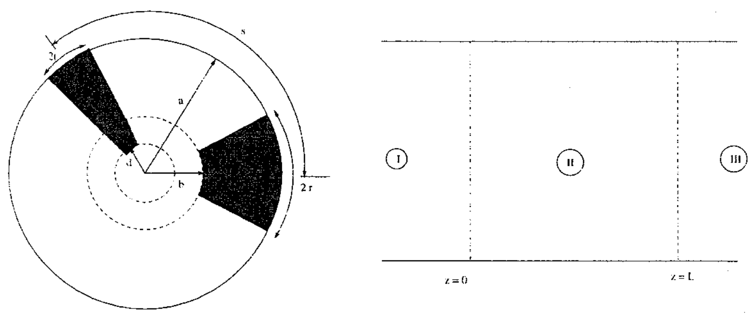
We purposely limit the analysis to the fundamental mode as it is the only propagating mode in the uniform regions between the different ridged sections as long as these are not too closely located. It is, however, worth mentioning that the analysis is straightforwardly extended to handle the case where modes other than the fundamental are propagating.

2 THEORY

The cross section of the ridged waveguide is shown in figure 1a. We assume that all metallic walls are lossless and that the ridges fit into the polar system of coordinates.

The eigenmodes of the structure can be divided into TE and TM modes whose cutoff frequencies are determined from the solution of Helmholtz equation. The electric and magnetic potentials are expanded in modal series in each of the subregions of figure 1a. A set of coupled integral equations for the electric field at the interfaces between the different regions are derived from the continuity of the magnetic field [1]. These are solved by the moment method using basis functions which include the edge conditions at the metallic wedges of the two ridges. The details can be found in reference [1].

To determine the scattering of the fundamental mode from a ridged section of finite length L, as depicted in figure 1b, we again derive two coupled vector integral equations for the



1a 1b

Figure 1: Cross section of asymmetric ridged circular waveguide (a) and ridged section of finite length (b)

transverse electric field at the two discontinuities at $z = 0$ and $z = L$ [2]. Let us assume that the transverse electric field at the two discontinuities are given by two unknow vector functions $\mathbf{Z}$ and $\mathbf{W}$. Using the orthogonality properites of the normal modes in the boundry conditions of the transverse electric field, the modal expansion coefficients are eliminated in favour of the vector functions $\mathbf{Z}$ and $\mathbf{W}$. Substituting the resulting expressions in the continuity condition of the transverse magnetic field at the two discontinuities, we obtain two coupled integral equations in these two vector functions. A moment method solution is applied to determine $\mathbf{Z}$ and $\mathbf{W}$ after which the reflected and transmitted waves follow straightforwardly. More precisely, we get the following coupled integral equations

$$
\begin{aligned}
\sum_m -\frac{\omega\epsilon_0}{k_{zm}^{TMI}}\left[\frac{\int_{S_2}\mathbf{Z}.\nabla\Phi_m^{TMI}ds}{\int_{S_1}\nabla\Phi_m^{TMI}\nabla\Phi_m^{TMI}}\right]\mathbf{a}_z\times\nabla\Phi_m^{TMI} + \sum_m \frac{k_{zm}^{TEI}}{\omega\mu_0}\left[\frac{\int_{S_2}\mathbf{Z}(\mathbf{a}_z\times\nabla\Phi_m^{TEI})ds}{\int_{S_1}\nabla\Phi_m^{TMI}\nabla\Phi_m^{TEI}ds}\right]\nabla\Phi_m^{TEI} \\
-j\sum_m \frac{\omega\epsilon_0}{k_{zm}^{TMII}}cot(k_{zm}^{TMII}L)\left[\frac{\int_{S_2}\mathbf{Z}.\nabla\Phi_m^{TMII}ds}{\int_{S_2}\nabla\Phi_m^{TMII}\nabla\Phi_m^{TMII}ds}\right]\mathbf{a}_z\times\nabla\Phi_m^{TMII} \\
+\sum_m j\frac{\omega\epsilon_0}{k_{zm}^{TMII}}\frac{1}{sin(k_{zm}^{TMII}L)}\left[\frac{\int_{S_2}\mathbf{W}.\nabla\Phi_m^{TMII}ds}{\int_{S_2}\nabla\Phi_m^{TMII}\nabla\Phi_m^{TMII}ds}\right]\mathbf{a}_z\times\nabla\Phi_m^{TMII} \qquad (1) \\
+\sum_m j\frac{k_{zm}^{TEII}}{\omega\mu_0}cot(k_{zm}^{TEII}L)\left[\frac{\int_{S_2}\mathbf{Z}.(\mathbf{a}_z\times\nabla\Phi_m^{TEII})ds}{\int_{S_2}\nabla\Phi_m^{TEII}\nabla\Phi_m^{TEII}ds}\right]\nabla\Phi_m^{TEII} \\
-\sum_m j\frac{k_{zm}^{TEII}}{\omega\mu_0}\frac{1}{sin(k_{zm}^{TEII}L)}\left[\frac{\int_{S_2}\mathbf{W}.(\mathbf{a}_z\times\nabla\Phi_m^{TEII})ds}{\int_{S_2}\nabla\Phi_m^{TEII}\nabla\Phi_m^{TEII}ds}\right]\nabla\Phi_m^{TEII} = 2\sum_m jk_{zm}^{TEI}D_m^I\nabla\Phi_m^{TEI}
\end{aligned}
$$

and

$$\sum_m -\frac{\omega\epsilon_0}{k_{zm}^{TMI}}[\frac{\int_{S_2}\mathbf{W}.\nabla\Phi_m^{TMI}ds}{\int_{S_1}\nabla\Phi_m^{TMI}\nabla\Phi_m^{TMI}}]\mathbf{a}_z\times\nabla\Phi_m^{TMI}+\sum_m\frac{k_{zm}^{TEI}}{\omega\mu_0}[\frac{\int_{S_2}\mathbf{W}(\mathbf{a}_z\times\nabla\Phi_m^{TEI})ds}{\int_{S_1}\nabla\Phi_m^{TMI}\nabla\Phi_m^{TEI}ds}]\nabla\Phi_m^{TEI}$$
$$-j\sum_m\frac{\omega\epsilon_0}{k_{zm}^{TMII}}cot(k_{zm}^{TMII}L)[\frac{\int_{S_2}\mathbf{W}.\nabla\Phi_m^{TMII}ds}{\int_{S_2}\nabla\Phi_m^{TMII}\nabla\Phi_m^{TMII}ds}]\mathbf{a}_z\times\nabla\Phi_m^{TMII}$$
$$+\sum_m j\frac{\omega\epsilon_0}{k_{zm}^{TMII}}\frac{1}{sin(k_{zm}^{TMII}L)}[\frac{\int_{S_2}\mathbf{Z}.\nabla\Phi_m^{TMII}ds}{\int_{S_2}\nabla\Phi_m^{TMII}\nabla\Phi_m^{TMII}ds}]\mathbf{a}_z\times\nabla\Phi_m^{TMII} \quad (2)$$
$$+\sum_m j\frac{k_{zm}^{TEII}}{\omega\mu_0}cot(k_{zm}^{TEII}L)[\frac{\int_{S_2}\mathbf{W}.(\mathbf{a}_z\times\nabla\Phi_m^{TEII})ds}{\int_{S_2}\nabla\Phi_m^{TEII}\nabla\Phi_m^{TEII}ds}]\nabla\Phi_m^{TEII}$$
$$-\sum_m j\frac{k_{zm}^{TEII}}{\omega\mu_0}\frac{1}{sin(k_{zm}^{TEII}L)}[\frac{\int_{S_2}\mathbf{Z}.(\mathbf{a}_z\times\nabla\Phi_m^{TEII})ds}{\int_{S_2}\nabla\Phi_m^{TEII}\nabla\Phi_m^{TEII}ds}]\nabla\Phi_m^{TEII}=-2\sum_m jk_{zm}^{TEI}C_m^{III}\nabla\Phi_m^{TEI}$$

The evident symmetry of these two integral equations in **Z** and **W** should be fruitfully exploited in the numerical solution by the moment method.

3 RESULTS

The present approach is applied to compute the reflection coefficient of the fundamental mode of arbitrary polarization at a ridged section of lengths $L = 10mm$ as a function of frequency. Figure 2 shows the reflection coefficients polarization S_{11cc}, S_{11cs}, and S_{11ss} as a function of frequency. The notation S_{11sc} stands for the reflection of the since polarization when only the cosine polarization is incident at port 1, the other terms follow by analogy. The dependence of the coupling between the two polarizations shows a substantial sensitivity to the frequency as exhibited by the dip at 9.80 GHz which corresponds to a phase of approximately 90 degrees for the lowest mode in the ridged section.

4 REFERENCES

1. S. Amari, S. Catreux, R. Vahldieck and J. Bornemann, "Application of a coupled-integral equations technique to circular ridged waveguides," to appear in IEEE Trans. Microwave Theory Tech.

2. S. Amari, J. Bornemann and R. Vahldieck, "Accurate analysis of scattering from multiple waveguide discontinuities using the coupled-integral- equations technique," Journal of Electromagnetic Waves and Applications, Vol. 8, No. 12, pp. 1642-1655, 1996.

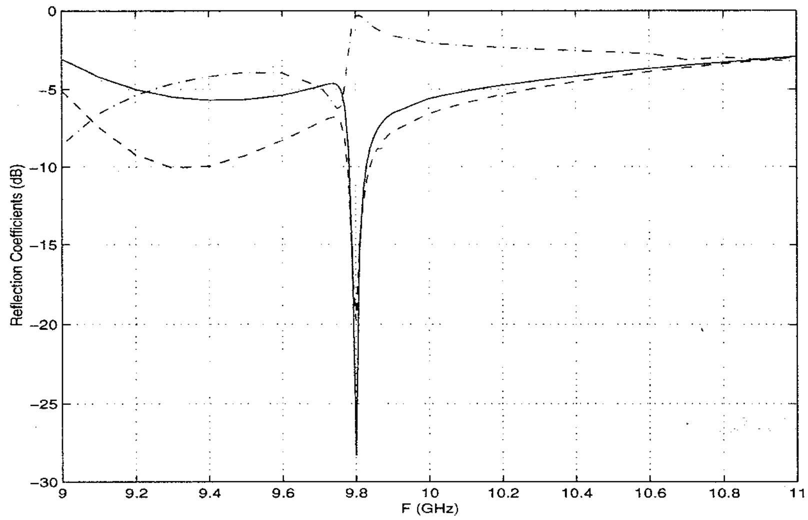


Figure 2: Reflection ocefficients S_{11cc}(solid line), S_{11ss} (dashed line) and S_{11sc} (dotted-dashed line) versus frequency of a ridged section of lengh L =10 mm. r =t = 3°, s = 135°, b=0.5a and d = 0.4a and a = 10 mm