Integral Equation Approach For Scattering Analysis Of Cascaded Waveguide Discontinuities

Jens Bornemann, *Senior Member, IEEE,* and Smain Amari, *Member, IEEE*

***Abstract*--An integral equation approach for waveguide discontinuity analysis is presented. The algorithm arrives at a highly sparse system matrix which is kept small due to the simultaneous incorporation of all edge conditions. Examples of circular and rectangular waveguide components demonstrate the feasibility of the method.**

***Index Terms*--Integral equations, numerical methods, filters, transformers, waveguide components.**

I. Introduction

Electromagnetic fields in waveguiding structures have been investigated for many years, and many different numerical techniques have been presented to visualize field propagation and/or extract performance-driven quantities such as scattering parameters. Components with arbitrary cross section or profile are usually analyzed by space-discretization techniques such as the frequency-domain finite-element method (FEM) or the finite-difference time-domain (FDTD) technique. Many of the so-called field solvers are now commercially available, e.g. [1], but they consume substantial computing resourses, especially when the structures under investigation exhibit frequency-sensitive responses.

Components with standard cross sections - rectangular, circular, elliptic and variations thereof - are preferably analyzed by modal field-matching techniques of which the mode-matching technique (MMT) is the most commonly used approaches, e.g. [2]. However, this method is plagued by a relatively slow convergence and the phenomenon of relative convergence [3] both of which have been linked to the inappropriate modelling of field singularities at metallic edges, e.g. [4], [5].

In this paper, we present an integral equation approach which, although using modal techniques for representing the electromagnetic field, incorporates the salient features of the method of moments (MoM) and permits the simultaneous incorporation of all field singularities through sets of edge-conditioned basis functions. The method is readily applicable to any cascaded waveguide discontinuity problem as will be demonstrated at examples in rectangular and circular waveguide technology covering X- to Ka-band frequency ranges. Comparisons with the standard MMT verify the theoretical approach.

Jens Bornemann is with the Department of Electrical and Computer Engineering, University of Victoria, P.O. Box 3055 Stn CSC, Victoria, BC V8W 3P6, Canada (e-mail: J.Bornemann@ieee.org).

Smain Amari is with Department of Electrical and Computer Engineering Royal Military College of Canada, Kingston, ON K7K 7B4, Canada (e-mail: Smain.Amari@rmc.ca).

II. Theory

Let us assume a structure consisting of a number of rectangular waveguide discontinuities as shown in Fig. 1. In each individual section of the component, the transverse electric and magnetic field components are expanded as

$$\begin{aligned}\vec{E}_T^i = \sum_q \left(\nabla T_{hq}^i \times \vec{e}_z\right)[F_{hq}^i \exp\{-jk_{zhq}^i z\} \\ + B_{hq}^i \exp\{+jk_{zhq}^i z\}] \\ + \sum_p \left(-\nabla T_{ep}^i\right)[F_{ep}^i \exp\{-jk_{zep}^i z\} \\ + B_{ep}^i \exp\{+jk_{zep}^i z\}]\end{aligned} \tag{1}$$

$$\begin{aligned}\vec{H}_T^i = \sum_q Y_{hq}^i \left(\nabla T_{hq}^i\right)[F_{hq}^i \exp\{-jk_{zhq}^i z\} \\ - B_{hq}^i \exp\{+jk_{zhq}^i z\}] \\ + \sum_p Y_{ep}^i \left(\nabla T_{ep}^i \times \vec{e}_z\right)[F_{ep}^i \exp\{-jk_{zep}^i z\} \\ - B_{ep}^i \exp\{+jk_{zep}^i z\}]\end{aligned} \tag{2}$$

where $T_{h,e}$ are cross-section functions, $Y_{h,e}$ are admittances, $k_{zh,e}$ are the phase constants for axial propagation, and F and B are the amplitudes of waves travelling in positive or negative axial direction $\vec{e}_z$, e.g. [2].

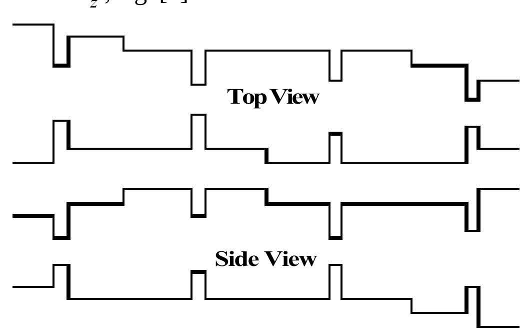

Fig. 1. Example of cascaded discontinuities in rectangular waveguide: A six-pole dual-mode filter.

At each discontinuity, the aperture field is expanded in a set of basis functions which includes the modal spectrum of the aperture and the appropriate terms to produce the required field singularities. At discontinuities, where waveguide walls are aligned, the respective mirror image of the non-aligned wall discontinuity is considered. In a cartesian cooordinate system, the aperture functions are

$$\vec{X}(x,y) = \sum_r \frac{\left[\nabla T^0_{[]hr}(x,y)\times\vec{e}_z\right]c_r}{\left[\left(1-\frac{x}{x_l}\right)\left(\frac{x}{x_u}-1\right)\left(1-\frac{y}{y_l}\right)\left(\frac{y}{y_u}-1\right)\right]^{1/3}} + \sum_s \frac{\left[-\nabla T^0_{[]es}(x,y)\right]c_s}{\left[\left(1-\frac{x}{x_l}\right)\left(\frac{x}{x_u}-1\right)\left(1-\frac{y}{y_l}\right)\left(\frac{y}{y_u}-1\right)\right]^{1/3}} \quad (3)$$

where $T^0_{[]hr}$ and $T^0_{[]es}$ are, respectively, the TE- and TM-mode cross-section functions of the common aperture between two connected sections, c_r and c_s are the coefficients of the basis functions, and x_l, x_u, y_l, y_u denote the positions of field singularities in the respective direction. In a circular-cylindrical system, the aperture functions for on-axis connected sections (Fig. 2) are

$$\vec{\Phi}(\boldsymbol{r},\boldsymbol{j}) = \sum_r \frac{\left[\nabla T^0_{()hr}(\boldsymbol{r},\boldsymbol{j})\times\vec{e}_z\right]c_r}{\left[1-\left(\boldsymbol{r}/r_0\right)^2\right]^{1/3}} + \sum_s \frac{\left[-\nabla T^0_{()es}(\boldsymbol{r},\boldsymbol{j})\right]c_s}{\left[1-\left(\boldsymbol{r}/r_0\right)^2\right]^{1/3}} \quad (4)$$

where the common (smaller) aperture is of radius r_0.

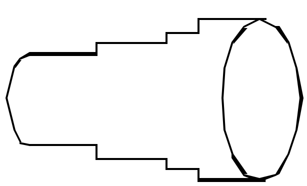

Fig. 2. On-axis discontinuities in circular waveguide: A waveguide transformer.

We now relate the basis function coefficients c^i at discontinuity i to those of their immediate neighbours, c^{i-1} and c^{i+1}. This is accomplished by combining TE and TM modes $(q,p\rightarrow k;\ r,s\rightarrow b)$ and expressing the z-dependent function (1) in terms of basis function coefficients

$$F_k^{i-1}\exp\left\{-jk_{zk}^{i-1}z\right\}+B_k^{i-1}\exp\left\{+jk_{zk}^{i-1}z\right\}=\sum_b W_{kb}^{I,i}c_b^i \quad (5)$$

$$F_k^{i}\exp\left\{-jk_{zk}^{i}z\right\}+B_k^{i}\exp\left\{+jk_{zk}^{i}z\right\}=\sum_b W_{kb}^{II,i}c_b^i \quad (6)$$

Here, W^I and W^{II} are matrices of inner products of the eigenfunctions of the waveguide left (*I*) and right (*II*), respectively, of a discontinuity with the edge-conditioned basis functions (3), (4) of the apertures. In cartesian coordinates, integration can be solved analytically whereas, in the cylindrical system, numerical integration is employed. Rearranging (5), (6), we arrive at the following differences of wave amplitudes as required in (2) for a specific discontinuity at location $z=0$

$$F_k^{i-1}-B_k^{i-1}=\sum_b\frac{jW_{kb}^{I,i}c_b^i}{\tan\{k_{zk}^{i-1}L_{i-1}\}}-\sum_b\frac{jW_{kb}^{II,i-1}c_b^{i-1}}{\sin\{k_{zk}^{i-1}L_{i-1}\}} \quad (7)$$

$$F_k^{i}-B_k^{i}=\sum_b\frac{-jW_{kb}^{II,i}c_b^i}{\tan\{k_{zk}^{i}L_{i}\}}-\sum_b\frac{jW_{kb}^{I,i+1}c_b^{i+1}}{\sin\{k_{zk}^{i}L_{i}\}} \quad (8)$$

where L denotes the length of a section. At the input (region 0) and output (region N) the above expressions change to

$$F_k^0-B_k^0=2\boldsymbol{d}_{kM}-\sum_b W_{kb}^{I,1}c_b^1 \quad (9)$$

$$F_k^N-B_k^N=\sum_b W_{kb}^{II,N}c_b^N-2\boldsymbol{d}_{kL} \quad (10)$$

where M and L denote the mode of excitation at the input and output, respectively.

In the last step, the magnetic field components (2) are matched at all discontinuities. Galerkin's method within the method of moments is applied in a way that produces the transposed matrices, W^t, whose entries have already been computed. The final matrix equation, e.g., for six discontinuities is of the form

$$\begin{bmatrix}\underline{A}_{11} & \underline{A}_{12} & \underline{0} & \underline{0} & \underline{0} & \underline{0}\\ \underline{A}_{12}^t & \underline{A}_{22} & \underline{A}_{23} & \underline{0} & \underline{0} & \underline{0}\\ \underline{0} & \underline{A}_{23}^t & \underline{A}_{33} & \underline{A}_{34} & \underline{0} & \underline{0}\\ \underline{0} & \underline{0} & \underline{A}_{34}^t & \underline{A}_{44} & \underline{A}_{45} & \underline{0}\\ \underline{0} & \underline{0} & \underline{0} & \underline{A}_{45}^t & \underline{A}_{55} & \underline{A}_{56}\\ \underline{0} & \underline{0} & \underline{0} & \underline{0} & \underline{A}_{56}^t & \underline{A}_{66}\end{bmatrix}\begin{bmatrix}\underline{c}_1\\ \underline{c}_2\\ \underline{c}_3\\ \underline{c}_4\\ \underline{c}_5\\ \underline{c}_6\end{bmatrix}=\begin{bmatrix}\underline{U}\\ \underline{0}\\ \underline{0}\\ \underline{0}\\ \underline{0}\\ \underline{0}\end{bmatrix};\begin{bmatrix}\underline{0}\\ \underline{0}\\ \underline{0}\\ \underline{0}\\ \underline{0}\\ \underline{V}\end{bmatrix} \quad (11)$$

where $\underline{U}$ and $\underline{V}$ are the input and output excitation vectors given by

$$\underline{U}=j2\left(\underline{W}^{I,1}\right)^t Diag\left\{Y^0\right\} \quad (12)$$

$$\underline{V}=j2\left(\underline{W}^{II,N}\right)^t Diag\left\{Y^N\right\} \quad (13)$$

The matrix system (11) has a number of distinct properties which make the method efficient and fast:

The size of matrix [$\underline{A}$] is N times the number of basis functions. Note that the number of basis functions per discontinuity can vary.

The modes enter the matrix elements through sums (matrix multiplications) which can individually be checked for convergence. This procedure eliminates the phenomenon of relative convergence known from mode-matching techniques.

Matrix [$\underline{A}$] is block-diagonal and symmetric. If the structure under investigation is symmetric in axial direction, then [$\underline{A}$] is also symmetric with respect to its minor diagonal. Hence only a limited number of matrix entries need to be computed.

An LU decomposition (under consideration of the block structure and symmetries) needs to be performed only once per frequency point.

For all different excitation vectors $\underline{U}$, $\underline{V}$, only the first and last coefficient vectors need to be computed.

The number of modes required in the generalized scattering matrix representation of the component under investigation is usually much smaller than those considered at the individual discontinuities within the component. If the component is symmetric, only one excitation vector is required to obtain the fundamental-mode scattering parameters.

The modal spectrum used to analyze a given structure is controlled by the sequence of cutoff frequencies. Therefore, the system can easily be adapted to specific modal requirements such as the TE_{m0} mode set for standard H-plane components or the TE/TM_{1n} mode set for standard E-plane systems.

In direct comparison with similar frequency-domain techniques, e.g. the mode-matching technique, CPU-time savings of at least one order of magnitude have been achieved. This makes the method ideally suited for fast and reliable design applications within optimization routines, e.g. [6].

III. RESULTS

Here we present examples of applications of the integral equation approach.

Fig.1 shows the basic layout of a six-pole dual mode filter which accommodates two electrical resonators in each of the three physical cavities. Cross coupling between the first and fourth and the third and sixth electrical resonators permits the creation of two transmission zeroes at finite frequencies. A typical response of such a filter, optimized for an X-band application, is depicted in Fig. 3. Up to 2000 modal terms are used in the field expansions, but the maximum number of basis functions at a certain discontinuity can be limited to 23.

The next example is a corrugated waveguide lowpass filter designed for K-band applications. The principle layout is shown at the top of Fig. 4. A two-section transformer is used to connect the standard K-band waveguide to the corrugated structure consisting of nine stubs. The analysis incorporates the field symmetries resulting in a $TE/TM_{(2m-1,\,2n)}$ mode set for the entire component. Fig. 4 also demonstrates excellent agreement with results obtained from the mode-matching technique. Note that this agreement extends over an amplitude rang of 300 dB corresponding to 15 orders of magnitude.

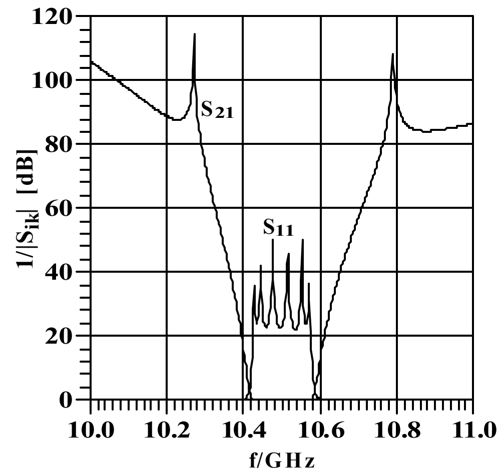

Fig. 3. Scattering parameter of optimized X-band dual-mode filter of Fig. 1.

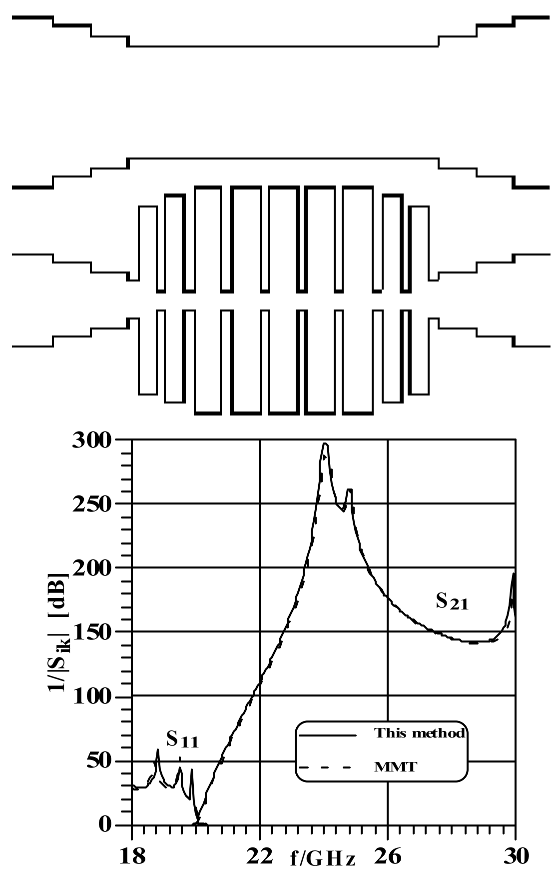

Fig. 4. Layout and performance of K-band corrugated waveguide lowpass filter and comparison with results obtained with the mode-matching technique (MMT).

The following two examples relate to on-axis circular waveguide components.

The performance of a transition from a 7.05mm-diameter to 8.9mm-diameter circular waveguide according to Fig. 2 is depicted in Fig. 5. Initial transformer designs, which apply rectangular waveguide E-plane design concepts [2] to circular waveguides, result in a return loss of 30 dB between 28 GHz and 40 GHz. After optimization, using the integral equation technique as a fast and reliable analysis tool and applying TE_{11}-mode field symmetry, the return loss is improved to beyond 40 dB over the same frequency range. For comparison and as a validation of the edge-conditioned basis function approach in circular-cylindrical coordinates, results obtained with the MMT are shown as dashed lines. Again, excellent agreement is observed.

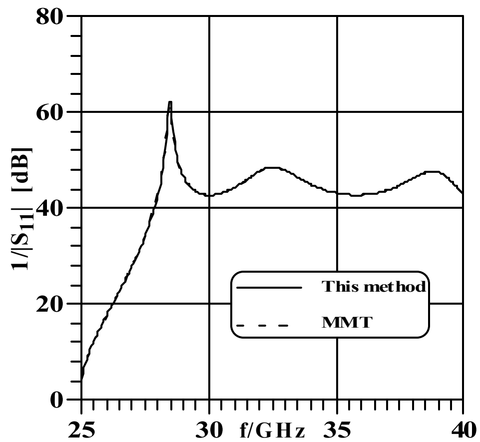

Fig. 5. Scattering parameter $|S_{11}|$ of a Ka-band circular waveguide transition according to Fig. 2 and comparison with results obtained with the mode-matching technique (MMT).

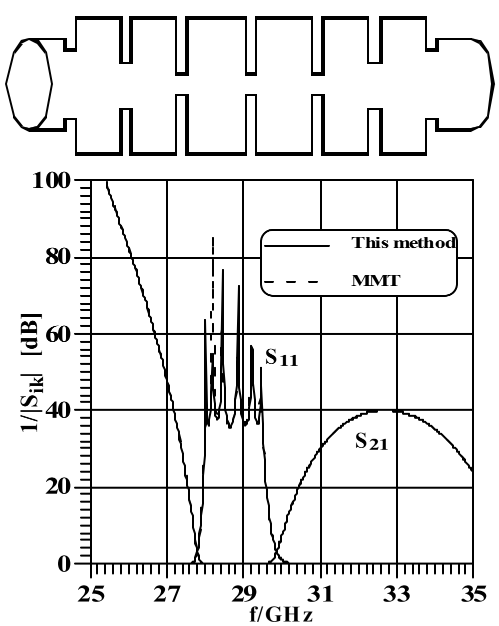

Fig. 6. Principal layout and performance of a four-resonator Ka-band circular waveguide filter and comparison with results obtained with the mode-matching technique (MMT).

A six-resonator circular waveguide iris filter for TE_{11}-mode operation is our last example. The layout is shown at the top of Fig. 6. All irises are 1 mm thick; the input/output waveguide diameters are 7.04 mm, and the diameter of the resonators is increased to 9 mm for a better stopband performance. The 35 dB return-loss bandwidth is 1.5 GHz centered at 28.75 GHz. A maximum of 199 modes and 19 basis functions is used in the final analysis. The computations using the integral equation approach show, again, excellent agreement with results obtained by the mode-matching technique (dashed line in Fig. 6).

IV. CONCLUSIONS

The integral equations approach presented in this paper presents a fast, powerful and effective alternative for the scattering analysis of cascaded waveguide discontinuities. Its most attractive features are the highly sparse system matrix, the simultaneous consideration of all edge conditions and the elimination of the relative convergence phenomenon known from mode-matching techniques. Example designs show good agreement with mode-matching results. However, the integral equation approach is at least one order of magnitude faster than the MMT.

ACKNOWLEDGMENT

The authors would like to thank Uwe Rosenberg of Marconi Communications, Backnang, Germany for providing an initial four-pole design of the six-pole dual-mode filter of Figs. 1 and 3. We are also in debt of Dr. Jaroslaw Uher of EMS Technologies, Montreal, Canada for providing the initial design of the corrugated lowpass filter shown in Fig. 4.

REFERENCES

[1] B. Veidt, "Selecting 3-D electromagnetic software", *Microwave J.*, Vol. 41, pp. 126-137, Sep. 1998.

[2] J. Uher, J. Bornemann and U. Rosenberg, *Waveguide Components for Antenna Feed Systems. Theory and CAD*, Artech House, Norwood 1993.

[3] T. Itoh ed., *Numerical Techniques for Microwave and Millimeter-Wave Passive Structures*, John Wiley & Sons, New York 1989.

[4] R. Sorrentino, M. Mongiardo, F. Alessandri and G. Schiavon, "An investigation of the numerical properties of the mode-matching technique," *Int. J. Numerical Modelling*, Vol. 4, pp. 19-43, March 1991.

[5] T. Rozzi and M. Mongiardo, "E-plane steps in rectangular waveguides," *IEEE Trans. Microwave Theory Tech.*, Vol. 30, pp. 1279-1288, Aug. 1991.

[6] J.W. Bandler and S.H. Chen, "Circuit optimization: The state of the art", *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 424-443, Feb. 1988.