FREQUENCY-DOMAIN ANALYSIS OF COUPLED LINES IN THE PRESENCE OF EXTERNAL RF FIELDS

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Abstract: A frequency-domain analysis of coupled lines in the presence of external RF fields is introduced. Assuming a proximity-coupled lines arrangement on a printed circuit board and quasi-TEM propagation along the lines, we present the derivation of the overall impedance matrix of the coupler. The external electromagnetic field is readily incorporated in the line voltages and currents, and a characteristic equation is obtained. The superposition of C and *Π* modes is used to derive the impedance matrix.

INTRODUCTION

Electromagnetic interference of external fields on printed circuit boards is a well-known problem in microwave/RF technology, e.g. [1]. As the demand for high-density packaging increases, so does the necessity of measuring and evaluating the influence of external fields, e.g. [2]. Modelling techniques for such phenomena have evolved only recently and are mainly based on time-domain approaches and transient analyses, e.g. [2], [3]. A frequency-domain technique is presented in [4], but the formulation is based on incorporating distributed sources in the general transmission-line model.

In this paper, we present a frequency-domain analysis, which accounts for the entire external electromagnetic field. The theory is presented for the example of coupled lines on printed circuit boards, where we assume quasi-TEM propagation.

FORMULATION

Fig.1 shows the general configuration of proximity-coupled lines. The external fields E and H can be regarded as interference from the near-by circuit. They may be caused by radiation from other circuitry and can be calculated from the respective currents (via potential functions) flowing on these circuits.

The behavior of two coupled lines in the presence of external electromagnetic fields can be described by the following set of equations:

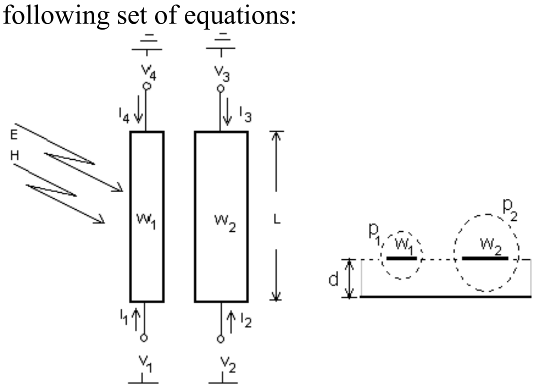

Fig.1 Coupled lines under the influence of external fields.

$$-\frac{dV_1}{dx} = z_1\left[I_1 - \int_{p_1} \vec{H}\cdot d\vec{l} \right] + z_m I_2 \tag{1}$$

$$-\frac{dV_2}{dx} = z_2\left[I_2 - \int_{p_2} \vec{H}\cdot d\vec{l} \right] + z_m I_1 \tag{2}$$

$$-\frac{dI_1}{dx} = y_1\left[V_1 + \int_0^d \vec{E}\cdot d\vec{y} \right] + y_m V_2 \tag{3}$$

$$-\frac{dI_2}{dx} = y_2\left[V_2 + \int_0^d \vec{E}\cdot d\vec{y} \right] + y_m V_1 \tag{4}$$

where z_j and y_j (j=1,2) are self-impedance and admittance per unit length of line, and z_m and y_m are mutual impedance and admittance per unit length.

Eliminating I_1 and I_2 from (1) to (4) gives the following set of coupled equations for voltages V_1 and V_2

$$\frac{d^2V_1}{dx^2} - a_1V_1 - b_1V_2 - \left(\left(z_m y_2 + y_1 z_1 \right) \int_0^d \vec{E}\cdot d\vec{y} + z_1 \int_{p_1} \frac{d\vec{H}\cdot d\vec{l}}{dx} \right) = 0 \tag{5}$$

$$\frac{d^2V_2}{dx^2} - a_2V_2 - b_2V_1 - \left((z_m y_1 + y_2 z_2) \int_0^d \vec{E} \cdot d\vec{y} + z_2 \int_{p_2} \frac{d\vec{H} \cdot d\vec{l}}{dx} \right) = 0 \tag{6}$$

where $a_1 = y_1z_1 + z_my_m$, $b_1 = z_1y_m + z_my_2$, $a_2 = y_2z_2 + z_my_m$, $b_2 = y_1z_m + z_2y_m$.
Using (5) and (6), we can write

$$\frac{d^4V_2}{dx^4} - (a_2 + a_1)\frac{d^2V_2}{dx^2} + (a_1a_2 - b_1b_2)V_2 + (a_1c_2 - b_2c_1) = 0 \tag{7}$$

where $c_1 = \left((z_m y_2 + y_1 z_1) \int_0^d \vec{E} \cdot d\vec{y} + z_1 \int_{p_1} \frac{d\vec{H} \cdot d\vec{l}}{dx} \right)$ and $c_2 = \left((z_m y_1 + y_2 z_2) \int_0^d \vec{E} \cdot d\vec{y} + z_2 \int_{p_2} \frac{d\vec{H} \cdot d\vec{l}}{dx} \right)$.

By assuming a variation of type $e^{-\gamma x + j\omega t}$ for V_1, V_2, $\vec{E}$ and $\vec{H}$, we can write the characteristic equation as

$$\gamma^4 - (a_1 + a_2)\gamma^2 + a_1(a_2 + c_2) - b_2(b_1 + c_1) = 0\,. \tag{8}$$

The solutions of equation (8)

$$\gamma_{c,\pi}^2 = \frac{a_1 + a_2}{2} \pm \frac{1}{2}\sqrt{(a_1 + a_2)^2 - 4\big(a_1(a_2 + c_2) - b_2(b_1 + c_1)\big)} \tag{9}$$

represent the forward and backward traveling waves of *C* and *Π* modes on asymmetrical coupled lines with propagation constants γ_c and γ_π, respectively, under the influence of external fields. For symmetrical lines and symmetric external fields, these modes become even and odd modes.
For both modes, the relationship between the line voltages may be determined from (5) and (6)

$$\frac{d^2V_1}{dx^2} - a_1V_1 - b_1V_2 - c_1 = 0 \qquad \frac{d^2V_2}{dx^2} - a_2V_2 - b_2V_1 - c_2 = 0 \tag{10}$$

so that the voltage ratios can be written in terms of the propagation constants (9)

$$\frac{V_{2c,\pi}}{V_{1c,\pi}} = \frac{c_2\left(\gamma_{c,\pi}{}^2 - a_1\right) + b_2c_1}{c_1\left(\gamma_{c,\pi}{}^2 - a_2\right) + b_1c_2} = R_{c,\pi} \tag{11}$$

The general solutions for the voltages on the two lines in terms of all four waves are given by

$$V_1 = A_1e^{-\gamma_c x} + A_2e^{\gamma_c x} + A_3e^{-\gamma_\pi x} + A_4e^{\gamma_\pi x} \tag{12}$$

$$V_2 = R_cA_1e^{-\gamma_c x} + R_cA_2e^{\gamma_c x} + R_\pi A_3e^{-\gamma_\pi x} + R_\pi A_4e^{\gamma_\pi x} \tag{13}$$

Substitution of (12), (13) into (1), (2) and solving for port currents I_1, I_2 yields

$$I_{c2} = \frac{\gamma_c(z_m - R_cz_1)}{R_c\left(z_m^2 - z_1z_2\right)}R_cA_1e^{-\gamma_c x} - \frac{\gamma_c(z_m - R_cz_1)}{R_c\left(z_m^2 - z_1z_2\right)}R_cA_2e^{\gamma_c x} + \frac{z_1z_m \int_{p_1} \vec{H} \cdot d\vec{l} - z_2z_1 \int_{p_2} \vec{H} \cdot d\vec{l}}{z_m^2 - z_1z_2} \tag{14}$$

$$I_{c1} = \frac{\gamma_c(z_2 - R_cz_m)}{\left(z_1z_2 - z_m^2\right)}A_1e^{-\gamma_c x} - \frac{\gamma_c(z_2 - R_cz_m)}{\left(z_1z_2 - z_m^2\right)}A_2e^{\gamma_c x} + \frac{z_1z_2 \int_{p_1} \vec{H} \cdot d\vec{l} - z_2z_m \int_{p_2} \vec{H} \cdot d\vec{l}}{z_1z_2 - z_m^2} \tag{15}$$

$$I_{\pi 2} = \frac{\gamma_\pi(z_m - R_\pi z_1)}{R_\pi\left(z_m^2 - z_1z_2\right)}R_\pi A_3e^{-\gamma_\pi x} - \frac{\gamma_\pi(z_m - R_\pi z_1)}{R_\pi\left(z_m^2 - z_1z_2\right)}R_\pi A_4e^{\gamma_\pi x} + \frac{z_1z_m \int_{p_1} \vec{H} \cdot d\vec{l} - z_2z_1 \int_{p_2} \vec{H} \cdot d\vec{l}}{z_m^2 - z_1z_2} \tag{16}$$

$$I_{\pi 1} = \frac{\gamma_\pi(z_2 - R_\pi z_m)}{\left(z_1z_2 - z_m^2\right)}A_3e^{-\gamma_\pi x} - \frac{\gamma_\pi(z_2 - R_\pi z_m)}{\left(z_1z_2 - z_m^2\right)}A_4e^{\gamma_\pi x} + \frac{z_1z_2 \int_{p_1} \vec{H} \cdot d\vec{l} - z_2z_m \int_{p_2} \vec{H} \cdot d\vec{l}}{z_1z_2 - z_m^2} \tag{17}$$

Equations (14)- (17) can be rearranged as follows

$$i_{c1} = I_{c1} - I_{s1} = Y_{c1}A_1e^{-\gamma_c x} - Y_{c1}A_2e^{\gamma_c x} \qquad i_{c2} = I_{c2} - I_{s2} = Y_{c2}R_cA_1e^{-\gamma_c x} - Y_{c2}R_cA_2e^{\gamma_c x} \tag{18}$$

$$i_{\pi 1} = I_{\pi 1} - I_{s1} = Y_{\pi 1}A_3e^{-\gamma_\pi x} - Y_{\pi 1}A_4e^{\gamma_\pi x} \qquad i_{\pi 2} = I_{\pi 2} - I_{s2} = Y_{\pi 2}R_\pi A_3e^{-\gamma_\pi x} - Y_{\pi 2}R_\pi A_4e^{\gamma_\pi x} \tag{19}$$

where

$$Y_{c1} = \frac{\gamma_c(z_2 - R_c z_m)}{z_1 z_2 - z_m^2}, \quad Y_{c2} = \frac{\gamma_c(z_m - R_c z_1)}{R_c(z_m^2 - z_1 z_2)}, \quad Y_{\pi 1} = \frac{\gamma_\pi(z_2 - R_\pi z_m)}{z_1 z_2 - z_m^2}, \quad Y_{\pi 2} = \frac{\gamma_\pi(z_m - R_\pi z_1)}{R_\pi(z_m^2 - z_1 z_2)}$$

and

$$I_{s1} = \frac{z_1 z_2 \int_{p_1} \vec{H}\cdot d\vec{l} - z_2 z_m \int_{p_2} \vec{H}\cdot d\vec{l}}{z_1 z_2 - z_m^2}, \quad I_{s2} = \frac{z_1 z_m \int_{p_1} \vec{H}\cdot d\vec{l} - z_2 z_1 \int_{p_2} \vec{H}\cdot d\vec{l}}{z_m^2 - z_1 z_2}$$

Finally, the impedance matrix of the four-port is found by solving for port voltages in terms of port currents. Since the port voltages and currents are given by

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ R_c & R_c & R_\pi & R_\pi \\ R_c e^{-\gamma_c L} & R_c e^{\gamma_c L} & R_\pi e^{-\gamma_\pi L} & R_\pi e^{\gamma_\pi L} \\ e^{-\gamma_c L} & e^{\gamma_c L} & e^{-\gamma_\pi L} & e^{\gamma_\pi L} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} \tag{20}$$

$$\begin{bmatrix} i_1 \\ i_2 \\ -i_3 \\ -i_4 \end{bmatrix} = \begin{bmatrix} Y_{c1} & -Y_{c1} & Y_{\pi 1} & -Y_{\pi 1} \\ R_c Y_{c2} & -R_c Y_{c2} & R_\pi Y_{\pi 2} & -R_\pi Y_{\pi 2} \\ R_c Y_{c2} e^{-\gamma_c L} & -R_c Y_{c2} e^{\gamma_c L} & R_\pi Y_{\pi 2} e^{-\gamma_\pi L} & -R_\pi Y_{\pi 2} e^{\gamma_\pi L} \\ Y_{c1} e^{-\gamma_c L} & -Y_{c1} e^{\gamma_c L} & Y_{\pi 1} e^{-\gamma_\pi L} & -Y_{\pi 1} e^{\gamma_\pi L} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} \tag{21}$$

the impedance matrix follows as

$$[Z] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ R_c & R_c & R_\pi & R_\pi \\ R_c e^{-\gamma_c L} & R_c e^{\gamma_c L} & R_\pi e^{-\gamma_\pi L} & R_\pi e^{\gamma_\pi L} \\ e^{-\gamma_c L} & e^{\gamma_c L} & e^{-\gamma_\pi L} & e^{\gamma_\pi L} \end{bmatrix} \begin{bmatrix} Y_{c1} & -Y_{c1} & Y_{\pi 1} & -Y_{\pi 1} \\ R_c Y_{c2} & -R_c Y_{c2} & R_\pi Y_{\pi 2} & -R_\pi Y_{\pi 2} \\ -R_c Y_{c2} e^{-\gamma_c L} & R_c Y_{c2} e^{\gamma_c L} & -R_\pi Y_{\pi 2} e^{-\gamma_\pi L} & R_\pi Y_{\pi 2} e^{\gamma_\pi L} \\ -Y_{c1} e^{-\gamma_c L} & Y_{c1} e^{\gamma_c L} & -Y_{\pi 1} e^{-\gamma_\pi L} & Y_{\pi 1} e^{\gamma_\pi L} \end{bmatrix}^{-1} \tag{22}$$

Assuming that the impedances of the connected lines are known, the four-port scattering parameters of the coupler are obtained straightforwardly.

CONCLUSION AND DISCUSSION

We present a frequency-domain analysis of asymmetrical coupled lines in the presence of external electromagnetic fields. Based on transmission-line theory and quasi-TEM propagation, a characteristic equation is obtained which provides solutions for *C* and *Π* modes and accounts for the influence of the external field. Superposition of the solutions is used to obtain the overall impedance matrix of the coupler, which can be readily converted to scattering parameters.

The presentation will focus on the fundamental steps in the theory and explain the extension of conventional *C*- and *Π*-mode analysis to include external fields. Moreover, some examples will be shown which demonstrate the influence of electromagnetic-field illumination on the coupler performance.

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