Mode Spectrum Eigenvalue Formulation for Irregular Waveguides and Its Application to Waveguide Component Design

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Abstract—**A new formulation to obtain the eigenmode spectra of irregular waveguides is presented. The method uses modified rectangular waveguide modes as expansion terms and leads to a classical eigenvalue problem. A two-dimensional negative rectangular step function is introduced to satisfy boundary conditions for TM modes. The procedure is combined with a mode-matching code and is used to analyse and design waveguide structures involving resonating components. The results are verified by measurements and comparison with existing full-wave modelling tools.**

Keywords – waveguide modes; irregular waveguides; classical eigenvalues; mode-matching techniques; computer-aided design.

I. Introduction

Modern waveguide technology employs a variety of components with irregular cross sections. An attempt to classify such structures is presented in [1].

In order to incorporate irregular waveguides into computer-aided design procedures, the mode sequences and related expansion coefficients are normally calculated from a series expansion that, after truncation and forming inner products with selected test functions, leads to a matrix equation. Earlier formulations resulted in singular-value equations, e.g. [2] – [4], which usually require new programming efforts for changing boundary conditions. Therefore, classical eigenvalue approaches have become more popular as they are easily adapted to changing contours of irregular waveguides.

A number of different approaches have been presented over the years and their solutions combined with some form of modal analysis. These include the boundary integral resonant mode expansion, e.g. [5], [6], the finite-element method, e.g. [7], [8], the electric and magnetic field integral equation technique, e.g. [9], and others such as, e.g., neural network models [10]. An attempt to use the mode distributions of the surrounding regular waveguides as expansion and testing functions is presented in [11]. However, it was found that in the presence of sharp edges, the method requires the use of edge-conditioned basis functions, which – in turn – limits general applicability to structures with predefined edge structure. An interesting approach, which leads to a classical symmetric eigenvalue matrix equation, is presented in [12], [13]. However, the use of polynomial approximations involving Gamma functions limits efficient code implementation and, therefore, only simple discontinuities have been presented so far [14], [15].

This paper focuses on a combination of the approaches used in [11] and [12], [13]. It will be shown that by using the mode composition of the surrounding regular waveguide, the TE-mode spectrum of an irregular waveguide is obtained straightforwardly. The TM-mode spectrum requires a modification of the regular waveguide modes to incorporate the boundary conditions of the irregular waveguide. The results demonstrate the general applicability of this approach.

II. Theory

The mode spectrum eigenvalue formulation is demonstrated for Cartesian coordinates using rectangular waveguides with a number arbitrarily positioned ridges. The only restriction is that individual ridges must be connected to the housing or to each other with at least one of them connected to the housing. A cross-section example is depicted in Fig. 1 with each ridge defined by its lower-right coordinates (e_i, d_i) and surface area $w_i \times t_i$.

According to [12], [13], the mode spectrum is obtained from

$$\nabla_T^2 \begin{Bmatrix} H_z \\ E_z \end{Bmatrix} + k_c^2 \begin{Bmatrix} H_z \\ E_z \end{Bmatrix} = 0 \tag{1}$$

where ∇_T is the transverse Laplacian operator, k_c are the eigenvalues, and H_z, E_z are the longitudinal field components of TE and TM modes, respectively. By using expansion functions for the z components

$$\begin{Bmatrix} H_z \\ E_z \end{Bmatrix} = \sum_{p=1}^{P} c_n \begin{Bmatrix} h_{zp} \\ e_{zp} \end{Bmatrix} \tag{2}$$

and truncating the series, a generalized eigenvalue equation of the form

$$K\underline{c} = \underline{k}_c^2 M\underline{c} \tag{3}$$

is obtained, where diagonal matrix $\underline{k}_c$ holds the P eigenvalues and matrix $\underline{c}$ the corresponding eigenvectors. The elements of matrices K and M are given by

$$K_{pq} = \int_S \nabla T_p \nabla T_q ds, \quad M_{pq} = \int_S T_p T_q ds \tag{4}$$

where T replaces h_z or e_z, and S represents the cross section of the irregular waveguide. Note that in order to maintain flexibility of the positions of all ridges, all surface integrals are computed as

$$\int_S f(s)ds = \int_{\text{housing}} f(s)ds - \sum_{i=1}^{N} \int_{\text{ridge(i)}} f(s)ds \tag{5}$$

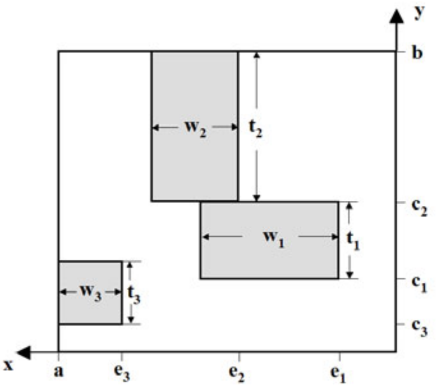


Figure 1. Cross section of a rectangular waveguide with three arbitrarily positioned ridges.

So far, this procedure is known. We are now using the mode spectrum of the housing as expansion functions. While the TE modes of the housing

$$h_{zp(m,n)} = \frac{\cos\left(\frac{m\pi}{a}x\right)}{\sqrt{1+\delta_{0m}}} \frac{\cos\left(\frac{n\pi}{b}y\right)}{\sqrt{1+\delta_{0n}}} \tag{6}$$

(δ_{0k} being the Kronecker delta) satify the bounday conditions for the TE modes in (4) immediately, those for the TM modes need to be forced to vanish over the cross sections of the ridges.

$$e_{zp(m,n)} = \prod_{i=1}^{N} U\left(\frac{x-e_i}{w_i}, \frac{y-d_i}{t_i}\right) \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) \tag{7}$$

This is achieved by a two-dimensional negative rectangular step function, which is defined as

$$U\left(\frac{x-x_0}{\Delta x}, \frac{y-y_0}{\Delta y}\right) = \begin{cases} 0, & \begin{cases} x_0 \le x \le x_0 + \Delta x \\ y_0 \le y \le y_0 + \Delta y \end{cases} \\ 1, & \text{everywhere else} \end{cases} \tag{8}$$

Note that the derivatives of (7) (with (8)) in (4) lead to delta functions. They are considered by using the sifting theorem

$$\int_{\upsilon_0-\Delta\upsilon}^{\upsilon_0+\Delta\upsilon} f(\upsilon)\delta(\upsilon_0)d\upsilon = f(\upsilon_0) \tag{9}$$

Once the eigenvalues and eigenvectors are obtained, the TE-to-TE- and TM-to-TM-mode coupling integrals from the regular (housing) to the irregular waveguide follow straightforwardly from matrices K as the housing modes are used as expansion functions. Only TM-to-TE-mode coupling needs to be computed separately. Power normalization of the irregular waveguide modes is accomplished by using matrices M. Note that functions U in (8) do not contribute to TM-to-TM-mode coupling since the boundary conditions of the transverse electric field of the regular waveguide force them to vanish on the faces and edges of the ridges.

One of the disadvantages of this method is that quite a few waveguide modes are required to compute the mode spectrum of the irregular waveguide with sufficient accuracy. Moreover, the symmetric matrices in (4) become ill-conditioned if the sum of the cross sections of the ridges occupy too much space of the cross section of the regular waveguide. An adaptive process is used in such cases to find a reasonable compromise between accuracy and computational efficiency.

The advantage of this method is that the ridges can be arbitrarily placed and, as we will show in the following section, can even be used for structures in which regular waveguides of different cross sections are created.

III. RESULTS

In this section, we will demonstrate the capabilities of the mode spectrum eigenvalue formulation and verify the results obtained. The examples are based on irregular rectangular waveguides with one or two ridges in the cross sections.

Fig. 2 shows a performance comparison for a cascaded double-ridge component. Excellent agreement is obtained with results obtained with the mode-matching technique (MMT) (c.f. [16]) as well as with S_{11} measurements presented in [3].

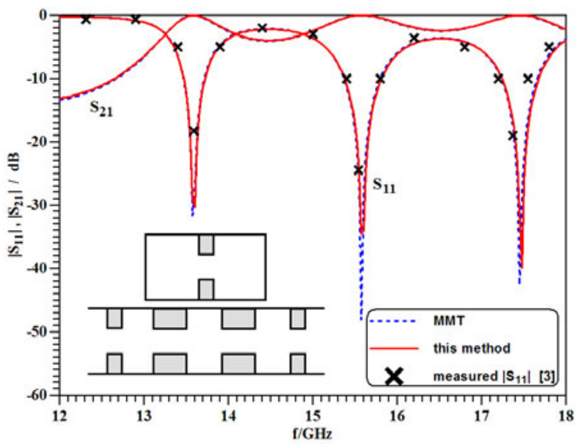
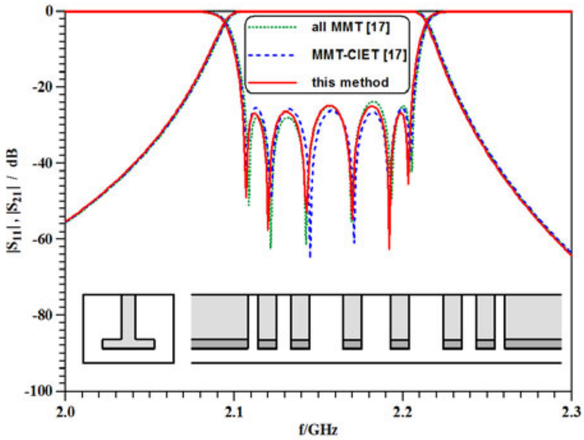


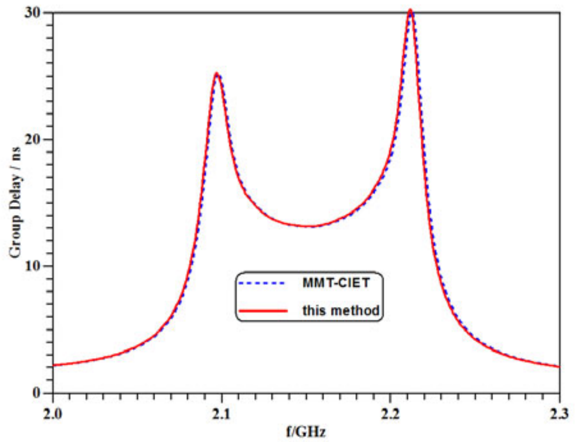
Figure 2. Computed and measured [3] performance of a component formed by cascaded rectangular and double-ridged waveguides.

A six-resonator below-cutoff T-septum waveguide filter for the lower Gigahertz range is presented in Fig. 3. The mode spectrum eigenvalue formulation is in excellent agreement with data obtained from the MMT and the coupled-integral-equation technique (CIET) in [17] (Fig. 3a). In order to demonstrate agreement with respect to phase calculations, the group-delay response is shown in Fig. 3b and also shows excellent agreement with a combined MMT-CIET approach.

The next two examples are to demonstrate that even in cases where the ridges are placed such that smaller regular waveguides are obtained, the eigenvalue formulation performs accurately. Fig. 4 shows the performance of an inductive iris filter where two ridges are placed such that they cover each approximately one quarter of the cross section of the rectangular waveguide. The eigenvalue formulation correctly computes the eigenvalues and eigenvectors of the apertures formed by the irises as demonstrated by excellent agreement with results obtained with the CIET.



(a)



(b)

Figure 3. S-parameter (a) and group-delay (b) performances of a six-resonator below-cutoff T-septum waveguide filter [17].

An interesting case is that of the metal-insert filter presented in Fig. 5. First of all, only one single ridge is used to satisfy the cross section while others are set to zero without any additional change to the code. Secondly, the eigenvalues appear in pairs – as expected – with each pair representing those of both the left and the right apertures. The excellent agreement with MMT results in Fig. 5 verifies the applicability of this technique to 'standard' waveguide problems.

Of course, the CPU times to compute the eigenvalues of the apertures in Fig. 4 and Fig. 5 are far beyond those of algorithms which use the analytically known eigenvalues of the rectangular apertures. Therefore, these examples only serve the purpose to demonstrate the flexibility of the method.

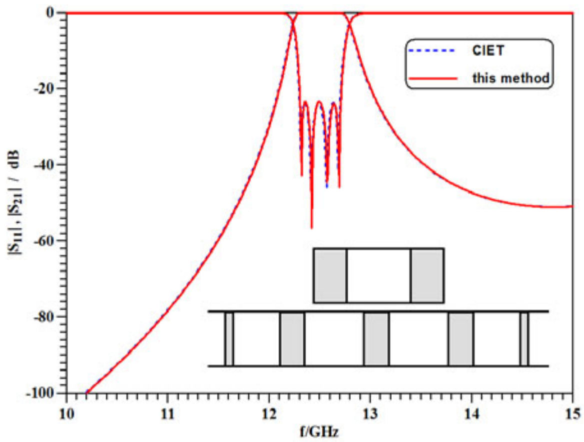


Figure 4. Performance comparison for a four-resonator inductive-iris filter.

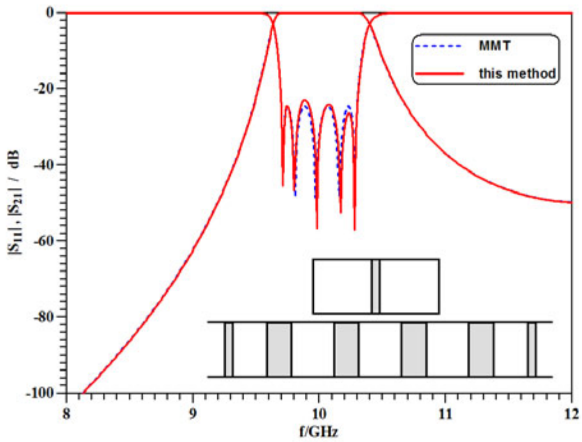


Figure 5. Performance comparison for a five-resonator metal-insert filter.

The following two examples involve below-cutoff ridge waveguide filters with slightly different shapes. Fig. 6 shows the performance of one with pedestals. The cutoff frequency of the pedestalled ridge cross section is close to 3.5 GHz. A six-resonator filter with wide stopband characteristics is obtained after optimization of individual section lengths.

The structure used in Fig. 7 is very simlar, but instead of the pedestal, this ridge is converted into a smaller ridge section which can serve – if more of such smaller sections are incorporated in the cross section – as a way of modelling rounded ridges. The performance of such a below-cutoff ridge waveguide filter is similar to that of Fig. 6, except for the reduced return loss at the lower midband frequency.

IV. CONCLUSIONS

The mode spectrum eigenvalue formulation for irregular cross sections presents a viable option for waveguide component analysis. The method is not necessarily faster than existing techniques but provides flexibility and easy implementation in existing routines based on modal matching approaches. Accuracy and flexibility are demonstrated by comparing the results for a variety of components with those obtained from measurements and other full-wave field solvers.

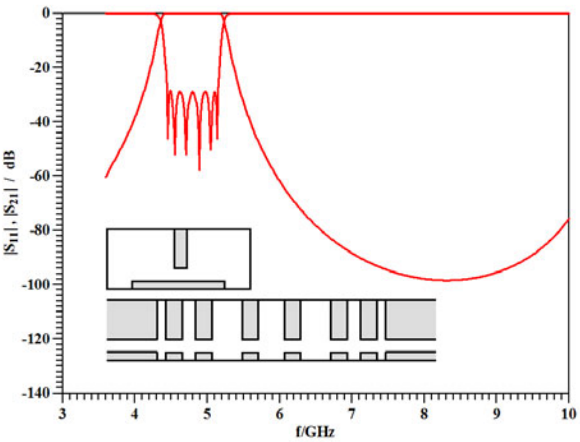


Figure 6. Performance of a six-resonator below-cutoff ridge waveguide filter with pedestals.

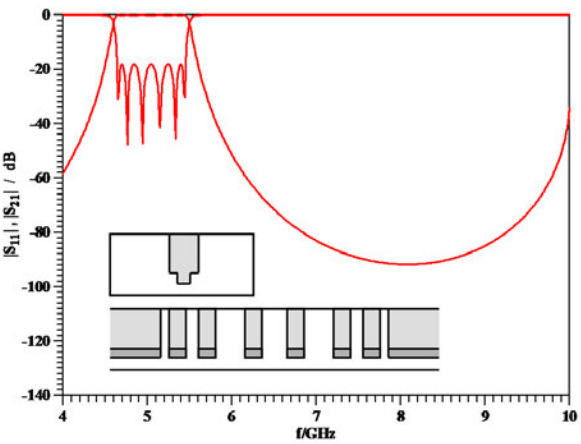


Figure 7. Performance of a six-resonator below-cutoff ridge waveguide filter with two different ridges in the cross section.

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