Scattering Matrix Subtraction Technique for Mode-Matching Analysis of Substrate Integrated Waveguide Junctions

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Abstract—A scattering matrix subtraction method is employed within the mode-matching technique (MMT) to model right-angle substrate integrated waveguide (SIW) junctions. The advantage of this approach lies in the fact that generalized scattering matrices of inline SIW waveguide circuits, including simple bi- and trifurcations are computed first. In a second step, the scattering matrices of regular waveguide corners are subtracted to yield the overall scattering matrices of right-angle SIW junctions. The method is demonstrated for a SIW corner with straight forward extension to T-junctions and cross junctions. Examples of a SIW corner filter, a SIW T-junction diplexer, a SIW cruciform hybrid and a SIW crossover demonstrate the validity of this technique.

Keywords—mode-matching techniques, generalized scattering matrix, substrate integrated waveguide, computer-aided design

I. INTRODUCTION

Analysis and design of substrate integrated waveguide (SIW) components are usually performed with commercially available field solvers such as CST and HFSS. While they offer the most general modeling capabilities, they are not always the fastest to complete circuit designs in a timely fashion. Therefore, other numerical techniques have been applied to passive SIW circuits, including the method of moments (MoM) [1], the boundary integral-resonant mode expansion (BI-RME) method [2], the mode-matching technique (MMT) [3, 4], and combinations of the MMT with the MoM [5] or the spectral-domain approach (SDA) [6].

Within the MMT, especially when modeling circular via holes by their equivalent squares, a fast and accurate analysis procedure has been developed [3, 4] that lends itself well to SIW design and optimization of inline circuits with input ports located left and output ports right. In empty waveguide technology, right-angle junctions such as corners, T- and cross junctions, are easily modeled by a method introduced a long time ago [7]. However, when a right-angle junction contains via holes or other elements, the standard MMT becomes difficult and cumbersome to apply [8].

Therefore, this paper focuses on a scattering-matrix subtraction technique [9] that allows right-angle SIW junctions to be computed within the standard MMT. It is based on modeling inline structures from whose *S* matrices those of regular waveguide corners are subtracted. The applicability of this method is successfully demonstrated at SIW corner, T-junction and cross-junction examples.

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II. THEORY

Fig. 1 demonstrates the basic principle of the scattering matrix subtraction technique for a SIW waveguide corner. Note that we use square vias which are easily converted to circular ones by a straightforward equivalence formula [4].

The generalized S matrix C of the SIW corner is defined as

$$b_1 = C_{11}a_1 + C_{12}a_2$$
, $b_2 = C_{21}a_1 + C_{12}a_2$ (1)

where a and b are the incident and reflected wave amplitude vectors containing propagating and evanescent modes. Within the MMT, it is straightforward to calculate the S matrix S of the inline configuration (upper right in Fig. 1)

$$b_1 = S_{11}a_1 + S_{13}a_3$$
, $b_3 = S_{31}a_1 + S_{13}a_3$ (2)

The waveguide corner S matrix Z is readily obtained by applying the resonator model in [7]

$$a_2 = Z_{22}b_2 + Z_{23}a_3$$
, $b_3 = Z_{32}b_2 + Z_{33}a_3$ (3)

By using (2) and (3) to express a_3 , b_3 as functions of a_1 , b_1 , a_2 , b_2 , the form of (1) is obtained, and the SIW corner submatrices are

$$C_{11} = S_{11} - S_{13}WS_{31}$$

$$C_{12} = S_{13}WZ_{32}Z_{22}^{-1} = C_{21}^{T}$$

$$C_{21} = Z_{22}^{-1}Z_{23}WS_{31} = C_{12}^{T}$$

$$C_{22} = Z_{22}^{-1} - Z_{22}^{-1}Z_{23}WZ_{32}Z_{22}^{-1}$$

$$(4)$$

In (4), exponents -1 and T denote inverse and transposed, respectively.

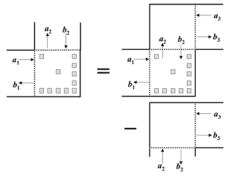


Fig. 1. Scattering matrix subtraction technique applied to a SIW corner.

Matrix W is given as

$$W = \left[S_{33} - Z_{33} + Z_{32} Z_{22}^{-1} Z_{23} \right]^{-1}$$
 (5)

While the number of modes considered in matrices S and Z is arbitrary and usually determined by convergence criteria, the S matrix subtraction must be performed with fewer modes due to the fact that the matrices to be inverted in (4) become nearly singular for a large number of modes. Therefore, all propagating modes plus a few lower-order evanescent ones are taken to compute C in (4) [9].

Fig. 2 presents those structures from which regular waveguide corners have to be subtracted to obtain a SIW T-junction or a SIW cross junction.

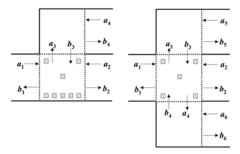


Fig. 2. Left: Subtract one corner to obtain a SIW T-junction; right: subtract two corners to obtain a SIW cross junction.

III. RESULTS

In order to verify the *S* matrix subtraction technique, we present four SIW examples. They contain more sensitive, frequency selective components to demonstrate the achievable accuracy in comparison with a field solver such as CST Microwave Studio.

Fig. 3 compares the MMT results with those of CST for a three-resonator filter with one corner resonator. Very good

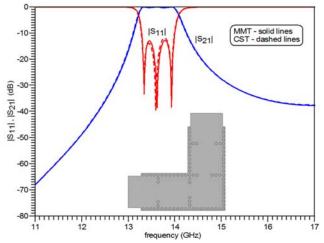


Fig. 3. Performance comparison between MMT with *S* matrix subtraction and CST at the example of a three-resonator SIW corner filter.

agreement is observed. The entire corner resonator with both coupling SIW irises is included in the corner region of Fig. 1. The filter is designed with the MMT including S matrix subtraction and recomputed using CST. The substrate is RT/duroid 6002 with ε =2.94 and h=508 μ m.

A 15 GHz diplexer and its performance are presented in Fig. 4, using the same substrate. The component is designed and optimized using the MMT with *S* matrix subtraction and reevaluated with CST. Excellent agreement between MMT and CST is observed. This not only validates the *S* matrix subtraction technique but also the conversion from square via holes used in the MMT [4] to circular ones in CST. Note that a similar diplexer, designed with the MMT at 19 GHz, has been prototyped in [10]. Measurements demonstrate good agreement with the MMT and CST results.

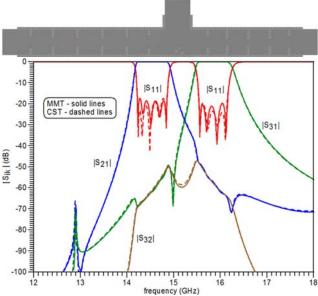


Fig. 4. Performance comparison between MMT with *S* matrix subtraction and CST at the example of a SIW T-junction diplexer with four- and five-resonator channel filters.

Our next example is a SIW cruciform hybrid designed with a planar circuit model and HFSS in [11] on ε =2.2 substrate. Fig. 5 shows the performance comparison of this cross-junction coupler between the MMT with *S* matrix subtraction and CST. Very good agreement is observed. Note that the sharp zero in reflection coefficient (S_{11}) and isolation (S_{41}) is well reproduced in the MMT results. Our results also validate those presented in [11]. According to Fig. 2 (right), the port allocation in Fig. 5 is: port 1 – left, port 2 – right, port 3 – up, port 4 – down.

Finally, Fig. 6 presents a SIW crossover design. It is based on a TE_{102} -mode resonator operating in both horizontal and vertical directions. Therefore, isolation better than 20 dB is obtained between the crossing paths. Since the component is entirely symmetric, other *S* parameters follow accordingly. (Port numbering is identical to that of Fig. 5). In this case, only the four corner vias are included in the *S* matrix subtraction. Very good agreement with results of CST is observed again.

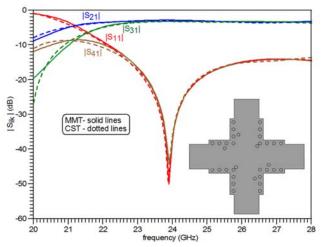


Fig. 5. Performance comparison between MMT with *S* matrix subtraction and CST at the example of a SIW cruciform hybrid; dimensions according to [11].

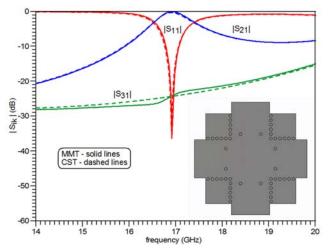


Fig. 6. Performance comparison between MMT with *S* matrix subtraction and CST at the example of a symmetric SIW crossover.

Note that in all these and previous comparisons (e.g. [4], [10]) between MMT and CST, depending on implementation, the MMT code is at least one order of magnitude faster than CST.

IV. CONCLUSION

The scattering matrix subtraction method in combination with the mode-matching technique presents a viable option for the analysis and design of SIW circuits with right-angle junctions. The overall approach is simple, fast, and provides excellent agreement with results of other field solvers, even though the *S* matrix subtraction is required to use a reduced mode set. Four examples containing frequency-dependent components and a SIW corner, SIW T-junction and two SIW cross junctions validate the numerical approach, thus providing a framework for the analysis and design of SIW components.

REFERENCES

- [1] E. Arnieri and G. Amendola, "Analysis of substrate integrated waveguide structures based on the parallel-plate waveguide Green's function," *IEEE Trans. Microw. Theory Tech.*, vol. 56, pp. 1615-1623, July 2008.
- [2] F. Mira, M. Bressan, G. Conciauro, B.G. Martinez, and V.E.B. Esbert, "Fast S-domain modeling of rectangular waveguides with radially symmetric metal insets," *IEEE Trans. Microw. Theory Tech.*, vol. 53, pp.1294-1303, Apr. 2005.
- [3] J. Bornemann, F. Taringou, and Z. Kordiboroujeni, "A mode-matching approach for the analysis and design of substrate-integrated waveguide components," *Frequenz*, vol. 65, pp. 287–292, Sep. 2011.
- [4] Z. Kordiboroujeni and J. Bornemann, "Mode-matching analysis and design of substrate integrated waveguide T-junction diplexer and corner filter," *Int. J. Numer. Model.*, vol. 28, pp. 497-507, Sep./Oct. 2015.
- [5] R. Rezaiesarlak, E. Mehrshahi, and H.R. Sadreazami, "Hybrid of moment method and mode matching technique to study substrate integrated waveguide," *Proc. Int. Conf. Microw. MM Wave Techn. (ICMMT)*, pp. 1980-1982, Chengdu, China, May 2010.
- [6] E.D. Caballero, H. Esteban, A. Belenguer, and V.E. Boria, "Efficient analysis of substrate integrated waveguide devices using hybrid mode matching between cylindrical and guided modes," *IEEE Trans. Microw. Theory Tech.*, vol. 60, pp. 232-243, Feb. 2012.
- [7] E. Kühn, "A mode-matching method for solving field problems in waveguide and resonator circuits," AEÜ, vol. 27, pp. 511–518, Dec. 1973.
- [8] V.A. Labay and J. Bornemann, "Generalized modal scattering matrix of discontinuity-distorted waveguide multiport junctions," *Int. J. Numer. Model.*, vol. 10, pp.153-167, May-June 1997.
- [9] R. Beyer and F. Arndt. "The generalized scattering matrix separation technique combined with the MM/FE method for the efficient modal analysis of a comprehensive class of 3D passive waveguide circuits," *IEEE MTT-S Int. Microw. Symp. Dig.*, pp. 277–280, Orlando, USA, May 1905
- [10] Z. Kordiboroujeni, J. Bornemann and T. Sieverding, "K-band substrate integrated waveguide T-junction diplexer design by mode-matching techniques," *Proc. Asia-Pacific Microwave Conf.*, pp. 1297-1299, Sendai, Japan, Nov. 2014.
- [11] I. Ohta, K. Toda, M. Kishihara, and T. Kawai, "Design of cruciform substrate-integrated waveguide hybrids based on H-plane planar circuit approach," *Proc. Asia-Pacific Microw. Conf.*, pp. 1-4, Bangkok, Thailand, Dec. 2007.