

**Figure 5** Normalized main beam gain as a function of parasitic element spacing  $d$

sive option. The analysis presented here is for a horizontally polarized feed, suitable for terrestrial communications operating in most common fading conditions. The limitation of this type of approach is the degradation in the radiation pattern for large offsets in feed position, leading to a maximum usable angular skew. Parasitic elements introduced into feed systems also lead to higher levels of feed blockage due to the increased physical size of the feed.

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## A TECHNIQUE FOR DESIGNING RING AND ROD DIELECTRIC RESONATORS IN CUTOFF WAVEGUIDES

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**ABSTRACT:** A simple expression for the length of a dielectric ring or rod resonator in a cutoff circular waveguide is given. Instead of using a

determinant equation to design the resonator, it is only required to determine the phase of the reflection coefficient of the resonating mode at a step discontinuity between an infinitely long rod or ring and an infinitely long cutoff waveguide. The technique is validated by comparison to an analytical solution, as well as previously published results from the literature. © 1999 John Wiley & Sons, Inc. Microwave Opt Technol Lett 23: 203–205, 1999.

**Key words:** dielectric resonators; ring resonators; circular waveguides

## I. INTRODUCTION

Dielectric resonators are the primary components in modern microwave filters, especially when a substantial reduction in size is required [1, 2]. Low-loss bandpass filters have been implemented by placing  $TE_{01\delta}$  high- $Q$  dielectric rod or ring resonators axially in a cutoff circular waveguide [3].

An accurate design of this type of filter was carried out using the mode-matching technique (MMT) by Kobayashi and Minegishi [4]. In this approach, the resonant frequency and the coupling coefficients are determined iteratively from a determinant equation whose size is increased until convergence is reached [4]. A similar approach was used by Zaki and Chen to determine the resonant modes and frequency spectrum of a dielectric rod in a circular cavity [5]. The resonant frequency of a dielectric cylinder placed between two metallic plates was determined by Jaworski and Pospieszalski using a variational approach [6]. In all of these reports, the resonant frequency is determined from a transcendental determinant equation. Despite the fact that, in most applications of dielectric resonators, one is interested in the design of dielectric resonators with a prescribed resonant frequency, most of the numerical techniques found in the literature address the analysis problem where the resonant frequencies of a given resonator are determined.

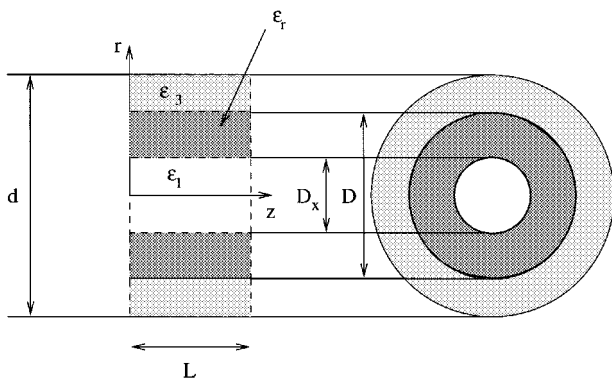
In this paper, we present an alternative technique which gives the length of the dielectric rod or ring to achieve resonance at a desired frequency in terms of the propagation constant of the resonant mode and the phase of its reflection coefficient at the interface between the dielectric-loaded section and the cutoff waveguide. When the MMT is used in the analysis of the assembled filter, the information on the phase of the reflection coefficient is already known. Admittedly, most of the numerical effort goes into this part; the design of a resonator is a welcome spin off of this effort.

## II. THEORY

We consider a dielectric rod or ring of dielectric constant  $\epsilon_r$  of axial length  $L$  in a lossless circular waveguide of inner radius  $a$ . Energy confinement is brought about by the cutoff section; no metallic end plates are needed. The frequency of operation is such that propagation takes place in the dielectric-loaded section, while the empty waveguide is cut off. These are, for example, the conditions for bandpass filters using  $TE_{01\delta}$ -mode resonators coupled through sections of a below-cutoff waveguide.

The problem we address in this paper is the following. *Given the cross section of the dielectric-loaded region, determine the length  $L$  such that the  $TE_{01\delta}$ -mode is resonant at a prescribed frequency  $f_0$ .*

Obviously, the problem can be solved by setting up a determinant equation in the expansion coefficients in the different regions labeled  $\epsilon_1$ ,  $\epsilon_r$ , and  $\epsilon_3$  in Figure 1. Such an approach was used in [4]. Here, we consider an alternative approach which does not require solving a determinant.



**Figure 1** Geometry of dielectric ring resonator in a cutoff circular waveguide

Consider a structure where the dielectric-loaded region extends in the region  $z < 0$  and the cutoff waveguide in the region  $z \geq 0$ . We assume that the propagation constant  $\beta(f_0)$  of the  $TE_{01}$ -mode in the dielectric-loaded region is known. It can be easily determined from a simple characteristic equation in the case of rod or ring resonators. We also assume that the reflection coefficient  $S_{11}(f_0)$  of this mode at the discontinuity in Figure 2, at  $z = 0$ , is also known. In this work,  $S_{11}$  was determined using a moment-method formulation of the MMT [7] to avoid the phenomenon of relative convergence. Note that the  $S_{11}(f_0)$  is calculated by accurately taking into account the coupling to higher order modes in both regions of Figure 2.

Since the empty waveguide is cut off, it follows that

$$|S_{11}(f_0)| = 1. \quad (1)$$

To determine the length of the resonator in Figure 1 for resonance to occur at  $f_0$ , we expand the transverse electric field into forward and backward traveling waves:

$$\mathbf{E}_t = (Ae^{-j\beta(f_0)z} + Be^{j\beta(f_0)z})\mathbf{e}_1. \quad (2)$$

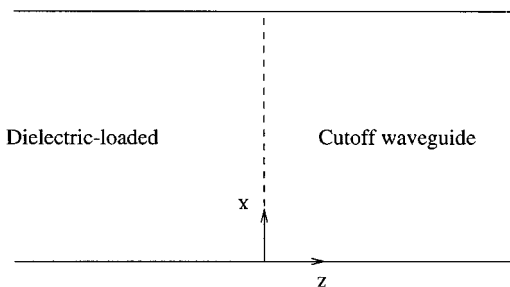
Here,  $\mathbf{e}_1$  is the transverse electric field of the mode in the dielectric-loaded region.

At  $z = 0$ , the reflection coefficient can be used to express  $A$  in terms of  $B$ :

$$A = S_{11}B. \quad (3)$$

Similarly, at  $z = L$ , we get

$$Be^{j\beta(f_0)L} = S_{11}Ae^{-j\beta(f_0)L} = S_{11}^2Be^{-j\beta(f_0)L}. \quad (4)$$



**Figure 2** Semi-infinite ring resonator in a cutoff circular waveguide. The dielectric ring has the same cross section as in Figure 1

Since the constant  $B$  is nonzero, this last equation can be rewritten as

$$e^{2j\beta(f_0)L} = S_{11}^2 = |S_{11}|^2 e^{2j\phi} = e^{2j\phi}. \quad (5)$$

Here, we used the fact that  $|S_{11}| = 1$  from (1);  $\phi$  is the phase of  $S_{11}$ . From this equation, we deduce the length of the resonator as

$$L = \frac{\phi}{2\pi} \lambda_g + n \frac{\lambda_g}{2}, \quad \lambda_g = \frac{2\pi}{\beta(f_0)}. \quad (6)$$

The value of  $n$  is chosen to give the smallest nonnegative value of  $L$ .

### III. ILLUSTRATIVE RESULTS

As a first example, we examine the limiting case when the resonator is terminated in short circuits at  $z = 0$  and  $z = L$ . In such a case,  $S_{11} = -1$  and  $\phi = \pi$ . The possible solutions are then

$$L = \frac{\lambda_g}{2} + n \frac{\lambda_g}{2}, \quad n = 0, 1, \dots \quad (7)$$

which is a known result.

We consider next a more realistic case of a ring resonator. The dimensions of the ring (see Fig. 1) are  $D = 4.91$  mm,  $D_x = 1.47$  mm, and  $d = 11.73$  mm. The dielectric constant is  $\epsilon_r = 24.3$ . The desired resonant frequency is 11.958 GHz. These are the same dimensions used in [4].

The propagation constants in the dielectric-loaded region are determined using a mode-matching formulation along the lines presented in [4]. The generalized scattering matrix of the corresponding structure (Fig. 2) was also determined using a variant of the MMT.

At  $f_0 = 11.958$  GHz, the phase of  $S_{11}$  was determined as  $\phi = 1.8967$  rad, and the propagation constant is such that  $\beta d/2 = 3.0813$ . Using (6), we get the smallest length of the resonator as  $L = 3.61$  mm, which is identical to the value given in [4]. When the technique is applied to the case of a dielectric rod with the same dimensions as those given in [4], we get a length  $L = 3.42$  mm, which is again identical to the value given in this reference. Table 1 summarizes these results.

As a final example, we consider the case when the resonator is a rod with the same radius as the empty waveguide. In this case, the reflection coefficient is given by the wave admittance of the  $TE_{01}$ -mode in the two regions as

$$S_{11} = \frac{Y_1 - Y_2}{Y_1 + Y_2}. \quad (8)$$

Since the mode is propagating in the dielectric-loaded region ( $Y_1$ ) and below cutoff in the empty waveguide ( $Y_2$ ),  $Y_1$  is real and  $Y_2$  is pure imaginary such that  $|S_{11}| = 1$ . The length of

**TABLE 1**  $F_0 = 11.958$  GHz,  $\epsilon_r = 24.3$ ,  $\epsilon_3 = 1.03$ ,  $\epsilon_1 = 1$ ,  $d = 11.73$  mm,  $D = 4.91$  mm

$D_x$ (mm)	$L$ (mm), This Work	$L$ (mm) [4]
1.47	3.61	3.61
0	3.42	3.42

the resonator is then given by

$$e^{2j\beta L} = \frac{(Y_1 - Y_2)^2}{(Y_1 + Y_2)^2}. \quad (9)$$

This result is readily verified from a complete analysis where the transverse electric and magnetic fields are matched at  $z = 0$  and  $z = L$ . Since the analysis is straightforward, it is not presented here.

Although the technique has been applied only to rod and ring resonators, it can be applied to more complex structures under the assumption listed at the beginning of Section II.

#### IV. CONCLUSIONS

A simple equation to design rod and ring dielectric resonators in a cutoff waveguide is presented. At a prescribed frequency, the length of the resonator is given explicitly in terms of the propagation constant of the resonant mode in the dielectric-loaded region and the phase of its reflection coefficient at the interface between the loaded and empty regions. The approach is validated both analytically and numerically.

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## A BROADBAND CIRCULAR MICROSTRIP ANTENNA WITH TWO OPEN-RING SLOTS

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**ABSTRACT:** This letter presents a new design of a probe-fed circular microstrip antenna with two open-ring slots for broadband operation. Due to the two embedded open-ring slots in the circular patch, two resonant modes of similar radiation characteristics are found to be

excited at frequencies in the vicinity of the fundamental resonant frequency ( $f_{11}$ ) of a corresponding simple circular microstrip antenna without slots. The two closely excited resonant modes can cause the enhancement of the impedance bandwidth of the proposed circular microstrip antenna, and experimental results show that the obtained bandwidth for the proposed antenna can be greater than two times that of a simple circular microstrip antenna. The proposed antenna design and obtained broadband performance are presented. © 1999 John Wiley & Sons, Inc. *Microwave Opt Technol Lett* 23: 205–207, 1999.

**Key words:** microstrip antenna; broadband operation

#### 1. INTRODUCTION

With some specific slots loaded in microstrip antennas, broadband operation has been achieved. For such slot-loaded broadband circular microstrip antennas, the specific slots loaded in the patch include a U-shaped slot [1], an arc-shaped slot [1], and two sectoral slots [2]. In the design with a U-shaped slot or an arc-shaped slot, broadband operation is demonstrated by incorporating a foam or an air substrate [1]. As for the case with two sectoral slots, the broadband antenna shown is fabricated on a regular electrically thin microwave substrate [2]. The presence of the two sectoral slots in the circular patch results in the excitation of two closely excited resonant modes for the antenna to have a wide impedance bandwidth. However, some distortions in the radiation patterns of the frequencies within the obtained wide impedance bandwidth have also been observed for such a design [2].

In this letter, we demonstrate another promising slot-loaded circular microstrip antenna with two open-ring slots for broadband operation. The broadband antenna to be studied is printed on an inexpensive FR4 microwave substrate. The present proposed broadband design is related to the dual-frequency operation of a circular microstrip antenna with an open-ring slot [3] in which two operating frequencies with similar radiation characteristics are obtained, and the frequency ratio between the two frequencies is within a region of about 1.23–1.32. It is then found that, by embedding two open-ring slots, the frequency ratio between the two operating frequencies can be significantly lowered to be close to unity, which leads to bandwidth enhancement for the proposed antenna. Moreover, for the proposed antenna, good broadside radiation patterns of the frequencies within the impedance bandwidth are observed. Details of the proposed antenna design are described, and measured radiation patterns and gains are given.

#### 2. ANTENNA DESIGN

The proposed slot-loaded broadband circular microstrip antenna is shown in Figure 1. The circular patch in this study has a disk radius of 23.36 mm, and is printed on an FR4 substrate of thickness 1.6 mm ( $h$ ) and relative permittivity 4.4 ( $\epsilon_r$ ). The outer open-ring slot is at a small distance of 1 mm away from the patch boundary. The two open-ring slots have the same narrow width, 0.5 mm, and the distance between the two slots is set to be 1 mm. Both of the two slots are placed symmetrically with respect to the  $y$ -axis in which an optimal probe feed for good impedance matching can be located. The distance between the probe feed and the disk center is denoted as  $d_p$  here. The inner open-ring slot has a small opening, and is fixed to be 2 mm in this study. The outer open-ring slot has a relatively large opening, and has an angle of  $\phi$ . In the proposed design, it is found that, by choosing a