Solution to Assignment 1

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I. Question 1

Find the response of the linear time invariant system h(k) to the input x(n), where n is time index and k is index of filter coefficient.

$$x(n) = \begin{cases} 1 & \text{for } n = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

$$h(k) = \begin{cases} 0.5 & \text{for } k = 0, 2 \\ 1.0 & \text{for } k = 1 \\ 0 & \text{otherwise} \end{cases}$$

Solution.

The response y(n) of an *LTI* system to the input signal x(n) is the sum of delayed inputs x(n-k) weighted by the impulse response h(k):

$$\begin{aligned} y(n) &= \sum_{k=-\infty}^{\infty} h(k)x(n-k) \\ &= \sum_{k=0}^{2} h(k)x(n-k) \\ &= h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) \\ &= 0.5x(n) + x(n-1) + 0.5x(n-2) \end{aligned}$$

n	x(n)	x(n-1)	x(n-2)	y(n)
0	1	0	0	0.5
1	1	1	0	1.5
2	0	1	1	1.5
3	0	0	1	0.5
4	0	0	0	0

II. Question 2

Repeat Question 1 for the same x(n) but

$$h(k) = \begin{cases} 1 & \text{for } k = 0 \\ -1 & \text{for } k = 1 \\ 0 & \text{otherwise} \end{cases}$$

Solution.

The response y(n) can be expressed as the same way in Question 1.

$$\begin{aligned} y(n) &= \sum_{k=0}^{1} h(k)x(n-k) \\ &= h(0)x(n) + h(1)x(n-1) \\ &= x(n) - x(n-1) \end{aligned}$$

n	x(n)	x(n-1)	y(n)
0	1	0	1
1	1	1	0
2	0	1	-1
3	0	0	0

III. Question 3

(a) Find y(n) for x(n) as above for the system y(n) = ay(n-1) + x(n-1)

(b) Draw a block diagram of this system.

Solution.

(a) In this case, the output is the sum of past outputs as well as the past inputs.

n	x(n)	x(n-1)	y(n)
0	1	0	0
1	1	1	1
2	0	1	a+1
3	0	0	a*(a+1)
4	0	0	a*a*(a+1)
.	.	.	...

(b) The block diagram of this system is as follows:

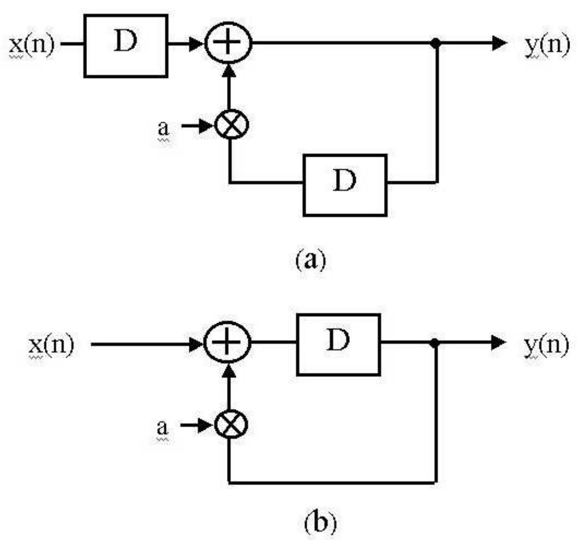

Fig. 1. Block diagram of the system

IV. Question 4

Repeat 1,2,3, using Matlab.

Solution.

(a) x(n), h(k) and y(n) for Question 1.

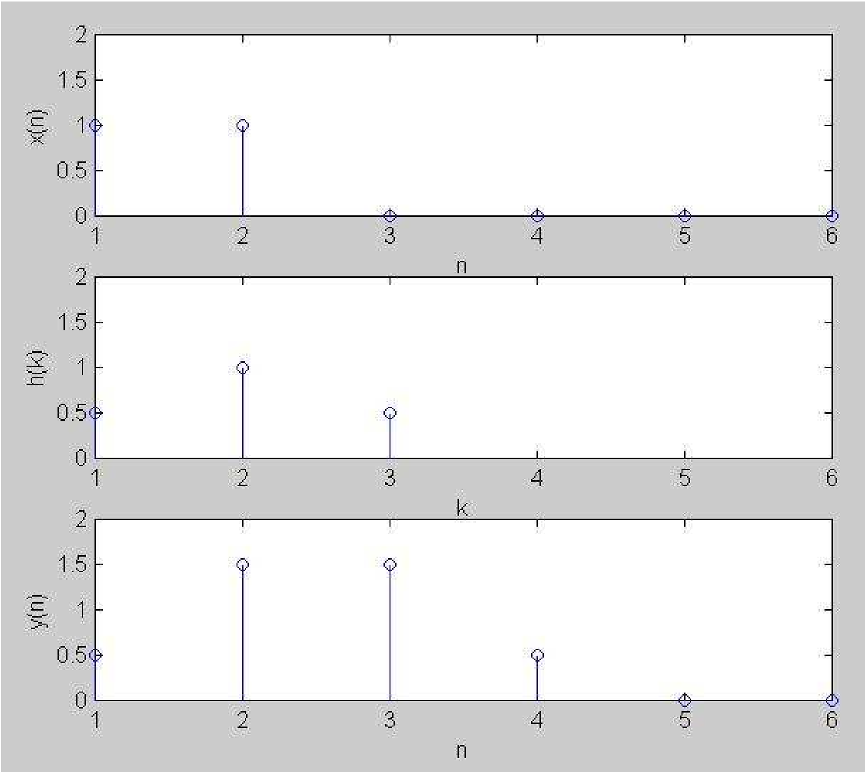

Fig. 2.

(b) x(n), h(k) and y(n) for Question 2.

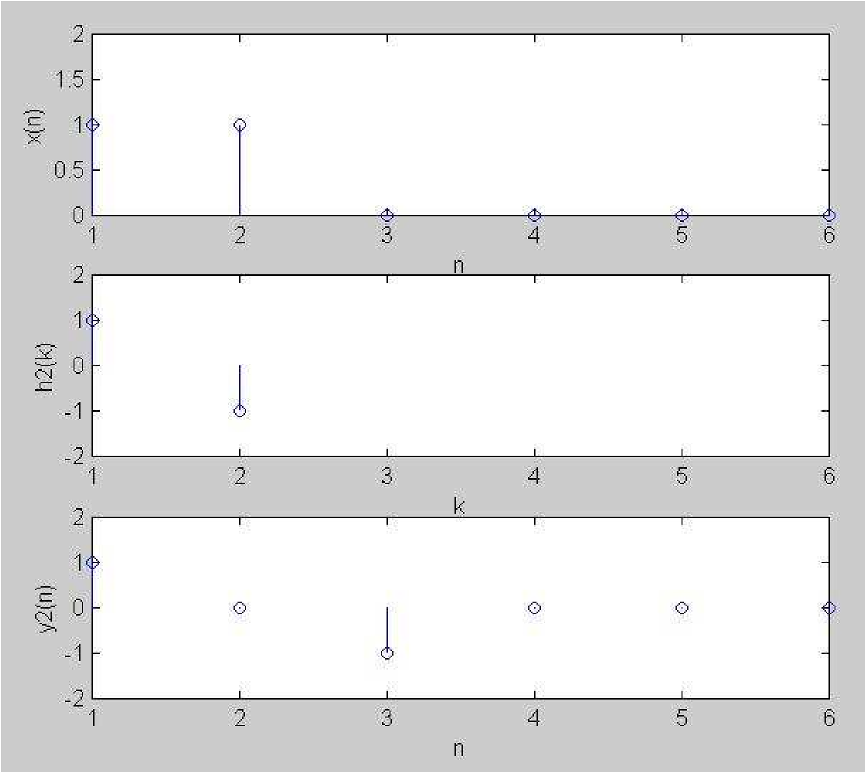

Fig. 3.

(c) x(n) and y(n) for Question 3. (Assume a = 0.5)

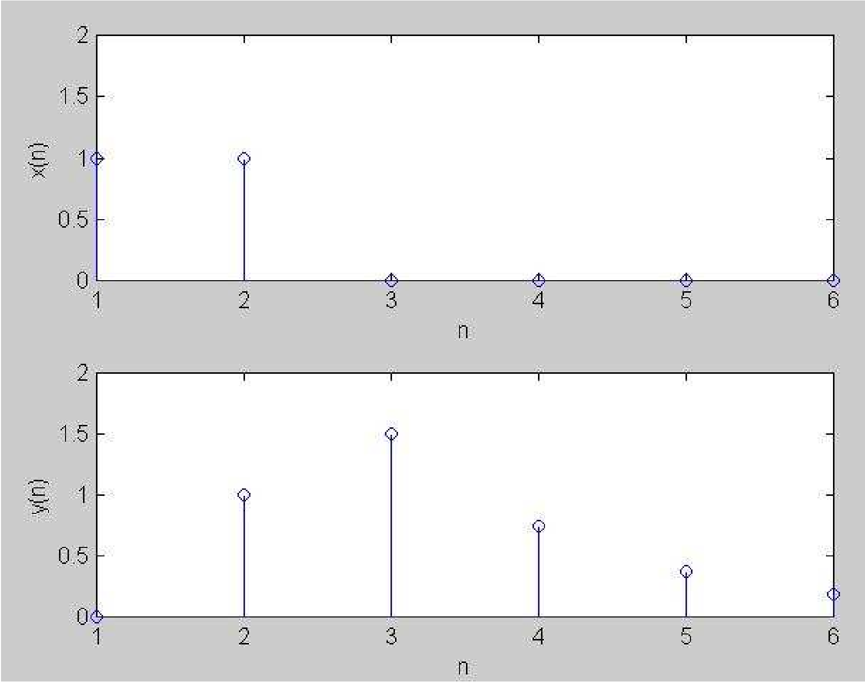

Fig. 4.