Design of a new time orthogonal multiuser line code

A.J. Al-Sammak, R.L. Kirlin and P.F. Driessen

Abstract: The design of a new multiuser line-coding scheme based on ideas from collaborative coding multiple access (CCMA) is presented. The new scheme yields good throughput in terms of user information bits per time slot (baud). Separate user data streams (or a single data stream demultiplexed into multiple separate streams) are combined in a binary adder channel. Trade-offs among the number of users, number of time slots per user symbol, and number of user information bits per user symbol are developed, and possible codes are tabulated. A maximum rate of 4.4 bits/baud can be achieved by using five time slots, with two symbols/user and 22 users.

1 Introduction

Collaborative coding multiple access (CCMA) techniques permit simultaneous transmission by multiple users over a common binary adder channel [1–3] and, more recently, on a shared radio channel [4]. An essential feature of CCMA is that each user is limited to using M so-called constituent codes of the T-user collaborative code, in order that coding and decoding may occur at separate locations for each user. Typically M is a power of 2 so that each user sends one of M codes for each user information symbol (group of $\log_2 M$ information bits). The maximum capacity of such codes in total information bits (over all users) per time slot is estimated in [1].

If all the user information symbols are available at the same location, then there is no need to limit each user to M codes. Instead, we design a scheme in such a way that for any combination of the U-users' information symbols, a unique point from a sum vector constellation diagram is sent. This scheme is applicable to multiple data streams entering a single encoder unit at one location, and received at a single decoder at a different location. Alternatively, a single data stream can be demultiplexed into multiple sub-streams thus creating the equivalent of multiple users. In this case, we have a new multilevel line code that occupies several time slots (bauds) for each information symbol. The code rate (capacity) in user information bits per time slot can exceed the CCMA bounds in [2] because all the user information symbols are available at the same location. This new line code is the subject of this paper.

In one example line code, using two time slots per symbol, a 2-D constellation diagram is given for the new scheme, which looks similar to the 16-point quadrature amplitude modulation (QAM), but it is different from it in three aspects. First, this scheme allows multiple users. Secondly, it employs simple addition of the same signal type in all of the different time slots. Thirdly, this code utilises time orthogonal rather than the phase orthogonal signals used in QAM.

This line code may also be used to define line codes for applications where we desire to send only one bit of information using multiple levels and multiple time slots, such as the multilevel analogue system (MAS) codes [5], 2B1Q [6], and other new codes [7]. In this case, only one user is needed to send the information, and the other 'users' send data in a pattern that defines the line code.

2 Problem description

Given a binary adder channel, U communication channel users [Note 1], each independently select one of M symbols coded as B bits (binary) from his independent alphabet, as indicated in Fig. 1. The U symbols might be added decimally either (*a*) in the transmitter or (*b*) at the receiver. We specifically are concerned with (*a*), addition in the transmitter, as in cable or point-to-point radio. For example, the sum of the first (or second, or..) bits of the U symbols may result in a number 0 through U. For some applications it is useful to define the vector V whose B elements are the individual bit sums. The receiver decoder must uniquely specify each users' symbol that contributed to the sum.

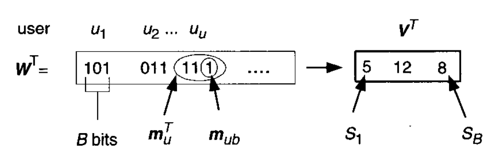

Fig. 1 *Makeup of U users' B-bit binary symbols m_u constituting their concatenated source symbol word, w, along with the associated sum vector* **V** *composed of B bit sums S_b from the U symbols*

2.1 Definitions

U = number of users

B = number of bits or time slots allowed for symbol transmission

S_b = sum of user bits in time slot b, $b = 1,2\ldots, B$, $0 \leq S_b \leq U$

Note 1: We choose to use the symbol U instead of the traditional T to avoid confusion with time and matrix transpose.

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V = the base 10, length B vector whose elements are the sums of each user B-bit word: $V = S_1 10^{B-1} + S_2 10^{B-2} + \ldots + S_B 10^0$

w = a length U^*B vector containing the U concatenated B-bit symbols

M = number of B-bit symbols used by each user

m_u = vector whose elements are the B bits selected by user u

m_{ub} = bth element in m_u

u_i = original (uncoded) users' symbols ($i = 1, 2, \ldots, U$), e.g. if $M = 2$, $u_i = 0$ or 1 independent of the value of B; if $M = 4$, $u_i = 00$, 01, 10, or 11, etc.

In terms of the parameters of block line codes such as 2B1Q, one can interpret U as the input block length, B as the output block length. The other variables define the attributes of the particular line code.

The above definitions and Fig. 1 lead to the following implications.

2.2 Implications

Given the above implementation, we may infer a number of facts about the design as follows:

(i) The number of dimensions in the signal space is B.

(ii) The maximum number of possible values of each S_b is U.

(iii) The number of possible unique values (levels) of the sum vectors V is $N_{V\max} = (U+1)^B$.

(iv) The maximum number of unique symbols available to all users is $M_{\max} = 2^B$, but not all may be used by all users if unique decodability is imposed.

(v) The number N_w possible values of w is $M_{\max}^U$. However for decoding (from V) uniqueness, the actual N_w words used must be less than or equal to N_V. This implies that a smaller number M might be used. Based on this, the following constraint can be formulated:

$$N_w = M^U \leq N_V = (U+1)^B \tag{1}$$

A most important point is that one can use the constraint two ways: for fixed U and B one can select an actual M small enough that $M^U \leq (U+1)^B$, or for a fixed M and U one may increase B so that $M^U \leq (U+1)^B$. For $U = 1$, this result reduces to the familiar $M \leq 2^B$.

Based on the above constraint, we will answer the questions:

(i) What is the maximum number U of users that can use B bits; alternatively, given U users, what is the minimum number B of bits that is required?

(ii) How can we uniquely determine the user symbols associated with each V?

3 Design methodology

In this Section we implement a design that answers the above questions, and several examples are given and tables are presented to demonstrate findings.

3.1 A primary example

A communication scheme can be designed in such a way that for any combination of the U users' data u_i, a unique point from a sum vector constellation diagram is sent. As an example, consider the 4-user ($U = 4$), 2-bit ($B = 2$) code. If $M = 2$ symbols/user, this code needs to use only 16 points from the $N_v = 25$ possible sums in two-dimensional space, i.e. from the grid of size $(U+1)^B = (4+1)^2 = 25$. Results are shown in the constellation maps in Fig. 2. There are many possibilities (15 out of 25 = 2042975) to select a constellation of 16 points out of the available 25. If we select a QAM-like constellation (to minimise probability of error), then this 4-user code may be viewed as a 4B2Q block line code in which 4 binary bits are converted to 2 quaternary bauds.

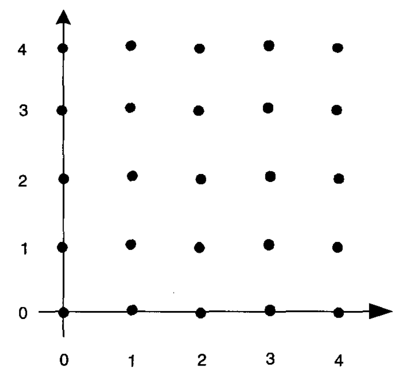

Fig. 2 *Two-dimensional signal space for $U = 4$ users, $B = 2$ time slots per symbol (dimensions $x = t_1$ and $y = t_2$) and $M = 2$ symbols/user (unipolar 0 or 1); all possible sum voltage levels*

The choice of M is subject to the constraint $M^U < (U+1)^B$. At each transmission time U users' selected symbols are assigned by lookup table to the B-bit sums according to a mapping such as in Fig. 3 and Table 1. Note

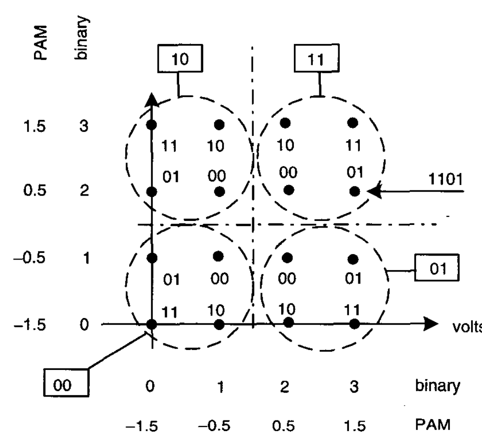

Fig. 3 *Two-dimensional signal space for $U = 4$ users, $B = 2$ time-slots per symbol and $M = 2$ symbols/using QAM-like constellation*

that each user does not select one of two B-bit symbols, which are then added in the channel, as would be done for a collaborative code. The receiver reverses the lookup and delivers the users' messages to U recipients. The dashed blocks in Fig. 4 are needed only in the case wherein there is only one user and this user's data stream is broken into U symbols in sequence, and these are encoded the same way.

To ease encoding, the number of symbols/user can be chosen to be $2, 4, 8, \ldots, 2^B$ and to minimise bit error rate the codewords are Gray-coded to the extent possible. The constellation diagram for the 4-user code is given in Fig. 3, while Table 1 gives the assignment of constellation points for the 4-user code. In Fig. 3, as well as Table 1, two sets of

Table 1: Constellations for 4-user code

Assignment number	4 users' simultaneous binary symbols $u_1\ u_2\ u_3\ u_4$	Constellation assignment (S_1, S_2)	
		Direct Tx Levels	Symmetric PAM
0	0000	1 1	−0.5 −0.5
1	0001	0 1	−1.5 −0.5
2	0010	1 0	−0.5 −1.5
3	0011	0 0	−1.5 −1.5
4	0100	2 1	0.5 −0.5
5	0101	3 1	1.5 −0.5
6	0110	2 0	0.5 −1.5
7	0111	3 0	1.5 −1.5
8	1000	1 2	−0.5 0.5
9	1001	0 2	−1.5 0.5
10	1010	1 3	−0.5 1.5
11	1011	0 3	−1.5 1.5
12	1100	2 2	0.5 0.5
13	1101	3 2	1.5 0.5
14	1110	2 3	0.5 1.5
15	1111	3 3	1.5 1.5

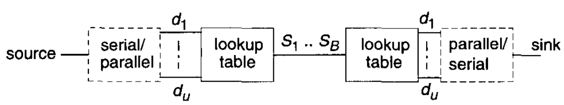

Fig. 4 *U users' symbol sums are assigned by lookup table; the receiver reverses the lookup and delivers the users' messages to U recipients*
Dashed blocks are needed only for the single-user case, where coding is applied to one user's data

constellation axes are given. One is for all-positive level symbols only (direct transmission, direct Tx), while the second uses 0-centred PAM (pulse amplitude modulation).

3.2 Tabular information

In this Section a number of useful tables demonstrate the possibilities under the inequality constraints or bound (1). First we define and evaluate a code's information rate and efficiency and tabulate these quantities for various numbers of users and bits/symbol with M fixed at 2 symbols/user. Later we tabulate maximum number of users U for given M and B, or maximum M for given U and B.

For $M=2$, Tables 2 and 3 show, respectively, for $B=2$ and 3 a code's information rate R for various U, along with efficiency η and efficiency–rate product ηR. Information rate is defined as

$$R = (U/B)\log_2 M \quad \text{user information bits/time-slot} \qquad (2)$$

The efficiency is defined as the symbol combinations used by the U users divided by the number of symbols available given B-bit symbols:

$$\eta = M^U/(U+1)^B \leq 1 \qquad (3)$$

Tables 4 and 5 show, respectively, for $B=2$ and 3 the maximum possible M for various U and the associated code rates and efficiencies.

Table 2: Code rate and efficiency for various *U*, *M*=2, *B*=2

U	$M^U=2^U$	$(U+1)^B$	Rate $R=(U/B)\log_2 M$	Efficiency $\eta=M^U/(U+1)^B$	ηR
2	4	9	1	0.444	0.444
3	8	16	1.5	0.5	0.75
4	16	25	2	0.64	1.28
5	32	36	2.5	0.889	2.222
6	64	49	bound (3) not satisfied		

Table 3: Code rate and efficiency for various *U*, *M*=2, *B*=3

U	$M^U=2^U$	$(U+1)^B$	Rate $R=(U/B)\log_2 M$	Efficiency $\eta=M^U/(U+1)^B$	ηR
2	4	27	2/3	0.148	0.099
3	8	64	1	0.125	0.125
4	16	125	4/3	0.128	0.171
5	32	216	5/3	0.148	0.247
6	64	343	2	0.187	0.373
7	128	512	7/3	0.25	0.583
8	256	729	8/3	0.351	0.936
9	512	1000	3	0.512	1.536
10	1024	1331	10/3	0.769	2.564
11	2048	1728	bound (3) not satisfied		

The example code of Table 1 corresponds to the $U=4$ entry in Tables 2 and 4. Note that the code R in bits per time slot can exceed the CCMA bounds in [2] because all the user information symbols are available at the same

Table 4: Maximum *M* for varying *U* with *B*=2

U	M calculated $M=(U+1)^{B/U}$	M Selected	M^U	$(U+1)^B$	Rate $R=(U/B)\log_2 M$	Efficiency $\eta=M^U/(U+1)^B$	ηR
2	3	2	4	9	1	0.444	0.444
3	2.52	2	8	16	1.5	0.5	0.75
4	2.24	2	16	25	2	0.64	1.28
5	2.048	2	32	36	2.5	0.889	2.222
6	1.91	–	64	49	–	$M<2^{\dagger}$	

†Each user must have at least two symbols for transmission

Table 5: Maximum M for varying U with $B=3$

U	$(U+1)^{B/U}$	M Selected	M^U	$(U+1)^B$	Rate $R=(U/B)\log_2 M$	Efficiency $\eta=M^U/(U+1)^B$	ηR
2	5.196	4	16	27	2*(2/3)	0.593	0.79
3	4	4	64	64	2*(1)	1	2
4	3.343	2	16	125	4/3	0.128	0.171
5	2.93	2	32	216	5/3	0.148	0.247
6	2.645	2	64	343	2	0.187	0.373
7	2.438	2	128	512	7/3	0.25	0.583
8	2.28	2	256	729	8/3	0.351	0.936
9	2.154	2	512	1000	3	0.512	1.536
10	2.053	2	1024	1331	10/3	0.769	2.564
11	1.96	–	–	–	$M<2$		

location, and due to the fact that there is no limitation on codeword selection for each user.

Tables 6 and 7 show, respectively, for $B=2$ and 3 the maximum possible U for $M=4$ and the associated code rates and efficiencies.

Table 6: Maximum U for $M=4$, $B=2$

U	$M^U=4^U$	$(U+1)^B$	Rate $R=(U/B)\log_2 M$	Efficiency $\eta=M^U/(U+1)^B$	ηR
2	16	9	bound (3) not satisfied		

Table 7: Maximum U for $M=4$, $B=3$

U	$M^U=4^U$	$(U+1)^B$	Rate $R=(U/B)\log_2 M$	Efficiency $\eta=M^U/(U+1)^B$	ηR
2	16	27	2*(2/3)	0.593	0.395
3	64	64	2*1	1	1
4	256	125	bound (3) not satisfied		

In the following Tables we find the upper limit on information rate that can be achieved using a given M bits per symbol. Tables 8, 9 and 10 show the maximum rates for various U and B for $M=2$, 4 and 8, respectively.

Table 8: Upper rate limits for $M=2$ symbols/user

B	U	$R=(U/B)\log_2 M$	$\eta=M^U/(U+1)^B$
2	5	2.5	0.889
3	10	10/3	0.769
4	16	4	0.785
5	22	4.4	0.6517

Table 9: Upper rate limits for $M=4$ symbols/user

B	U	Rate	η
3	3	2	1
4	5	2.5	0.79
5	7	2.8	0.50

Table 10: Upper rate limits for $M=8$ symbols/user

B	U	Rate	η
4	2	1.5	0.79
5	3	1.8	0.5
6	4	2	0.26
6	5	2.5	0.7

From Tables 8–10 it is clear that the best rate among those examined, which cover all practical variations, is achieved when using just $M=2$ symbols/user.

4 Conclusions

This paper presents a new design methodology for block line codes based on ideas from collaborative code multiple access (CCMA). This methodology achieves a high rate in terms of user information bits per time-slot. The new code may be used as a single user line code by demultiplexing the input data block into U blocks. Alternatively, it may be used with multiple users when the encoding is done at one location.

The scheme utilises time orthogonal functions to obtain higher code rates (throughput) measured in total information bits per time slot. The paper gives the upper limits achieved for various combinations of the number of users (demultiplexed substreams), number of time slots per user symbol, and number of information bits per user symbol and gave detailed examples of a 4-user code, with 2 bits/time-slot. The code rates achieved here can exceed the CCMA capacity bounds in [2] because all the user information symbols are available at the same location.

An advantage of the new scheme is that it can allow a transmission rate of more than 1.0 (normally achieved using simple TDM). With $B=2$ dimensions the scheme can achieve up to $R=2.5$ bits/time-slot. Increasing the dimensional space to $B=3$, the code rate reaches $R=10/3$ (3.33) bits/time-slot for the 10-user case, $M=2$ symbols/user). A maximum rate of $R=4.4$ can be achieved when using $B=5$, with 2 symbols/user ($M=2$) and $U=22$ users.

Encoding is achieved using a lookup table, which maps any combination of users' data to a corresponding constellation point for transmission over the channel. At the receiver, a reverse process is carried out for decoding the received message into its originally transmitted data symbols.

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