PN CODE ACQUISITION FOR ASYNCHRONOUS CDMA COMMUNICATIONS BASED ON INTERFERENCE CANCELLATION

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ABSTRACT

In this paper, a new PN code acquisition technique based on interference cancellation is developed for asynchronous CDMA communication systems. The acquisition scheme is robust because the near-far problem is alleviated by the cancellation process. Two schemes are proposed. One is based on a one-by-one cancellation, in which the estimated strongest signal is removed from the current composite signal at one stage and never be re-estimated in the following stages. The signals are removed one by one until the weakest signal is detected. The other scheme is based on a simultaneous estimation and cancellation technique, in which all the signals so far detected are estimated together and removed from the original composite signal at each stage. The latter scheme has a better acquisition performance. However the former one requires less computations.

1. INTRODUCTION

It is a well-known fact that a CDMA system suffers the near-far effects when each receiver is equipped with a conventional matched filter which corresponds to a specific signature sequence. Two types of near-far resistant receivers have been proposed. The first type is based on the optimal multi-user detector and its sub-optimal detectors. The second type is based on the interference cancellation detectors. The first one is able to turn the strong interference signals into useful information for detecting weak signals, and thus completely near-far resistant. The second one first detects the strongest interference signals, then subtract them from the compound signals and finally detect the desired signal message from the residual signal. Since the subtraction cannot be made perfect even when the channel noise (assumed to be AWGN) reduces to zero, the interference cancellation receiver is not completely near-far resistant (at least theoretically). However, the cancellation technique can be a useful tool for a receiver to complete the acquisition process.

No matter what type of receivers is used, the acquisition process in such a system becomes more crucial because a receiver has to know exactly the phases of all users' PN codes or at least the phases of those codes which have larger signal powers than the desired code. Unfortunately, the near-far effects cannot be avoided during the acquisition of those PN codes. While the *equal power* assumption can be accepted when analyzing the bit-error-rate performance since the implementation of automatic power control is possible, the same assumption will not be appropriate for the acquisition process, simply because the power control technique cannot work without knowing each user's code. In this paper, we develop a novel acquisition scheme which can alleviate the near-far effects during acquisitions using interference cancellation techniques iteratively until all the PN codes or the desired PN code are captured (K-user system is assumed).

2. ACQUISITION MODEL

In this section, we describe two acquisition schemes based on cancellation techniques. Signals are assumed to be in baseband form. Although this may not represent the real signal, the acq schemes developed in this paper can be modified for the carrier modulated signal.

2.1. Estimation Scheme

We conduct the acquisition process in a time interval of MT, where T is the duration of a single data bit and M is a positive integer number. For an asynchronous CDMA system, there will be $M+1$ data bits for each user in general. Thus the received signal can be written as

$$r(t) = S(t,b) + n(t) \tag{1}$$

where

$$S(t,b) = \sum_{i=1}^{M+1}\sum_{k=1}^{K} b_k(i)A_k c_k(t-iT-\tau_k) \tag{2}$$

where K is the number of users in the system, $b_k(i)$ is the ith data bit of user k in the observation block. A_k is the signal amplitude of user k and $c_k(t)$ is the unit-amplitude signature waveform of user k and is zero outside the interval $[0,T]$.

Using conventional matched filters, it is likely that the strongest signal is detected. Without loss of generality, we assume that signals are numbered such that their amplitudes satisfy $A_1 \geq A_2 \geq \cdots \geq A_K$ and the delay τ_1 is zero. Then $r(t)$ can be rewritten as

$$\begin{aligned} r(t) &= \sum_{i=1}^{M} b_1(i)A_1c_1(t-iT) \\ &+ \sum_{i=1}^{M+1}\sum_{k=2}^{K} b_k(i)A_k c_k(t-iT-\tau_k) + n(t) \end{aligned} \tag{3}$$

or in the discrete form

$$r = H_1\theta_1 + J_1 + n \tag{4}$$

where

$$\theta_1 = b_1A_1 = A_1[b_1(1)\ b_1(2)\ \dots\ b_1(M)]^T,$$

$$H_1 = [c_1(1)\ c_1(2)\ \dots\ c_1(M)] \in \{-1,0,1\}^{(mM)\times M},$$

$$c_1(i) = [\mathbf{0}^T_{(i-1)m}\ c_{1,1}\ c_{1,2}\ \dots\ c_{1,m}\ \mathbf{0}^T_{(M-i)m}]^T,$$

where $c_{1,p}$ is the pth sample of the total m samples of PN code $c_1(t)$, and vector $\mathbf{0}_L$ is a column vector with L zero elements. The interference term J_1 with respect to the signal $H_1\theta_1$ at the first cancellation stage is written as

$$J_1 = H_{J_1}\theta_{J_1} \tag{5}$$

where the interference parameter θ_{J_1} is

$$\theta_{J_1} = B_{J_1}A_{J_1}$$

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where the interference amplitude vector is

$$A_{J_1} = [A_2\ A_3\ \dots\ A_K]^T,$$

and the interference data matrix can be written as

$$B_{J_1} = \begin{bmatrix} \mathrm{diag}(b_{J_1}(1)) \\ \mathrm{diag}(b_{J_1}(2)) \\ \vdots \\ \mathrm{diag}(b_{J_1}(M+1)) \end{bmatrix} \quad (6)$$

with $b_{J_1}(i) = [b_2(i)\ b_3(i)\ \dots\ b_K(i)]^T$, $i = 1, 2, ..., M+1$. The interference sequence matrix is defined as

$$H_{J_1} = [c_2(1)\ c_3(1)\ \dots\ c_K(1)\ c_2(2)\ c_3(2)\ \dots\ c_K(2)\ \dots\ c_2(M+1)\ c_3(M+1)\ \dots\ c_K(M+1)] \in \{-1,0,1\}^{mM\times(K-1)(M+1)},$$

where $c_k(i)$ depends on the delay τ_k. For example, if τ_2 leads to the number of PN code samples to be p_2 for user 2 in the first bit of the observation, we have

$$c_2(1) = [c_{2,m-p_2+1}\ \dots\ c_{2,m}\ \mathbf{0}^T_{(mM-p_2)}]^T \in \{-1,0,1\}^{mM\times 1},$$

$$c_2(2) = [\mathbf{0}^T_{p_2}\ c_{2,1}\ \dots\ c_{2,m}\ \mathbf{0}^T_{m(M-1)-p_2}]^T \in \{-1,0,1\}^{mM\times 1},$$

and

$$c_2(i) = [\mathbf{0}^T_{(i-2)m+p_2}\ c_{2,1}\ \dots\ c_{2,m}\ \mathbf{0}^T_{m(M-i+2)-p_2}]^T \in \{-1,0,1\}^{mM\times 1},$$

and so on.

Using the conventional matched filter, the first user's signature code is detected by an acquisition algorithm called Automatic Threshold Control (ATC) acquisition [2]. Thus H_1 is known in (4). Then we conduct the estimation on θ_1. Applying the Least-Squares (LS) method to the estimation, we get [1]

$$\hat{\theta}_1 = (H_1^T H_1)^{-1} H_1^T r = \frac{1}{m} H_1^T r \quad (7)$$

which turns out to be the outputs of the conventional matched filter 1 at the M sampling instants.

To see how the estimation behaves, we calculate its mean and covariance. Substitute r of (4) into (7), then we get

$$\begin{aligned} \hat{\theta}_1 &= \frac{1}{m} H_1^T (H_1\theta_1 + J_1 + n) \\ &= \theta_1 + \frac{1}{m} H_1^T H_{J_1}\theta_{J_1} + \frac{1}{m} H_1^T n \\ &= \theta_1 + \frac{1}{m} R_{1J_1}\theta_{J_1} + \frac{1}{m} H_1^T n, \end{aligned} \quad (8)$$

where the cross-correlation matrix R_{1J_1} between user 1 and other users is given by

$$R_{1J_1} = H_1^T H_{J_1} = \begin{bmatrix} r^T_{1J_1}(0) & r^T_{1J_1}(1) & 0 & \cdots & 0 \\ 0 & r^T_{1J_1}(0) & r^T_{1J_1}(1) & & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & r^T_{1J_1}(0) & r^T_{1J_1}(1) \end{bmatrix}, \quad (9)$$

where

$$\begin{aligned} r^T_{1J_1}(0) &= [c_1(1)^T c_2(1)\ \ c_1(1)^T c_3(1)\ \dots\ c_1(1)^T c_K(1)] \\ &= [r_{12}(0)\ r_{13}(0)\ \dots\ r_{1K}(0)], \end{aligned}$$

$$\begin{aligned} r^T_{1J_1}(1) &= [c_1(1)^T c_2(2)\ \ c_1(1)^T c_3(2)\ \dots\ c_1(1)^T c_K(2)] \\ &= [r_{12}(1)\ r_{13}(1)\ \dots\ r_{1K}(1)]. \end{aligned}$$

Once the delays τ_k are fixed, R_{1J_1} is a constant matrix (unknown). The estimation $\hat{\theta}_1$ is a random vector because of the randomness of θ_{J_1} (random data) and the AWGN n. Since each of the data bits in θ_{J_1} is either -1 or 1 with equal probability, the mean of θ_{J_1} is a zero vector. It is obvious that $\bar{\hat{\theta}}_1 = \theta_1$, which means that $\hat{\theta}_1$ is an unbiased estimation.

The variance of $\hat{\theta}_1$ depends on the covariances of $\frac{1}{m}R_{1J_1}\theta_{J_1}$ and $\frac{1}{m}H_1^T n$ (the multi-user interference and the AWGN are independent), which is calculated as

$$\begin{aligned} \mathrm{Cov}(\hat{\theta}_1) &= \frac{1}{m^2}\mathrm{E}\{(R_{1J_1}\theta_{J_1})(R_{1J_1}\theta_{J_1})^T\} \\ &\quad + \frac{1}{m^2}\mathrm{E}\{(H_1^T n)(H_1^T n)^T\} \end{aligned} \quad (10)$$

where the second term is easily found to be

$$\frac{\sigma^2}{m} I_M.$$

where I_M is a $M \times M$ identical matrix. To calculate the first term of (10), we first calculate a matrix defined as $D_{J_1} = \mathrm{E}\{\theta_{J_1}\theta^T_{J_1}\}$. This is easily obtained by noticing that all the data bits in θ_{J_1} are i.i.d. 1 or -1 random variables with equal probabilities. Thus D_{J_1} is found to be a diagonal matrix

$$D_{J_1} = \begin{bmatrix} (\mathrm{diag}(A_{J_1}))^2 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & (\mathrm{diag}(A_{J_1}))^2 \end{bmatrix} \quad (11)$$

Thus the first term of (10) is given by

$$\frac{1}{m^2} R_{1J_1} D_{J_1} R^T_{1J_1} = \begin{bmatrix} \sigma_1^2 & \rho & 0 & \cdots & \cdots & 0 \\ \rho & \sigma_1^2 & \rho & & & \vdots \\ 0 & \ddots & \ddots & \ddots & & \vdots \\ \vdots & & & & \ddots & 0 \\ \vdots & & & \ddots & \ddots & \rho \\ 0 & \cdots & \cdots & 0 & \rho & \sigma_1^2 \end{bmatrix} \quad (12)$$

where

$$\begin{aligned} \sigma_1^2 &= \frac{1}{m^2}[r^T_{1J_1}(0)(\mathrm{diag}(A_{J_1}))^2 r_{1J_1}(0) \\ &\quad + r^T_{1J_1}(1)(\mathrm{diag}(A_{J_1}))^2 r_{1J_1}(1)] \\ &= \frac{1}{m^2}\sum_{k=2}^{K}(r^2_{1k}(0) + r^2_{1k}(1))A_k^2, \end{aligned} \quad (13)$$

$$\rho = \frac{1}{m^2} r_{1J_1}^T(0)(\text{diag}(A_{J_1}))^2 r_{1J_1}(1)$$
$$= \frac{1}{m^2}\sum_{k=2}^{K} r_{1k}(0) r_{1k}(1) A_k^2. \quad (14)$$

The ith error variance of the estimation $\hat{\theta}_1$ is given by

$$\text{Var}(\hat{\theta}_1(i)) = \sigma_1^2 + \frac{\sigma^2}{m^2} \quad (15)$$

We have seen that the unknown asynchronous multi-user interference introduces correlation ρ among the M estimations. For a synchronous CDMA system, $r_{1k}(1) = 0$, then $\rho = 0$, i.e. the correlation is zero.

In most cases, the estimation $\hat{b}_1 = \text{sign}(\hat{\theta}_1)$ is good enough [1] for the acquisition since $A_1 \geq A_k$, $k \neq 1$. However, the amplitude $\hat{A}_1$ depends on $r_{1k}(0)$, $r_{1k}(1)$ and A_k, $k \neq 1$. To further improve the estimation of A_1, we apply the LS estimation conditioned on that b_1 is estimated correctly, or at least during the M bits data block. Then r in (4) can be written as

$$r = H_1 b_1 A_1 + J_1 + n$$
$$= \bar{H}_1 A_1 + J_1 + n \quad (16)$$

where $\bar{H}_1 = H_1 b_1$. The unknown variable A_1 is then estimated as

$$\hat{A}_1 = (\bar{H}_1^T \bar{H}_1)^{-1}\bar{H}_1^T r$$
$$= \frac{1}{mM}\bar{H}_1^T r$$
$$= A_1 + \frac{1}{Mm}\bar{H}_1^T J_1 + \frac{1}{Mm}\bar{H}_1^T n \quad (17)$$

The variance of $\hat{A}_1$ is

$$\text{Var}(\hat{A}_1) = \frac{1}{M^2 m^2}\text{E}\{b_1^T R_{1J_1}\text{E}\{\theta_{J_1}\theta_{J_1}^T\} R_{1J_1}^T b_1\}$$
$$+ \frac{1}{M^2 m^2}\text{E}\{b_1^T H_1^T \text{E}\{nn^T\} H_1 b_1\}$$
$$= \frac{\sigma_1^2}{M} + \frac{\sigma^2}{Mm} \quad (18)$$

Compared with $\text{Var}(\hat{\theta}_1(i))$ in (15), we conclude that the variance has been reduced M times by the second LS estimation. In practice, it is very easy to get $\hat{A}_1$ because it is just the absolute average value of the M samples from the output of matched filter 1.

2.2. Acquisition Scheme 1

The most straightforward way for acquiring all the PN codes is to apply the above estimation repeatedly until all the stronger signals are removed one by one and the weakest signal is given the chance to be captured. We call this scheme the one-by-one (OBO) cancellation.

First the K matched filters try to detect their own PN codes using the ATC acquisition scheme in a certain time period. Some of the matched filters may obtain acquisitions, but the strongest signal is assumed to be the one which corresponds to the largest average correlation output. Then we form an estimated signal for the signal detected $\hat{S}(\hat{\theta}_1) = H_1\hat{\theta}_1$. The signal is then removed or cancelled from the original signal, and the residual signal is $r - \hat{S}(b_1)$. For this signal, the left $K-1$ matched filters continue to detect their own codes. Once the strongest one is acquired, it will be removed in the same way. This procedure is repeated until all the PN codes are captured. If at the moment when the left matched filters are not able to detect any signals, the signals are assumed to be absent or too weak, and the receiver will pick up another block of data and continue the acquisition, at the same time it will start the demodulation for the captured signals.

[1] The performance of $\hat{b}_1$ is often given by the required bit-error rate, a system design specification.

2.3. Acquisition scheme 2

In the OBO acquisition scheme, signals from different users are removed one by one by estimating their waveforms. Since the estimation on any user's signal cannot be perfect, especially when there exists multi-user interference, the updated signal will contain a residue of the cancelled signal. This remainder will be stored with the updated signal until the last detected signal. The residual signals from each stage form a harmful interference to the undetected signals and sometimes make it difficult to capture the relatively weak signals even when the AWGN is negligible.

The shortcoming of the OBO scheme can be alleviated by the second acquisition scheme which is a simultaneous estimation and cancellation (SEC) technique. At each stage, all the signals so far detected are estimated simultaneously and then removed from the original compound signal. It is expected that the cancellation error will be getting smaller as more signals are detected correctly. Assuming that signature sequences of user 1 to user i have been detected, the received signal can be expressed as

$$r = H_i'\theta_i' + J_i' + n \quad (19)$$

where the matrix H_i' contains the information of PN sequences of user 1 to user i and θ_i' is the signal data bits and amplitudes of those i users, J_i' is the interference waveform exclusively from the left $K-i$ undetected users. Then the estimation on θ_i' can be obtained from

$$\hat{\theta}_i' = [H_i'^T H_i']^{-1} H_i'^T r \quad (20)$$

The estimated waveform is then obtained

$$\hat{S}(\hat{\theta}_i') = H_i'\hat{\theta}_i', \quad (21)$$

and the residual signal after the cancellation based on the estimation is $r - \hat{S}(\hat{\theta}_i')$.

3. ACQUISITION PROBABILITIES

In this section, we compute the probabilities of acquisitions at all detection stages given the *actual* SNRs of signals and an observation data block. To make this analysis tractable, several assumptions are made as follows. First, Gaussian approximation is made for those samples from the outputs of matched filters. Then, successive outputs from each of the matched filters are assumed to be statistically independent and acquisition decisions on successive bits are approximately independent. Finally, probabilities of acquisitions not in the order of signal power strengths are neglected. Therefore, we only consider the case in which the waveforms are detected in the order of the power strengths from large to small. The signal flow graph of acquisition process at different stages is shown in figure 1, where $P_{acq}(i)$ is the probability of the successful acquisition at stage i. It is clearly shown in the figure that the acquisition at a stage depends upon the detections at the previous stages. When the receiver cannot detect any signal at any stage, it will take a new block of data and repeat the acquisition process.

Following the definition in the traditional acquisition process, we use H^0 to denote the hypotheses at non-sync epoches and H^1 to denote the hypotheses at sync epoches.

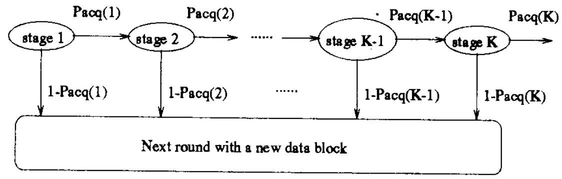

Figure 1. Signal flow graph of acquisition process.

Using the ATC acq scheme for the matched filter i and assuming that the detected data bit is +1, the sampled outputs of the matched filter yields the decision variables which may be expressed as

$$Z_i(0) = \tilde{A}_i + N_i(0) \qquad \text{at } H^1 \tag{22}$$

$$Z_i(n) = N_i(n), \qquad n = 1,\ 2,\ ...,\ m-1 \ \text{ at } H^0 \tag{23}$$

where $\tilde{A}_i$ is the *effective* signal amplitude, $N_i(0)$ and $N_i(n)$ ($n = 1, 2, ..., m-1$) are mutually independent Gaussian random variables with zero means and variances $\sigma_i^2(0)$ and $\sigma_i^2(n)$ respectively. The values of $\tilde{A}_i$, $\sigma_i^2(0)$ and $\sigma_i^2(n)$ will be evaluated in [4]. The decision is made in favor of the signal corresponding to the largest $|Z_i(n)|$ in a window of length m, while the sign of this term is used to decide whether +1 or −1 was transmitted. In the acquisition process, the sign of the term does not affect the detection result as long as the epoch is picked up. Thus the acq epoch is detected if $|Z_i(0)| > |Z_i(n)|$ or $Z_i(0) > |Z_i(n)|$ and $Z_i(0) < -|Z_i(n)|$ for $n = 1, 2, ..., m-1$. However the probability of $Z_i(0) < -|Z_i(n)|$ for $n = 1, 2, ..., m-1$ is very small and can be neglected when +1 was transmitted. Therefore the probability of an acquisition detection in a window for the ith matched filter can be written as

$$\begin{aligned} P_d(i) &= \Pr\{|Z_i(n)| < |Z_i(0)|,\ n = 1,\ ...,\ m-1.\} \\ &\approx \Pr\{|Z_i(n)| < Z_i(0)|Z_i(0) > 0,\ n = 1,\ ...,\ m-1.\} \end{aligned} \tag{24}$$

By the second assumption, we have

$$P_d(i) = \prod_{n=1}^{m-1} \Pr\{|Z_i(n)| < Z_i(0)|Z_i(0) > 0\} \tag{25}$$

The acquisition for the ith user's signal is obtained if N out of M' such windows have successful detections. The probability of the successful acquisition for user i's signal will be

$$P_{acq}(i) = \sum_{n=N}^{M'} \binom{M'}{n} P_d(i)^n (1 - P_d(i))^{M'-n} \tag{26}$$

The probability that a detector successfully passes stage i is given by

$$P_{pass}(i) = \prod_{k=1}^{i} P_{acq}(k) \tag{27}$$

The average number of data blocks a receiver needs to achieve the acquisition at stage i is given by by

$$\begin{aligned} N(i) &= \sum_{k=1}^{\infty} k(1 - P_{pass}(i))^{k-1} P_{pass}(i) \\ &= 1/P_{pass}(i) \end{aligned} \tag{28}$$

Table 1. Acq probabilities and average No. of acq blocks for case 1.

Stage	OBO scheme		SEC scheme	
No.(i)	$P_{acq}(i)$	$N(i)$	$P_{acq}(i)$	$N(i)$
1	1.0000	1	1.0000	1
2	1.0000	1	1.0000	1
3	0.9999	1	0.9999	1
4	0.8309	1.2	0.9317	1.1
5	0.3480	2.9	0.5867	1.7
6	0.0573	17.5	0.2133	4.7

$P_d(i)$ was derived completely in [4] and reproduced here

$$\begin{aligned} P_d(i) &= \int_0^{\infty} p(z_0) \prod_{n=1}^{m-1} \text{erf}\left(\frac{z_0}{\sqrt{2\sigma_i^2(n)}}\right) dz_0 \\ &= \frac{1}{\sqrt{\pi}} \int_{-\sqrt{\gamma_i}}^{\infty} e^{-v^2} \prod_{n=1}^{m-1} \text{erf}(\alpha_n v + \alpha_n \sqrt{\gamma_i}) dv \end{aligned} \tag{29}$$

where $\alpha_n = \sqrt{\sigma_i^2(0)/\sigma_i^2(n)}$, and $\gamma_i = \tilde{A}_i^2/(2\sigma_i^2(0))$ is the *effective* SNR for the main correlation peak.

4. NUMERICAL RESULTS

We compare the two acquisition schemes by computing their acquisition probabilities and the average number of blocks required to complete the ith stage acquisition. A 6-user CDMA system is considered. Each user uses a m-sequence of length 31. The 6 users have their actual SNRs of 24, 20, 16, 10, 8 and 6 dB and relative phases of 0, 20, 10, 6, 10 and 3 respectively. The observation block contains 10 bit symbols and $(M', N) = (5, 3)$. The results are illustrated in table 1. It can be seen from this table that, to capture all the 6 users in this numerical example, the OBO scheme needs about 18 data blocks while the SEC scheme only needs about 5 blocks which is faster than the OBO scheme. Besides, simulation results achieved in [4] are very close to the theoretical results.

5. CONCLUSION

Any acquisition processes for CDMA communications must be operating in the environment of the near-far effects. This makes those relatively weak signals very difficult to be detected. Therefore the only chance for some of the weak signals to be captured is when the channel is relatively quite or all the other users are not active. The throughput and delay of the system will be too high due to this effect. The new acquisition schemes developed in this paper can be used to greatly reduce the near-far effect. Implementation of the acq schemes would be a more interesting research topic.

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