III. OUTPUT NOISE AND THE 2-D NOISE MATRIX

Under zero initial conditions, the formula to compute the output noise $e_y(i, j)$ of (2) is given [1, eq. (11)] by

$$e_y(i, j) = C \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} A(i, j) w_{eb}(k-i, l-j) + w_{cd}(k, l)$$

(3)

where

$$A(i, j) = A^{-1}(i, j) \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix} + A^{i-1}(i, j) \begin{bmatrix} 0 & 0 \\ 0 & I_n \end{bmatrix}$$

(4)

and the matrix $A^{i-j}$ (denoted by $A_{ij}$ in [2]) is recursively computed as follows [3]:

$$A^{i-j} = A^{i-j} = A^{i-j} = A^{i-j} = I, \quad (i, j) > (0, 0)$$

(5a)

$$A^{0,0} = A^{0,0} = A^{0,0} = A^{0,0} = I$$

(5b)

Since $A(0,0) = I$, (5) can be written as

$$e_y(i, j) = C \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} A(i, j) w_{eb}(k-i, l-j) + w_{cd}(k, l).$$

(6)

To derive (3) or (6), we only need to follow the same procedure as given in [3] to derive the general response formula for $y(k, l)$ of (1) [3, p. 3].

On the other hand, the formula given in [2, eq. (13)] to compute the output noise is

$$y(k, l) = C \sum_{0 < i < k} \sum_{0 < j < l} \left( A^{i-j} \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix} \right) w_{eb}(k-i, l-j) + w_{cd}(k, l).$$

(7)

Obviously, (7) is incorrect and both $w_{eb}(k-i-1, l-j)$ and $w_{eb}(k-i, l-j-1)$ should be $w_{eb}(k-i, l-j)$. From (3), we can obtain the 2-D noise matrix [1, eq. (17)]:

$$W = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} A(i, j) C(CA(i, j).$$

(8)

On the other hand, (7) results in the following incorrect formula for the 2-D noise matrix [2, eq. (18)]:

$$W = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (A^{i-j}) C(CA^{i-j}).$$

(9)

It can be easily seen that the 2-D noise matrix $W$ given by (8) has the dual form to the 2-D covariance matrix [1, eq. (22)], [2, eq. (24)]:

$$K = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \hat{A}(i, j) BB' \hat{A}(i, j).$$

(10)

while the matrix $W$ given by (9) does not.

IV. CONCLUDING REMARKS

It has been shown that the derivation of the formulas to compute the output noise and the 2-D noise matrix given in [2] is incorrect. Moreover, the formula for the 2-D noise matrix obtained in [2] does not have the dual form to the formula for the 2-D covariance matrix.

It should be noted that from the viewpoint of 2-D system theory, the 2-D noise matrix (2-D observability Gramian) $W$ and the 2-D covariance matrix (2-D controllability Gramian) $K$ are related to 2-D local observability and 2-D local controllability which are dual notions to each other. Therefore, the duality between $W$ and $K$ is reasonable. Moreover, the duality makes any algorithms for computing $K$ (such as the algorithms based on the contour integral [1] and the frequency dependent Lyapunov equation [4]) be able to be applied directly to computing $W$.

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REFERENCES


Author's Reply on the 2-D Unit Noise Matrix

W.-S. LU AND A. ANTONIOU

We thank Lin and Kawamata (pp. 610–611, this issue) and Hinamoto, Hamaoka, and Maekawa (pp. 609–610, this issue) for their interest in the results reported in [1].

In the one-dimensional (1-D) case, matrix $W_1$ is defined as

$$W_1 = \sum_{i=0}^{\infty} g_{i} g_{j}$$

(1)

where $\{ g_i = a, \quad i = 0, 1, 2, \cdots \}$ is the impulse response from state to output [2]. A general analysis of roundoff noise in 1-D state-space digital filters was carried out by Kwang [3], [4], yielding the same matrix $W_1$ given by (1) (see (13) of [4]). With a few minor modifications, the derivation of the 2-D noise model

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and the computation of the output-noise power given by [1] are essentially the same as those reported in [3], [4].

A natural 2-D extension of matrix \( W_i \) was proposed by Metzios [5] as

\[
W = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} g(i, j) g(i, j)
\]

\[ (2a) \]

with

\[
g(i, j) = c A^{i-j}, \quad i, j \geq 0
\]

\[ (2b) \]

representing the impulse response of the 2-D filter from state to output.

After re-examination of our noise model, the random error \( \alpha(i, j) + \beta(i, j) \) generated by product quantization can also be expressed as

\[
\begin{bmatrix}
\tau(i+1, j) \\
\tau(i, j+1)
\end{bmatrix}
\]

\[ (3) \]

where \( \tau(i+1, j) \) and \( \tau(i, j+1) \) are the random error generated in the computations of \( v(i+1, j) \) and \( h(i, j+1) \), respectively.

Evidently, the error model given by (3) reflects the nature of error propagation in a 2-D digital filter. With this modification, the error model in (12a) of [1] should be written as

\[
\begin{bmatrix}
\Delta v(i, j) \\
\Delta h(i, j)
\end{bmatrix} = A_{10} \begin{bmatrix}
\Delta v(i-1, j) \\
\Delta h(i-1, j)
\end{bmatrix} + A_{01} \begin{bmatrix}
\Delta v(i, j-1) \\
\Delta h(i, j-1)
\end{bmatrix}
\]

\[ + \begin{bmatrix}
\tau(i, j) \\
\tau(i, j)
\end{bmatrix}
\]

or

\[
\delta(i, j) = A_{10} \delta(i-1, j) + A_{01} \delta(i, j-1) + \tau(i, j)
\]

\[ (4) \]

where

\[
\delta(i, j) = \begin{bmatrix}
\Delta v(i, j) \\
\Delta h(i, j)
\end{bmatrix}
\quad \text{and} \quad \tau(i, j) = \begin{bmatrix}
\tau(i, j) \\
\tau(i, j)
\end{bmatrix}
\]

It now follows from (4) that equations (16)—(18) of [1] can readily be obtained where, (18) is the same as equation (18) of [5].

In conclusion, our formula for the 2-D unit noise matrix is not only correct but it is also consistent with other formulas used in [2]—[5]. While the formulas in [1] and [6] are different, they apply to different noise models. The correct relationship between the two forms of the 2-D unit noise matrix is discussed by Hinamoto, Hamanaka, and Maekawa (pp. 609—610, this issue).

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