Cyclic Prefixed Single Carrier Transmission in Ultra-wideband Communications

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Abstract—This letter proposes cyclic prefixed single carrier transmission with frequency domain equalization (SC-FDE) as an alternative physical layer solution for UWB communications. The performance of SC-FDE over IEEE 802.15.3a UWB channel models is analyzed, simulated and compared with that of impulse based single carrier UWB (SC-UWB) and multicarrier UWB employing orthogonal frequency division multiplexing (OFDM-UWB). The impact of channel coding on the performance of OFDM and SC-FDE in UWB is also studied. Our results demonstrate performance advantage of the SC-FDE scheme, especially when implementation issues such as low complexity and low power consumption for UWB are taken into consideration. The performance of SC-FDE with diversity combining using oversampling is also investigated.

Index Terms—Ultra-wideband (UWB), Single carrier transmission with frequency domain equalization (SC-FDE), Orthogonal frequency division multiplexing (OFDM), Impulse based single carrier UWB (SC-UWB).

I. INTRODUCTION

Ultra-wideband (UWB) technology, aiming at providing high data rate communication with low power consumption, has attracted enormous interest in recent years. Both multicarrier UWB employing orthogonal frequency division multiplexing (OFDM-UWB) and impulse based single carrier (SC) transmission have been proposed to IEEE 802.15.3a as the potential physical layer technology [1], [2]. OFDM-UWB [3], [4] has a simple receiver structure where a one-tap frequency domain equalizer can sufficiently eliminate the multipath effects. However, the inherent high peak-to-average power ratio (PAPR) at an OFDM transmitter becomes a serious issue when it comes to implementation. For impulse based SC-UWB, most research work focused on impulse radio with rake structures and time domain equalization [5]. Although single carrier transmission allows a simple transmitter structure, it can lead to high implementation complexity in the receiver when a large number of rake fingers and time domain equalization taps are required for very high data rate transmission in the UWB channels.

Single carrier block transmission with frequency domain equalization (SC-FDE), where data is transmitted block-wisely in the time domain while equalization is carried out in the frequency domain, has been proved to have similar complexity as OFDM, yet provides overall advantages in performance and implementation [6]–[8]. In this letter, we apply the SC-FDE scheme to UWB communications, as an alternative to the current UWB physical layer proposals. SC-FDE has better error performance than the impulse based SC-UWB and OFDM in many UWB environments. From the implementation perspective, SC-FDE results in a simpler receiver structure and possibly a lower sampling rate for the analog-to-digital converter (ADC) relative to impulse based SC-UWB. When compared with OFDM-UWB, the overall system complexity is similar; however, a lower PAPR value of the transmitted signal is a crucial advantage of SC-FDE over OFDM [6]. Despite the many favorable properties suitable for high speed, low-cost and low-power system, SC-FDE has rarely been investigated for UWB communications.

In this letter, we investigate the performance of SC-FDE over UWB channels. The remainder of this letter is organized as follows. Section II presents the SC-FDE system model, followed by the error probability analysis of the SC-FDE with linear MMSE equalization. Section III compares the performance and implementation complexity of the three transmission schemes, i.e., SC-FDE-UWB, OFDM-UWB, and the impulse based SC-UWB. In Section IV, we briefly describe a diversity scheme by using multiple samples of the received signal. Section V concludes the letter.

II. SC-FDE SYSTEM MODEL AND PERFORMANCE ANALYSIS

We consider the cyclic prefixed single carrier block transmission with frequency domain equalization over UWB channels. A block of signals $\mathbf{x} = [x_0, x_1, \ldots, x_{W-1}]^T$ is transmitted with block length $N$. Cyclic prefix (CP) is inserted between blocks to mitigate the inter-block-interference (IBI). As long as the duration of CP is longer than that of the channel impulse response, IBI effects can be ignored. The same block can be transmitted more than once to increase the received signal-to-noise ratio or one symbol in a block is repeated several times.

Suppose that the equivalent symbol spaced channel impulse response is of order $L$ with taps $\mathbf{h} = [h(0), \ldots, h(L-1)]^T$, where $T$ denotes the matrix transpose. Assuming timing is acquired, the received signal $\mathbf{y}$ can be expressed in a matrix form as [9]

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n},$$

(1)
that the circulant matrix \( \tilde{H} \) is zero padded to length \( N \) [9]. Each element in the noise vector \( \mathbf{n} \) is a real Gaussian random variable (R.V.) with variance \( \frac{N_0}{2} \), where \( N_0 \) is the one-sided noise power spectral density. At the receiver, after serial-to-parallel (S/P) transformation, the sampled received signal is converted to the frequency domain by the Discrete Fourier Transform (DFT) operator, followed by the frequency domain MMSE equalization, with the equalizer taps given by [7]

\[
C_k = \frac{H_k^*}{|H_k|^2 + N_0/(2E_b)},
\]

(2)

where \( E_b \) is the energy per transmitted bit, \( H_k \) is the \( k \)th DFT coefficient of the channel impulse response and \( (\cdot)^* \) denotes complex conjugate. After the frequency domain equalization and IDFT, signal detection is performed in the time domain. For simplicity and without loss of generality, we consider the BPSK modulation and the decision variable becomes \( \hat{x} = \text{sgn}(z) \), where \( z \) is the signal vector after the frequency domain equalization and IDFT, and \( \text{sgn}(\cdot) \) is the algebraic sign function.

Next, we analyze the performance of the BPSK modulated SC-FDE over UWB channels, based on the assumption that the CP duration is long enough to eliminate IBI. It is known that the circulant matrix \( \tilde{H} \) can be decomposed as [9]

\[
\tilde{H} = \mathbf{F}H \Lambda \mathbf{F}^H
\]

(3)

where \( (\cdot)^H \) denotes the complex conjugate transpose, \( \mathbf{F} \) is the DFT matrix and \( F_{l,k} = \frac{1}{\sqrt{N}} \exp(-j \frac{2\pi l k}{N}) \), \( 0 \leq l,k \leq N-1 \). Moreover, \( \Lambda = \text{diag}[H_0, H_1, \ldots, H_{N-1}] \). The received time domain signal \( y \) is then transformed to the frequency domain as

\[
Y = FY = \Lambda FX + Fn.
\]

(4)

After equalization and IDFT, the received signal \( z \) becomes

\[
z = \mathbf{F}^H CY = \mathbf{F}^H C \Lambda FX + \mathbf{F}^H CFn.
\]

(5)

where \( C \) is an \( N \times N \) diagonal matrix with its \( k \)th diagonal element as the frequency domain equalizer tap \( C_k \), for \( k = 0, 1, \ldots, N-1 \). To analyze bit error rate, we will, without loss of generality, consider the detection of bit \( x_0 \). The corresponding \( z_0 \) given by (5) can be expressed as

\[
z_0 = \frac{1}{N} \sum_{k=0}^{N-1} \eta_k x_0 + \frac{1}{N} \sum_{l=1}^{N-1} \sum_{k=0}^{N-1} \eta_l e^{-j \frac{2\pi l k}{N}} x_l + \tilde{n}
\]

(6)

where \( \eta_k = \frac{|H_k|^2}{|H_k|^2 + N_0/(2E_b)} \) and \( \tilde{n} = \frac{1}{N} \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} e^{-j \frac{2\pi l k}{N}} C_k n_l \), where \( n_l \) is the \( l \)th element in \( n \) and \( \tilde{n} \) is the real Gaussian noise after equalization and IDFT with variance

\[
\sigma_{\tilde{n}}^2 = \mathbb{E}[|\tilde{n}|^2] = \frac{N_0}{2N^2} \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} C_k e^{-j \frac{2\pi l k}{N}}.
\]

(7)

The MMSE receiver is not ISI free, where the second term in (6) is the interference. Denote \( S_l = \frac{1}{N} \sum_{k=0}^{N-1} \eta_l e^{-j \frac{2\pi k l}{N}} \). It follows that

\[
z_0 = \frac{1}{N} \sum_{k=0}^{N-1} \eta_k x_0 + I + \tilde{n},
\]

(8)

where \( I \) is the residual ISI term and \( I = \sum_{l=0}^{N-1} S_l x_l \). Conditional on \( I \) and the channel impulse response \( h \), the probability of error for SC-FDE is given by

\[
P_{\text{MMSE}}(e|h, I) = \Pr(z_0 > 0|x_0 = -1, h) = Q \left( \frac{1}{\sigma_{\tilde{n}}} \sum_{k=0}^{N-1} \eta_k - I \right)
\]

(9)

where \( Q(\cdot) \) is the Gaussian tail function. Using the techniques in [10], the probability of error unconditioned from \( I \) can then be obtained as

\[
P_{\text{MMSE}}(e|h) = \frac{1}{2} - \frac{1}{\pi} \sum_{m=0}^{M} \left[ \exp\left(-\frac{m^2 \sigma_{\tilde{n}}^2}{2} \sin(m \omega_0) \right) \prod_{l=1}^{N-1} \cos(m \omega_0 l) \right] + \alpha + \beta
\]

(10)

where \( \omega = \frac{\pi m}{N}, \quad g_0 = \frac{\pi m \eta_0}{\sigma_{\tilde{n}}}, \quad g_1 = \frac{\pi m \eta_1}{\sigma_{\tilde{n}}} \). Also, \( \alpha \) and \( \beta \) represent the approximation error and truncation error. The BER calculated from (10) can approach the exact value as accurately as desired by adjusting \( M \) and \( T_1 \). Alternatively, using the Gaussian approximation (GA) for the residual ISI term \( I \), we have

\[
P_{\text{MMSE}}(e|h) = Q \left( \frac{1}{N} \sum_{l=0}^{N-1} \frac{\eta_l}{E_b \sum_{l=0}^{N-1} |S_l|^2 + \sigma_{\tilde{n}}^2} \right).
\]

(11)

Our calculation using GA in (11) for ISI matches (10) very well, in agreement with the Gaussianity of multiple access interference (MAI) analyzed in [11], [12]. The unconditional probability of error for SC-FDE is then obtained by averaging over various UWB channel realizations of a particular channel model.

III. SIMULATION RESULTS AND DISCUSSIONS

A. Performance Comparison of SC-FDE, impulse based SC-UWB and MC-UWB employing OFDM

The performance of the MMSE based SC-FDE over UWB channels is evaluated using the formulas derived in the previous section and compared with the performance of OFDM-UWB and impulse based SC-UWB employing rake receiver and time domain MMSE equalization. Simulation is carried out assuming perfect channel state information (CSI) available at the receiver, due to the slow time-varying nature of the UWB channel. The UWB channels used are CM1-CM4 models proposed by the IEEE 802.15.3a Working Group [13], where the channel RMS delay spread ranges from around 5 ns to 25 ns. In all simulations, the transmitter impulse shaping filter, the receiver matched filter and one of the 100 channel realizations for a particular UWB channel model are convolved and sampled to form 100 different equivalent channel realizations over which the BER’s are averaged. For fair comparison, we evaluate the performance of three systems at the same effective system data rate of 400 Mbps. For both SC-FDE and OFDM-UWB, we use a data block length \( N = 256 \), CP length of 64, symbol duration of 2 ns and the root raised cosine (RRC) pulse with roll off factor \( \beta = 0.5 \). For the impulse SC-UWB, the performance is simulated using a 2.5 ns symbol...
duration with a 0.5 ns long the second derivative of a Gaussian pulse \( p(t) = \left(1 - 4\pi^2 t^2 / \tau^2 \right) e^{-2\pi^2 t^2 / \tau^2} \), where \( \tau = 0.18 \) ns. The oversampling factor 16 of the 0.5 ns pulse is used to perform rake reception, where the maximal ratio combining is utilized, with 10 rake fingers for CM1-CM3 [5] and 20 rake fingers for CM4, and different number of equalizer taps (10 for CM1 and CM2, 20 for CM3 and 30 for CM4). These parameters are chosen as a performance/complexity tradeoff where the investigated system can yield a reasonable performance with implementability complexity. Complexity issues will be detailed in Section III-D.

Figs. 1 and 2 show the performance comparison of impulse based SC-UWB, OFDM-UWB, and SC-FDE under UWB channel models CM1-CM4. It can be observed that SC-FDE substantially outperforms OFDM-UWB for each channel model. The reason lies in the fact that for single carrier transmission, the energy of an individual bit is distributed over the whole available frequency spectrum and therefore already enjoys frequency diversity [8]. In OFDM-UWB, however, the receiver decisions are made independently on each subcarrier, the detection of data symbols on the subcarriers with spectral nulls will be unreliable and dominate the performance degradation, which can be compensated by effective coding schemes as in Section III-B. Comparing the impulse SC-UWB and SC-FDE, an impulse SC-UWB system with 10 rake fingers and 10 time domain MMSE equalizer taps outperforms SC-FDE under CM1, where the small number of multipaths (10 major paths and maximum 60 resolvable paths) and the line-of-sight (LOS) channel condition validate the effectiveness of rake reception. However, this advantage diminishes for CM2-CM4 when the channel condition becomes worse (i.e., the maximum number of resolvable paths increases to 120-400 for CM2-CM4 with 16-50 major paths for a 0.5 ns pulse), where SC-FDE outperforms the impulse based SC-UWB even with a larger number of time domain equalizer taps and rake fingers. For instance, the impulse SC-UWB yields inferior performance to SC-FDE even when 20 rake fingers and 30 time domain equalizer taps are utilized for CM4, as in Fig. 1. Increasing the number of rake fingers will lead to improved BER performance, yet at the expense of high implementation complexity, as will be illustrated in III-D.

B. Impact of Channel Coding

The convolutional code of rate \( R = 1/3 \) with generator polynomial \( g_0 = 133_8, g_1 = 145_8, g_2 = 175_8 \) and constraint length of 7 is used as an example for performance comparison of coded OFDM-UWB and SC-FDE UWB. For simplicity purpose, equalization and decoding are completely separated in the design. The receiver employs Viterbi algorithm to perform maximum-likelihood decoding of the coded symbol block by block. For SC-FDE, the soft-decision metric, or the Euclidean distance between a block of the received signal and the transmitted data is given by \( D = | z - x |^2 = \sum_{k=0}^{N-1} | z_k - x_k |^2 \), where \( z \) is the equalized signal sequence. The Viterbi decoder then searches through the trellis to find the maximum likelihood path with the smallest metric and performs decoding. For coded OFDM, it is straightforward to derive the soft-decision metric as \( D' = \sum_{k=0}^{N-1} | H_k |^2 | z_k - x_k |^2 \). Since unreliable detection due to a severe channel attenuation on a particular subcarrier can be compensated by the weighted CSI, an improved coding gain for OFDM will be realized. Performance comparisons between the coded SC-FDE and OFDM over UWB channels CM1 and CM3 at a low code rate 1/3 are shown in Fig. 3. It can be observed that both OFDM and SC-FDE with channel coding achieve a significant performance gain over the uncoded systems, with perfect CSI assumed in the decoding metric of OFDM-UWB.

C. Impact of CP length and FFT size

The results presented so far are based on the FFT size of 256 and CP length of 64, achieving a system data rate of 400 Mbps. Our study shows that the same error performance can be obtained by employing a 128 point FFT with CP length of 32 for UWB channels CM1 and CM2 at data rates up to 400 Mbps. For channel models with larger delay spread, as in CM3 and CM4, an FFT size of 128 and CP length of 32 is only sufficient to achieve a system data rate of 200 Mbps. At higher data rates, Fig. 4 shows both OFDM-UWB and SC-FDE suffer performance degradation due to insufficient CP, with SC-FDE more sensitive to it. The reason lies in the fact that the IBI primarily distorts the time domain symbols at the beginning of a block. In SC-FDE, since detection is also performed in the time domain, a few severely distorted time domain symbols lead to overall substantial performance degradation. In OFDM-UWB, however, detection is performed in the frequency domain. The energy of the distortion is distributed to all subcarriers by the subsequent FFT operator, thus making the performance degradation in each subcarrier less severe. The higher sensitivity of the SC-FDE to the IBI effect, however, can be compensated by effective coding schemes. Utilizing 128 FFT/IFFT and 32 CP in an coded SC-FDE is sufficient to cover the worst UWB channel models (CM3 and CM4) at high data rates and yields comparable performance to the coded OFDM-UWB, as shown in Fig. 4.

D. Implementation Advantages of SC-FDE UWB over impulse based SC-UWB and OFDM-UWB

Compared to the impulse SC-UWB, SC-FDE scheme has a simpler receiver structure and lower power consumption. To implement rake fingers in impulse radio, either higher sampling rate than symbol rate is required, or multiple (L) symbol rate samplers like in [5] are necessary. As in the simulation for impulse SC-UWB when oversampling is used to perform rake reception, 32 GHz sampling rate is utilized to achieve a reasonable performance, which is too costly (if possible) for the current CMOS technology, with severe power consumption issue. On the other hand, to achieve symbol rate sampling, each finger requires a high speed analog correlator and an analog-to-digital converter (ADC). Using more rake fingers to capture multipath energy increases circuit complexity, power dissipation and storage requirement. In addition, a large number of channel taps and delays has to be estimated. Moreover, severe ISI becomes the dominant detrimental factor to performance at high SNR’s, especially for high data rate transmission. Substantial number of time
domain equalizer taps required in channels with large delay spread (e.g., 80 taps for CM4 at data rate of 400 Mbps) increases the complexity significantly. Assume that perfect CSI is available at the receiver. For the impulse SC-UWB, time domain MMSE equalization involves a $K \times K$ matrix inverse to calculate the equalizer taps, where $K$ is the number of equalizer taps. Cholesky decomposition can be utilized to perform the matrix inverse at a reduced computational complexity of $O(K^3/6)$ [14]. As shown in Fig. 1, $L = 20$ rake fingers and $K = 30$ equalizer taps used for CM4 still lead to inferior performance to SC-FDE, although the receiver complexity is already very high. SC-FDE, however, yields a much lower computational complexity compared to the impulse SC-UWB. To calculate the frequency domain equalizer taps, N complex multiplications are required. The frequency domain equalization performs FFT/IFFT, at complexity of $O(N \log_2 N)$ [6]. The overall computational complexity is then $O(\log_2 N)$ per symbol, and is independent of the number of multipaths. Therefore, SC-FDE UWB has a lower implementation and power consumption than the impulse based SC-UWB using rake receiver and time domain equalization in very high speed UWB communications.

Compared to the OFDM-UWB, SC-FDE transmission enables a similar overall complexity but solves the major implementation issue in OFDM transmitters where high peak-to-average power ratio values lead to significantly higher RF front-end costs and higher power consumption. As a result, SC-FDE makes it possible for the low cost, low power consumption front end device to be implemented.

IV. SC-FDE WITH DIVERSITY COMBINING

The simulation results in Section III are based on the assumption that the receiver matched filter output is sampled once every $T$ interval, where $T$ is the symbol duration. To further improve the performance of SC-FDE, diversity combining schemes can be employed by oversampling the received signal to form multiple branches. Fig. 5 shows the block diagram of the oversampling system which has a similar structure to the SC-FDE with spatial diversity in [15], while no multiple receiver antennas are required.

The matched filter output is oversampled at time instants $kT + t_m$, $m = 0, 1, \ldots, M - 1, k = 0, 1, \ldots, N - 1$ and $0 < t_m < T$. Assume timing offsets $t_m$’s are chosen to yield the best possible performance. Multiple branches of the received signal in one symbol are hence obtained. After DFT and equalization, the signal is combined in the frequency domain as $\mathbf{v} = \frac{1}{N} \sum_{m=1}^{M} \mathbf{w} e^{j2\pi nt_m}$ and the detection rule for BPSK signals becomes $\hat{s} = \text{sgn}(\mathbf{v}^H \mathbf{u})$. Fig. 6 shows the performance of the SC-FDE under UWB channels with 3 branch oversampling diversity combining, using an FFT size of 256 and CP length 64. It can be observed that a 3 dB performance gain is achieved by using the oversampling diversity. Note that the additional diversity branches obtained through oversampling have decreasing average SNR’s and are not independent and identically distributed (i.i.d). Therefore the performance gain is less substantial than that having i.i.d branches as in the spatial diversity.

V. CONCLUSION

Cyclic prefixed single carrier block transmission with frequency domain equalization has been proposed as an alternative physical layer technology for UWB communications. SC-FDE UWB achieves better performance than OFDM-UWB with no coding and comparable performance with channel coding at coding rate 1/3 or 1/2. The critical advantage of SC-FDE UWB over OFDM-UWB is the lower peak-to-average ratio. Compared to the impulse based SC-UWB for very high data transmission, SC-FDE is more effective in collecting multipath energy and combating ISI. SC-FDE has been shown to achieve better performance than impulse SC-UWB in a majority of realistic UWB channels with reasonable receiver complexity in the highly frequency selective UWB channels where rake structure and time domain equalization become too complex. Moreover, oversampling of the matched filter output has been investigated to increase the received signal-to-noise ratio and a 3 dB gain has been achieved with a 3 branch combining. With the overall advantages compared to impulse based SC-UWB and OFDM-UWB, SC-FDE block transmission is a promising candidate to meet the implementation requirements of high speed, low power and low cost UWB communications.

REFERENCES

Fig. 1. Performance comparison of SC-FDE, OFDM-UWB, and impulse based SC-UWB under the UWB channel CM1 and CM4. The impulse based SC-UWB uses 10 rake fingers, 10 equalizer taps for CM1 and 20 rake fingers, 30 equalizer taps for CM4.

Fig. 2. Performance comparison of SC-FDE, OFDM-UWB, and impulse based SC-UWB under the UWB channel CM2 and CM3. The impulse based SC-UWB uses 10 rake fingers, 10 equalizer taps for CM2 and 20 equalizer taps for CM3.

Fig. 3. Performance of coded OFDM and SC-FDE under the UWB channel CM1 and CM3 with code rate 1/3, FFT size of 256, CP length of 64, at system data rate 400 Mbps.

Fig. 4. Performance of coded OFDM and SC-FDE under the UWB channel CM3 with code rate 1/3, FFT size of 128, CP length of 32, at system data rate 400 Mbps.

Fig. 5. SC-FDE system with diversity combining

Fig. 6. Performance of SC-FDE with 3 diversity branches, with FFT size of 256 and CP length of 64.