LOW-ORDER KALMAN FILTERS FOR CHANNEL ESTIMATION

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ABSTRACT

This paper addresses the design of low-order Kalman filters to estimate radio channels with Rayleigh fading. Rayleigh fading cannot be perfectly modelled with any finite order auto-regressive (AR) process. Previously, only first and second order Kalman filters were used for channel estimation since higher order Kalman filters were found to not significantly improve accuracy. This is due to mismatches in the statistics of the AR models of the Kalman filters and the true Rayleigh fading. In this paper, the coefficients of the AR models for the Kalman filter are calculated by solving for the minimum square error solutions of an over-determined linear systems. The AR models generated have statistics closely matching the Rayleigh fading process. The Kalman filter using these AR models can accurately estimate the Rayleigh fading process. The accuracy of the new Kalman filters is demonstrated in the tracking of simulated Rayleigh fading processes of different bandwidths.

1. INTRODUCTION

A difficulty when designing wireless communications systems is managing the variation of the radio channel over time. Current radio communications systems use estimates and predictions of the radio channel state for power control and data symbol decoding to mitigate the negative effects of the varying radio channel. The effectiveness of these techniques is determined by the accuracy of the radio channel estimation system. Several types of adaptive filters have been proposed for tracking of radio channels [1–3].

The advantages of Kalman filtering over other filter algorithms is that the estimation algorithm computes the covariance of the channel estimation error which is useful when performing data symbol decoding. The Kalman filter also can adapt to changing levels of measurement noise. The Kalman filter requires knowledge of the time evolution process for the radio channel in the form of a finite auto-regressive (AR) model for the radio channel. It is well known that channel fading cannot be perfectly represented by any finite order AR model. This causes the calculations of parameters for finite order AR models which closely match the radio channel to be ill-conditioned [4, 5]. The previous literature on the use of Kalman filters for radio channel estimation is limited to Kalman filters of order 1 or 2 since the Kalman filters of higher order based on AR models using parameters from the ill-conditioned calculations did not provide significant improvements in estimation accuracy. This paper presents a new method for calculating the parameters of AR models of orders from 2 to 10 by developing over-determined equations for the AR model coefficients. The use of over-determined equations reduces the effect of the ill-conditioning problem. How these AR models are used to develop Kalman filters is presented. It is demonstrated how these low order Kalman filters provide accurate estimation and prediction of the radio channel fading process.

Section 2 presents the analytical model for the radio channel measurements. Section 3 describes the radio channel estimation procedure and methods for calculating the parameters of the Kalman filter. Section 4 presents results on the use of the this algorithm. Section 5 presents the conclusions of the paper and ideas for future research.

2. SIGNAL MODEL

For this paper, the CDMA signalling for UMTS will be used, which creates correlation properties in the transmitted signal which allow radio receivers to resolve the different radio propagation paths with delays differing by more than a chip period [2]. For the rest of this paper, it will be assumed that there are P resolvable radio propagation paths.

The spreading sequences and chip waveforms in UMTS have been designed so that if the delays of the propagation paths are estimated perfectly, the individual received sequences for the branch of the receiver corresponding to propagation path p is then approximately equal to

$$\mathrm{r}_p[k] = s_k\, \mathrm{g}_p[k] + \mathrm{n}_p[k] \qquad (1)$$

where the contributions of all other propagation paths other than p and symbols other than k are almost perfectly removed by the filtering and despreading procedures, and $\mathrm{n}_p[k]$ is the remaining white noise process after filtering and despreading [2]. The noise processes for different propagation paths will be independent. For the remainder of this paper, it is assumed that the propagation delays have been perfectly estimated and the received signal on each path of the receiver is given by (1).

The channel fading processes $\mathrm{g}_p[k]$ for $p = 0, 1, ..., P-1$ are modelled as independent complex Gaussian processes with independent and identically distributed imaginary and real components. If a given $\mathrm{g}_p[k]$ has a zero mean, the process is a Rayleigh fading process. If a given $\mathrm{g}_p[k]$ has a non-zero mean, the process is Rician fading. This paper will concentrate on channel estimation for Rayleigh fading, since Rayleigh fading results in worse receiver performance. The autocorrelation function for each channel gain process, $\mathrm{g}_p[k]$ is given by

$$\mathrm{E}\left\{\mathrm{g}_p[k]\,\mathrm{g}_p^*[n]\right\} = {\sigma_p}^2 \mathcal{J}_0\left(2\pi f_d T_s (n-k)\right) \qquad (2)$$

where ${\sigma_p}^2$ is the mean power gain of the p^{th} propagation path, $\mathcal{J}_0(\cdot)$ is the zero-order Bessel function of the first kind, f_d is the maximum Doppler frequency and superscript $*$ denotes complex conjugation [5]. The maximum Doppler frequency is given by $f_d = \frac{|v|}{\lambda}$ where v is the velocity of the mobile terminal and λ is the wavelength of the radio frequency carrier. If the wireless network obtains the speed of the mobile terminal from the mobile terminal location estimation system, the value v is known and the Doppler frequency, f_d, is available to the channel estimation system.

3. CHANNEL ESTIMATION

This section describes different methods for estimating the channel gain coefficients. The basis for channel estimation in CDMA systems is that if symbol s_k is known, it is possible to measure the channel gains for symbol k based on (1) as

$$\hat{\mathrm{g}}_p[k] = \frac{\mathrm{r}_p[k]}{s_k} = \mathrm{g}_p[k] + \frac{\mathrm{n}_p[k]}{s_k}. \qquad (3)$$

To decode symbol s_k, it is necessary to have estimates for the channel coefficients, $\mathrm{g}_p[k]$, which is calculated based on measurements $\hat{\mathrm{g}}_p[j]$ from previously received known symbols. Section 3.1 will discuss methods for estimating $\mathrm{g}_p[k]$ using Finite Impulse Response (FIR) filtering of past channel measurements. Section 3.2 discusses the extension of FIR methods to Kalman filtering of past channel measurements to obtain higher accuracy of estimates of the channel gains.

3.1. FIR Channel Estimation

FIR filters estimate use a finite number of past measurements of a random sequence to estimate the current value. For an order M FIR filter, the channel gain coefficient $\mathrm{g}_p[k]$ is estimated from the channel gain measurements for M previous channel measurements. k_1, k_2, ..., k_N using the linear equation $\tilde{\mathrm{g}}_p[k] = \boldsymbol{w}^T\boldsymbol{x}$ where $\boldsymbol{w}$ is a vector of weight coefficients, $\boldsymbol{x} = \left[\hat{\mathrm{g}}_p[k_1], \hat{\mathrm{g}}_p[k_2], ..., \hat{\mathrm{g}}_p[k_M]\right]^T$, and superscript T denotes matrix transposition. Since the channel gain processes are zero mean wide sense stationary complex Gaussian processes, the Minimum Square Error (MSE) estimate of $\mathrm{g}_p[k]$ given the M past channel gain measurements for symbols $k_1, k_2, ..., k_M$ is obtained using the weight vector which is the solution of the Weiner-Hopf linear equation $\boldsymbol{C}\boldsymbol{w} = \boldsymbol{\rho}$ where $\boldsymbol{C} = \mathrm{E}\left[\boldsymbol{x}\boldsymbol{x}^T\right]$ and $\boldsymbol{\rho} = \mathrm{E}\left[\mathrm{g}_p[k]\boldsymbol{x}\right]$ [6]. The entries of $\boldsymbol{C}$ and $\boldsymbol{\rho}$ can be calculated using from the autocorrelation of the fading process given in Section 2 and adding the effect of the independent additive white Gaussian noise processes from (3). A disadvantage of FIR filters is that the weight vector $\boldsymbol{w}$ must be adjusted for different levels of measurement noise which is computationally expensive. The next section will present the use of the Kalman filter for channel estimation which adjusts easily to changing measurement noise levels.

3.2. Kalman Filtering Channel Estimation

The Kalman filter is an estimation technique which uses a dynamic model of a system to optimally system states from noisy linear measurements. The system state at time j is denoted as $\boldsymbol{x}[j]$ and the dynamic model of system evolution is given as $\boldsymbol{x}[j+1] = \boldsymbol{\Phi}\boldsymbol{x}[j] + \boldsymbol{w}[j]$ where $\boldsymbol{\Phi}$ is the state transition matrix, and $\boldsymbol{w}[j]$ is a white complex Gaussian vector random process called the process noise with $\mathrm{E}\{\boldsymbol{w}[j]\boldsymbol{w}[j]^*\} = \boldsymbol{Q}$. If linear measurements of the system state at time j, are available as $\boldsymbol{y}[j] = \boldsymbol{H}[k]\boldsymbol{x}[j] + \boldsymbol{v}[j]$ where $\boldsymbol{H}[j]$ are known matrices, and $\boldsymbol{v}[j]$ is a zero mean white complex Gaussian random process with a covariance of $\boldsymbol{R}[j]$. An advantage of Kalman filtering for channel estimation is that $\boldsymbol{R}[j]$ can vary from sample to sample.

We will describe the application of the Kalman filter to the estimation of the channel gains for one propagation path p. For an order M Kalman filter, the state vector contains the last M measurements of the channel gain coefficients where measurements are made every S symbols $\boldsymbol{x}_p[j] = \left[\mathrm{g}_p[jS], \mathrm{g}_p[(j-1)S], ..., \mathrm{g}_p[(j-M+1)S]\right]^T$. The measurement for sample j is given by $\boldsymbol{y}[j] = \hat{\mathrm{g}}_p[jS]$ where substituting from (3) it can be seen that $\boldsymbol{H}[j] = s_{jS}\,[1\ 0\ ...\ 0]$ and $\boldsymbol{v}[j] = n_p[jS]/s_{jS}$ which has a covariance $\boldsymbol{R}_p[j]$ that is a function of the signal to noise ratio of the radio channel. For the remainder of this paper, we will assume that $\boldsymbol{R}_p[j]$ is known. The Kalman filter calculates estimates of the system state, $\hat{\boldsymbol{x}}[j]$, and error covariance, $\boldsymbol{P}[j] = \mathrm{Cov}\{\hat{\boldsymbol{x}}[j] - \boldsymbol{x}[j]\}$. The estimated state vector and error covariance given all measurements until sample m are denoted $\hat{\boldsymbol{x}}[j|m]$ and $\boldsymbol{P}[j|m]$. Given an initial state estimate $\hat{\boldsymbol{x}}[0|0]$ and error covariance estimate $\boldsymbol{P}[0|0]$, it is possible to recursively calculate other estimates of the channel state using the Kalman

$$
\begin{aligned}
\hat{\boldsymbol{x}}_p[j|j-1] &= \boldsymbol{\Phi}\hat{\boldsymbol{x}}_p[j-1|j-1] \\
\boldsymbol{P}_p[j|j-1] &= \boldsymbol{\Phi}\boldsymbol{P}_p[j-1|j-1]\boldsymbol{\Phi}^T + \boldsymbol{Q} \\
\hat{\boldsymbol{y}}_p[j|j-1] &= s_{jS}\,[1\;0\;...\;0]\,\hat{\boldsymbol{x}}_p[j|j-1] \\
\boldsymbol{M}_p[j|j-1] &= s_{jS}\,[1\;0\;...\;0]\,\boldsymbol{P}_p[j|j-1]\,[1\;0\;...\;0]^T\,{s_{jS}}^* \\
&\quad +\boldsymbol{R}_p[j] \\
\boldsymbol{z}_p[j] &= \boldsymbol{y}_p[j] - \hat{\boldsymbol{y}}_p[j|j-1] \\
\boldsymbol{K}_p[j] &= \boldsymbol{P}_p[j|j-1][1\;0\;...\;0]^T {s_{jS}}^* \\
&\quad \times (\boldsymbol{M}_p[j|j-1])^{-1} \\
\hat{\boldsymbol{x}}_p[j|j] &= \hat{\boldsymbol{x}}_p[j|j-1] + \boldsymbol{K}_p[j]\boldsymbol{z}_p[j] \\
\boldsymbol{P}_p[j|j] &= (\boldsymbol{I} - \boldsymbol{K}_p[j]s_{jS}\,[1\;0\;...\;0])\,\boldsymbol{P}_p[j|j-1]
\end{aligned}
$$

Fig. 1. Kalman Filter Equations

filter equations given in Figure 1 if the dynamic model is known.

The dynamic models used for channel estimation in the literature are based on AR models of the channel evolution process. The standard method of creating the state transition matrix based on an AR model of order M is

$$
\boldsymbol{\Phi} = \begin{bmatrix} w_1 & w_2 & ... & w_{M-1} & w_M \\ 1 & 0 & ... & 0 & 0 \\ 0 & 1 & ... & 0 & 0 \\ \vdots & 0 & \ddots & 0 & 0 \\ 0 & 0 & ... & 1 & 0 \end{bmatrix} \tag{4}
$$

where $\boldsymbol{w}$ is a vector of prediction weights calculated by solving the Weiner-Hopf equation for the prediction of $\mathrm{g}_p[jS]$ from noise free measurements. The process noise covariance is given by

$$
\boldsymbol{Q}_{11} = {\sigma_p}^2 \left[1 - \sum_{j=1}^{M} w_j \mathcal{J}_0\left(2\pi f_m j\right)\right] \tag{5}
$$

with all other entries of $\boldsymbol{Q}$ being 0 based upon the error calculation for FIR prediction filters [6]. The frequency f_m is given by $f_d T_s S$ where T_s is the symbol period. Previous efforts to use the Kalman filter for channel estimation have been limited to using dynamic models based an AR models of order 1 or 2. This is due to the channel fading process being an irrational process that cannot be perfectly modelled by any finite AR model. This has caused the linear system that needs to be solved to obtain $\boldsymbol{w}$ for the state transition model in (4) to be ill-conditioned. Generated AR models of order 2-10 did not model the fading process well and thus did not give good estimation performance. This problem can be reduced by altering the covariance matrix used when computing the weight vector. The covariance matrix is set to $\boldsymbol{C} = \boldsymbol{C}_0 + \lambda\boldsymbol{I}$ where $\boldsymbol{C}_0$ is the covariance of noise free measurements of $\boldsymbol{x}[j]$, $\boldsymbol{I}$ is an appropriately sized identity matrix, and λ is a small positive value [4]. This method works well to calculate AR models of orders higher than 10.

In this paper, we present a method for generating good AR-models of orders less than 10 by calculating $\boldsymbol{w}$ using an over-determined system. The vector $\boldsymbol{w}$ of length M which gives the minimum square error when substituted into the Weiner-Hopf equation is calculated when $\boldsymbol{C}$ is an $(M+L)\times M$ matrix and ρ is a vector of length $M+L$. The entries of $\boldsymbol{C}$ and $\boldsymbol{\rho}$ are given by

$$
\begin{aligned}
\boldsymbol{C}_{ij} &= {\sigma_p}^2 \mathcal{J}_0\left(2\pi f_m(i-j)\right) + \lambda\delta(i-j) \text{ and} \qquad (6) \\
\rho_j &= {\sigma_p}^2 \mathcal{J}_0\left(2\pi f_m j\right) \qquad (7)
\end{aligned}
$$

where $\delta(j) = \begin{cases} 1 & j=0 \\ 0 & \text{otherwise} \end{cases}$. Several excellent commercial and free software packages are available which can solve such over-determined linear systems. Once a weight vector $\boldsymbol{w}$ has been calculated, the state transition matrix $\boldsymbol{\Phi}$ and process noise covariance $\boldsymbol{Q}$ can be calculated using (4) and (5). It has been found that using $\lambda = 10^{-12}$ and $L = 5$ generated good AR models of order 2 to 10 which gave excellent results when used for Kalman filtering. The application of the Kalman filtering to the estimation of the complex channel coefficients are presented in the next section.

4. RESULTS

This section presents the application of the new Kalman filters to the estimation and prediction of the channel gain for a single propagation path. For the simulations, the mean power of the fading process is 1 and the power of the noise process is 0.01, for a signal to noise ratio of 20 dB. The Rayleigh fading process to be tracked is generated using an Inverse Discrete Fourier Transform method [7]. For each filter, average squared error of the predicted value of $\mathrm{g}_p[j]$ is reported. For the Kalman filters this is calculated as the average value of $\left[\mathrm{g}_p[jS] - \hat{\mathrm{g}}'_p[jS]\right]\left[\mathrm{g}_p[jS] - \hat{\mathrm{g}}'_p[jS]\right]^*$ where $\hat{\mathrm{g}}'_p[jS]$ is the first entry of $\hat{\boldsymbol{x}}[j|j-1]$. For the optimal FIR filter the error is $\left[\mathrm{g}_p[jS] - \hat{\mathrm{g}}''_p[jS]\right]\left[\mathrm{g}_p[jS] - \hat{\mathrm{g}}''_p[jS]\right]^*$ where $\hat{\mathrm{g}}''_p[jS]$ is calculated using the optimal FIR filter to estimate $\mathrm{g}_p[jS]$ with the inputs $\hat{\mathrm{g}}_p[mS]$ for $m = \{j-1, j-2, ..., j-M+1\}$. Figure 2 shows the new Kalman filter when applied to the estimation of when $f_m = 0.001$. The error of the new Kalman filter is labelled as the 'Overdetermined Model' Kalman filter on the results graph. The errors for a Kalman filter where the ill-conditioning of the linear equation to calculate the AR weight coefficients is reduced by adding λ to the diagonal entries of the covariance matrix $\boldsymbol{C}$ as suggested in [4] are plotted in the results, labelled as 'Reduced Condition'. The errors results for a Kalman filter with an unadjusted AR model are also plotted. It can be seen that

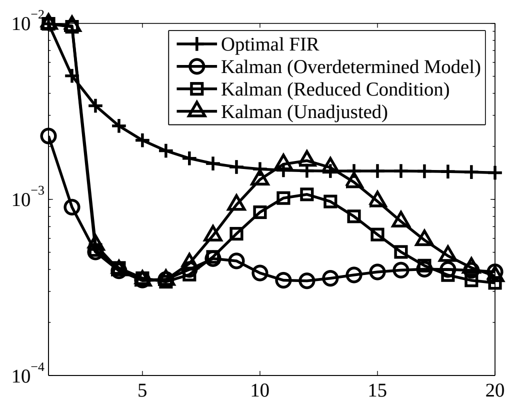

Fig. 2. Kalman Filter Performance ($f_m = 0.001$)

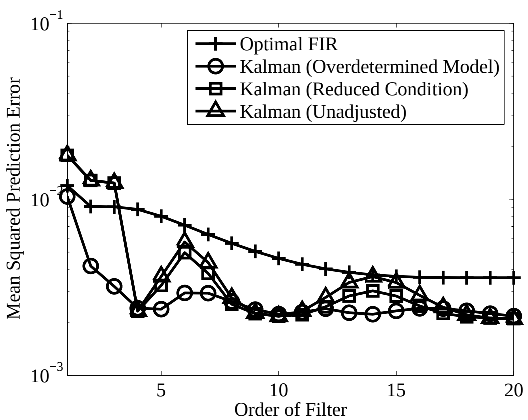

Fig. 3. Kalman Filter Performance ($f_m = 0.01$)

the new Kalman filter provides the best performance over all of the shown filters. An FIR filter of order greater than 200 is required to match the performance of the Kalman filter of order 5. For Kalman filters using AR models that are not calculated from the over-determined system, the mismatch between the AR models and the true channel process causes the error performance with increasing filter order to not monotonically decrease. The new filter gives the best performance for order 5 but Kalman filters of lower order also gave excellent accuracy improvements over the FIR filter. Figure 3 shows the results for a faster fading process with $f_m = 0.01$. The new Kalman filter again has the best performance with greater stability of the average error than the other Kalman filters as the order of the filter increases.

5. CONCLUSIONS

This paper presents a method for creating low-order Kalman filters to accurately track the Rayleigh fading radio channel. This method is based on the calculation of low order AR models with statistics closely matching those of the Rayleigh fading process. Simulation results are presented which show that the new Kalman filters can accurately track the radio channel for fading processes of different bandwidths. It is also shown that the new Kalman filter gives better performance than previously presented Kalman filter algorithms and gives performance as good as optimal high-order FIR filters with much lower computational cost. For future work, the application of these filters to joint channel estimation and symbol detection schemes will be investigated.

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6. REFERENCES

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